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BLACK HOLES AS SOLITONS

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BLACK HOLES AS SOLITONS *

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ABSTRACT

We remark that exact classical Schwarzschild-like solutions to Einstein's (and possibly f gravity) equations provide examples of realistic solitons.

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1. Under the broadest definition, any non-trivial solution to a system of classical non-linear equations, which is confined to a finite region of space and which carries a finite energy, may be considered a soliton. The problem is to discover to what extent such classical objects can approximate to the quantum systems encountered in particle physics. Are they stable? What conserved quantities can be associated with them? How do they interact with "ordinary" particles described by quantized fields?

It is the purpose of this note to point out that a whole class of solitons is apparently available among the known exact classical Schwarzschild-like solutions to Einstein's equations. These solutions are stationary, possess finite energy and can be localized. The stability of some of these solutions depends on rather general considerations but, in any case, the association of a finite and conserved energy with asymptotically flat (i.e. localized) solutions follows from the existence of the so-called energy-momentum pseudo-tensor.

2. For solutions to Einstein's equations, the conservation of energy and momentum results from appropriate symmetries in the equations of motion. This is unlike the conservation of magnetic charge in 't Hooft's theory,[1] where electromagnetism is part of a spontaneously broken non-abelian gauge system. In such theories the conservation is not dependent on a symmetry of the Lagrangian - and, moreover, magnetic charge is confined to sites of topological significance. There has been some controversy as to whether such topological significance is the hallmark of a soliton (see, for example, T.D. Lee [2], for the contrary viewpoint).

It is worth remarking in this context that, although classically stable, the gravitational solitons may be able to decay through vacuum effects. Thus, Hawking [3] has shown that the Schwarzschild solution radiates like a black body whose temperature is inversely proportional to its mass. Conceivably, vacuum effects could be relevant for non-gravitational solitons; for example, stability may be lost for solitons carrying topological conserved numbers ('t Hooft's monopole) through a phase transition induced by a high temperature or strong external electromagnetic environment.

3. Gravitational forces should not be significant in particle physics at foreseeable energies.*) However, the mathematically analogous f gravity [4]

*) But note that masses up to the Planck value $\sim 10^{19}$ GeV are already being contemplated for unified models of weak and electromagnetic interactions.

associated with strongly interacting 2^+ mesons could be relevant. Indeed, it was suggested some years ago that f gravity black holes - now to be called solitons - might represent hadrons.[4),5)] For such black holes the "Schwarzschild radius" of an f gravity soliton would be comparable to its Compton wavelength. An interesting possibility is that the well-known property of the black hole, whereby nothing can escape from it, could have implications for the confinement problem. The colour selectivity for saturation of strong forces (i.e. the colour singlet character of ordinary matter) could result from an $SL(6,C)'$ structure for f gravitons [6)], where the relevant internal symmetry is $SU(3)'$ of colour.

At present there are no exact known solutions to the two-tensor f-g generally covariant equations and no solutions known even in the flat space approximation $g_{\mu\nu} = \eta_{\mu\nu}$. However, if the mass of the f meson is discarded then the equations for $f_{\mu\nu}$ become identical in this approximation with Einstein's equations for $g_{\mu\nu}$. We conjecture that it may be possible to restore the effects of the mass term by means of the replacement $1/r \to e^{-mr}/r$ in the Schwarzschild solution - at least in some favourable co-ordinate system.

4. The most interesting solution of the pure Einstein-Maxwell equations is the charged Kerr-Newman rotating system [7)] described by the gravitational and electromagnetic fields,

$$g_{\mu\nu}(x)\, dx^\mu\, dx^\nu = -(\Delta/\rho^2)\,(dt - a\sin^2\theta\, d\phi)^2 + (\sin^2\theta/\rho^2)\left((r^2 + a^2)\, d\phi - a\, dt\right)^2 + (\rho^2/\Delta)\, dr^2 + \rho^2\, d\theta^2 \;,$$

$$F_{\mu\nu}(x)\, dx^\mu \wedge dx^\nu = Q\,\rho^{-4}\,(r^2 - a^2\cos^2\theta)\, dr \wedge (dt - a\sin^2\theta\, d\phi) + 2\,Q\,\rho^{-4}\, ar\cos\theta\sin\theta\, d\theta \wedge \left((r^2 + a^2)\, d\phi - a\, dt\right) \;,$$

where $\Delta = r^2 - 2Mr + a^2 + Q^2$ and $\rho^2 = r^2 + a^2\cos^2\theta$. (These are the Boyer-Lindquist [8)] or generalized Schwarzschild co-ordinates.) The energy and angular momentum carried by this soliton can be deduced from the asymptotic form of the metric fields. Thus, using the definition of the energy-momentum pseudotensor given by Landau and Lifshitz [9)],

$$\theta^{\nu\kappa}_{total} = \frac{\partial}{\partial x^\lambda} h^{\nu\kappa\lambda} = \frac{1}{16\pi}\frac{\partial^2}{\partial x^\lambda \partial x^\mu}\left[\sqrt{-g}\,(g^{\nu\kappa}g^{\lambda\mu} - g^{\nu\lambda}g^{\kappa\mu})\right] ,$$

the space integrals for P^μ and $M^{\mu\nu}$ can be reduced to two-dimensional surface integrals over a sphere of infinite radius. One finds a total energy equal to M and angular momentum equal to Ma. The linear momentum vanishes. In a similar fashion one deduces from the asymptotic form of $F_{\mu\nu}$ that the total charge is Q and magnetic moment Qa. (Notice that the gyromagnetic ratio takes the Dirac value, Q/M.)

In addition to these, there are other known classical solutions; for exemple Curzon's solution [10)] of Einstein's equations:

$$ds^2 = e^{-2m/\rho}\,(dt)^2 - e^{2m/\rho}\left\{e^{-m^2r^2/2\rho^4}\,(dr^2 + dz^2) + d\phi^2\right\} ,\; \rho^2 = r^2 + z^2 ,$$

and Singh's solution [11)] of a scalar tensor theory:

$$ds^2 = \left(1 - \frac{2m}{r}\right)^{\frac{b}{\sqrt{b^2 - \frac{3}{4}}}} dt^2 - \left(1 - \frac{2m}{r}\right)^{-\frac{b}{\sqrt{b^2 - \frac{3}{4}}}} \left[dr^2 + \left(1 - \frac{2m}{r}\right) r^2\,(d\theta^2 + \sin\theta^2\, d\phi^2)\right] ,$$

$$\phi = \left(1 - \frac{2m}{r}\right)^{\frac{1}{2\sqrt{b^2 - \frac{3}{4}}}} ,$$

this latter is related to Schwarzschild's solution. With these four-dimensional soliton solutions available, we feel that the recent work on particle production near black holes affords a particularly favourable testing ground for ideas in soliton physics.

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