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IN A SUPERSYMMETRIC MODEL

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A MASSLESS NON-GOLDSTONE NEUTRINO IN A SUPERSYMMETRIC MODEL *

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ABSTRACT

A supersymmetric model (with supersymmetry spontaneously broken) is exhibited which contains two massless neutrinos and two conserved lepton numbers. Only one of the two neutrinos is a Goldstone fermion with amplitudes which exhibit low-energy theorems.

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I. INTRODUCTION

The massless Goldstone spinor which accompanies spontaneous breakdown of supersymmetry has been shown to satisfy a number of low-energy theorems[1) -] similar to massless pions associated with chiral symmetry breakdown. It has been argued that such low-energy theorems do not appear to be satisfied for β decay, so that the electron-type neutrino is unlikely to be a Goldstone spinor, and thus not the fermion associated with spontaneous breakdown of supersymmetry[2)].

Empirically, no one knows how many massless or near massless neutrinos there are in nature. At present there seem to be two: the electron type, ν_e, and the muon type, ν_μ. Perhaps ν_μ is a Goldstone particle. There does not yet appear to be any good experimental reason for rejecting this possibility. To state it in more general terms, the real world may contain several distinct types of neutrino and, if the underlying dynamics is supersymmetric, just one of these should be a Goldstone particle with the attendant low-energy peculiarities.*) At the present time we can only be sure that this special particle is not to be identified with the electron-type neutrino.

In the following we shall exhibit a model **) in which there are two massless neutrinos and two conserved lepton numbers. One of the two neutrinos is a Goldstone particle, subject to low-energy theorems, while the other is not. This point is illustrated by the calculation of two decay processes, one involving the second neutrino but not the Goldstone neutrino and one involving both. We wish to emphasise that the model is not a realistic one - the use of familiar names like electron and muon is intended only as a suggestive book-keeping measure. The main purpose of this paper is to establish the principle that ordinary massless neutrinos can coexist in a supersymmetric context with the Goldstone spinor.

Before we exhibit the model, it is useful to distinguish three general types according to the relative balance between right- and left-handed spinor fields.

a) Perfect balance

When the numbers of left- and right-handed fields are equal, then before symmetry breakdown it is feasible for every spinor particle to be massive. The spontaneous breakdown of supersymmetry, however, will force the appearance of one massless lepton. If in addition there is a conserved lepton-number then a second massless lepton of opposite helicity is necessarily present. (Our model is of this type.)

*) This remark applies to conventional supersymmetries with one conserved supercurrent. If more complicated supersymmetries with more than one supercurrent are invented the situation could change.

**) This is an extension of a model due to Fayet[2)].

b) Surplus of one

If there is one extra left-handed spinor field, then the conservation of lepton-number would imply at least one massless lepton even when supersymmetry is preserved. If supersymmetry breaks, then either this lepton becomes itself the Goldstone spinor [2] or another massless pair must appear.

c) Surplus of two

With lepton-number conservation there will be (at least) two massless particles, one of which may be the Goldstone spinor and therefore subject to low-energy theorems. (It is not always possible to distinguish models of this type from those of type a): a re-assignment of lepton-number can sometimes point up their equivalence.)

If lepton-number is spontaneously broken, for cases a) and c), there must exist a Majorana-Goldstone spinor plus a second Majorana spinor which may or may not be massless.

II. THE MODEL *)

The Lagrangian admits the usual local symmetry of weak and electromagnetic interactions, $SU(2)_I \times U(1)_Y$. In addition to the gauge fields with (I,Y) content,

$$\Psi_0 \sim (0,0) \quad , \quad \underset{\sim}{\Psi}_1 \sim (1,0) \quad ,$$

we introduce six chiral matter fields:

$$\Phi_{0-} \sim (0,0) \quad , \quad \Phi_{1+} \sim (\tfrac{1}{2},-1) \quad , \quad \Phi_{2+} \sim (\tfrac{1}{2},1) \quad , \quad \Phi_{3-} \sim (1,0) \quad ,$$
$$\Phi_{4+} \sim (\tfrac{1}{2},-1) \quad , \quad \Phi_{5+} \sim (\tfrac{1}{2},1) \quad .$$

The Lagrangian is given by the expression

$$\mathcal{L} = \frac{1}{8} (\bar{D}D)^2 \Big[|\Phi_{0-}|^2 + \Phi^\dagger_{1+} \exp\left(2g_1\Psi_1 - 2g_0\Psi_0\right) \Phi_{1+} + $$
$$+ \Phi^\dagger_{2+} \exp\left(2g_1\Psi_1 + 2g_0\Psi_0\right) \Phi_{2+} + \frac{1}{2}\,\mathrm{Tr}\left\{\Phi^\dagger_{3-} \exp\left(2g_1\Psi_1\right) \Phi_{3-} \; \left(-2g_1\Psi_1\right)\right\} +$$
$$+ \Phi^\dagger_{4+} \exp\left(2g_1\Psi_1 - 2g_0\Psi_0\right) \Phi_{4+} + \Phi^\dagger_{5+} \exp\left(2g_1\Psi_1 + 2g_0\Psi_0\right) \Phi_{5+} \Big]$$

*) To make this paper self-contained, some of the results f Ref.[3] are recapitulated.

$$- \frac{1}{2} \bar{D}D \Big[\frac{1}{4} \bar{\underset{\sim}{\Psi}}_{1-} \underset{\sim}{\Psi}_{1+} + \frac{1}{4} \bar{\Psi}_{0-} \Psi_{0+} + \frac{\xi}{2} \bar{D}D\, \Psi_0 + \Phi^*_{0-}\left(s + ih\, \Phi^T_{1+}\, \tau_2\, \Phi_{2+}\right) +$$
$$+ \underset{\sim}{\Phi}^*_{3-} \left(ik_1\, \Phi^T_{1+}\, \tau_2 \underset{\sim}{\tau}\, \Phi_{5+} + ik_2\, \Phi^T_{2+}\, \tau_2 \underset{\sim}{\tau}\, \Phi_{4+}\right) + \text{h.c.} \Big] \quad , \qquad (2.1)$$

where $\Psi_1 = \underset{\sim}{\Psi}_1 \cdot \underset{\sim}{\tau}$ and $\Phi_{3-} = \underset{\sim}{\Phi}_{3-} \cdot \underset{\sim}{\tau}$. The "Yang-Mills" field strengths $\underset{\sim}{\Psi}_{1\pm}$, $\Psi_{0\pm}$ are defined in Ref.4. Together with the four gauge spinors (which are all left-handed, there are altogether eight left-handed and eight right-handed spinors in the theory. As will be seen later, after spontaneous symmetry breakdown, they will represent four massive charged and three massive neutral leptons plus two massless (one left-handed and one right-handed) neutrinos.

In order to exhibit the detailed structure of this Lagrangian, it is convenient to use the Wess-Zumino gauge where the Lagrangian takes a polynomial form. We can write it as a sum of distinct kinetic terms $\mathcal{L}_k$, which include the gauge interactions, and a self-interaction $\mathcal{L}_m$, among the matter fields,

$$\mathcal{L} = \mathcal{L}_k(\Phi_{0-}) + \mathcal{L}_k(\Phi_{1+}) + \mathcal{L}_k(\Phi_{2+}) + \mathcal{L}_k(\Phi_{3-}) + \mathcal{L}_k(\Phi_{4+}) +$$
$$+ \mathcal{L}_k(\Phi_{5+}) + \mathcal{L}_k(\Psi_1) + \mathcal{L}_k(\Psi_0) + \mathcal{L}_m \quad .$$

The singlet Φ_{0-} has no gauge interactions so that

$$\mathcal{L}_k(\Phi_{0-}) = |\partial A_{0-}|^2 + \bar{\psi}_{0-}\, i\not{\partial}\, \psi_{0-} + |F_{0-}|^2 \quad .$$

The matter doublets $\Phi_{1,2,4,5}$ have similar kinetic terms, typically,

$$\mathcal{L}_k(\Phi_{1+}) = \nabla_\mu A^\dagger_{1+} \nabla_\mu A_{1+} + \bar{\psi}_{1+}\, i\not{\nabla}\, \psi_{1+} + F^\dagger_{1+} F_{1+}$$
$$+ \left(i\sqrt{2}\, A^\dagger_{1+} \left(g_1 \bar{\underset{\sim}{\lambda}}_{1-} \cdot \underset{\sim}{\tau} - g_0 \bar{\lambda}_{0-}\right) \psi_{1+} + \text{h.c.}\right) + A^\dagger_{1+}\left(g_1 \underset{\sim}{D}_1 \cdot \underset{\sim}{\tau} - g_0 D_0\right) A_{1+} \quad ,$$

where the covariant derivatives are defined by

$$\nabla_\mu A_{1+} = \left(\partial_\mu - ig_1\, \underset{\sim}{W}_{1\mu} \cdot \underset{\sim}{\tau} + ig_0 W_{0\mu}\right) A_{1+} \quad ,$$

and similarly for ψ_1.

The gauge field kinetic terms are of the usual Yang-Mills form,

$$\mathcal{L}_k(\Psi_0) \;=\; -\tfrac{1}{4}\, W_{0\mu\nu}^2 + \bar{\lambda}_{0-}\; i\not{\partial}\; \lambda_{0-} + \tfrac{1}{2}\, D_0^2 + \xi D_0 \;,$$

$$\mathcal{L}_k(\Psi_1) \;=\; -\tfrac{1}{4}\, \underset{\sim}{W}_{1\mu\nu}^2 + \bar{\underset{\sim}{\lambda}}_{1-}\; i\not{\nabla}\; \underset{\sim}{\lambda}_{1-} + \tfrac{1}{2}\, \underset{\sim}{D}_1^2 \;,$$

where

$$W_{0\mu\nu} = \partial_\mu W_{0\nu} - \partial_\nu W_{0\mu} \;,$$

$$\underset{\sim}{W}_{1\mu\nu} = \partial_\mu \underset{\sim}{W}_{1\nu} - \partial_\nu \underset{\sim}{W}_{1\mu} + g_1\, \underset{\sim}{W}_{1\mu} \times \underset{\sim}{W}_{1\nu} \;,$$

$$\nabla_\mu \underset{\sim}{\lambda}_{1-} = \partial_\mu \underset{\sim}{\lambda}_{1-} + g_1\, \underset{\sim}{W}_{1\mu} \times \underset{\sim}{\lambda}_{1-} \;.$$

Finally, the matter field interaction term is given by

$$\begin{aligned}
\mathcal{L}_m = {} & F_{0-}^{*}\left(s + ihA_{1+}^{T}\tau_2 A_{2+}\right) \;+\; \underset{\sim}{F}_{3-}^{*}\left(ik_1 A_{1+}^{T}\tau_2 \underset{\sim}{\tau} A_{5+} + ik_2 A_{2+}^{T}\tau_2\underset{\sim}{\tau} A_{4+}\right) \\
& - \bar{\psi}_{0-}\left(ihA_{1+}^{T}\tau_2\psi_{2+} \;-\; ihA_{2+}^{T}\tau_2\psi_{1+}\right) \\
& - \bar{\underset{\sim}{\psi}}_{3-}\left(ik_1 A_{1+}^{T}\tau_2\underset{\sim}{\tau}\psi_{5+} + ik_1 A_{5+}^{T}\tau_2\underset{\sim}{\tau}\psi_{1+} + ik_2 A_{2+}^{T}\tau_2\underset{\sim}{\tau}\psi_{4+} + ik_2 A_{4+}^{T}\tau_2\underset{\sim}{\tau}\psi_{2+}\right) \\
& + A_{0-}^{*}\left(ihA_{1+}^{T}\tau_2 F_{2+} \;-\; ihA_{2+}^{T}\tau_2 F_{1+} \;+\; ih\psi_{1+}^{T}\tau_2 C^{-1}\psi_{2+}\right) \\
& + \underset{\sim}{A}_{3-}^{*}\Big(ik_1 A_{1+}^{T}\tau_2\underset{\sim}{\tau} F_{5+} \;+\; ik_1 A_{5+}^{T}\tau_2\underset{\sim}{\tau} F_{1+} \;+\; ik_1\psi_{1+}^{T}\tau_2\underset{\sim}{\tau} C^{-1}\psi_{5+} \;+ \\
& \qquad + ik_2 A_{2+}^{T}\tau_2\underset{\sim}{\tau} F_{4+} \;+\; ik_2 A_{4+}^{T}\tau_2\underset{\sim}{\tau} F_{2+} \;+\; ik_2\psi_{2+}^{T}\tau_2\underset{\sim}{\tau} C^{-1}\psi_{4+}\Big) \\
& + \text{h.c.}
\end{aligned}$$

The details of notation are explained in Ref.4. The theory has been constructed in accordance with the prescriptions given in that paper so as to conserve an overall lepton-number. However, in addition to this number, so far as the Lagrangian is concerned, the superfields Φ_{3-} , Φ_{4+} and Φ_{5+} admit of an extra global U(1) gauge transformation which we associate with the conservation of an "electron-number".

In the tree approximation, spontaneous symmetry breakdown occurs, with the spectrum of masses being given as follows:

1) Vectors

$$W_\mu^{\pm} = \frac{1}{\sqrt{2}}\,(W_\mu^1 \mp iW_\mu^2) \;, \qquad m(W^{\pm}) = \sqrt{2g_1^2(a_1^2 + a_2^2)} \;,$$

$$Z_\mu^0 = W_\mu^3\cos\theta - W_\mu^0\sin\theta \;, \qquad m(Z^0) = \sqrt{2(g_1^2 + g_0^2)\;(a_1^2 + a_2^2)} \;,$$

$$A_\mu = W_\mu^3\sin\theta + W_\mu^0\cos\theta \;, \qquad m(A) = 0 \;.$$

2) Spinors

$$\mu^- = \psi_{1+}^- + i\lambda_-^- \;, \qquad m(\mu^-) = 2g_1 a_1 \;,$$

$$E^- = \psi_{4+}^- - \psi_{3-}^- \;, \qquad m(E^-) = \sqrt{2}\,k_2 a_2 \;,$$

$$e^+ = \psi_{5+}^+ + \psi_{3-}^+ \;, \qquad m(e^+) = \sqrt{2}\,k_1 a_1 \;,$$

$$M^+ = \psi_{2+}^+ + i\lambda_-^+ \;, \qquad m(M^+) = 2g_1 a_2 \;.$$

$$\mu^0 = \psi_{1+}^0\cos\delta - \psi_{2+}^0\sin\delta - i\lambda_-^3\cos\theta + i\lambda_-^0\sin\theta \;, \qquad m(\mu^0) = \sqrt{2(g_1^2 + g_0^2)\;(a_1^2 + a_2^2)} \;,$$

$$M^0 = \psi_{1+}^0\sin\delta + \psi_{2+}^0\cos\delta + \psi_{0-} \;, \qquad m(M^0) = h\sqrt{a_1^2 + a_2^2} \;,$$

$$E^0 = \psi_{4+}^0\sin\delta' + \psi_{5+}^0\cos\delta' + \psi_{3-}^0 \;, \qquad m(E^0) = \sqrt{k_1^2 a_1^2 + k_2^2 a_2^2} \;,$$

$$\nu = \psi_{4+}^0\cos\delta' - \psi_{5+}^0\sin\delta' + \lambda_-^3\sin\theta + \lambda_-^0\cos\theta \;, \qquad m(\nu) = 0 \;.$$

3) Scalars

$$\phi_1^+ = A_{2+}^+\cos\delta + A_{1+}^{-*}\sin\delta \;, \qquad m_{\phi_1^+} = \sqrt{2g_1^2(a_1^2 + a_2^2)}$$

$$\phi_2^+ = -A_{2+}^+\sin\delta + A_{1+}^{-*}\cos\delta \;, \qquad m_{\phi_2^+} = 0$$

$$\phi_3^+ = A_{3-}^+ \;, \qquad m_{\phi_3^+} = \sqrt{2k_1^2 a_1^2 - 2g_1^2(a_1^2 - a_2^2)}$$

$$\phi_3^- = A_{3-}^- \;, \qquad m_{\phi_3^-} = \sqrt{2k_2^2 a_2^2 + 2g_1^2(a_1^2 - a_2^2)}$$

$$\phi_4^- = A_{4+}^- \;, \qquad m_{\phi_4^-} = \sqrt{2k_2^2 a_2^2 - 2g_1^2(a_1^2 - a_2^2)}$$

$$\phi_5^+ = A_{5+}^+ \;, \qquad m_{\phi_5^+} = \sqrt{2k_1^2 a_1^2 + 2g_1^2(a_1^2 - a_2^2)}$$

$$\phi_1^0 = A_{1+}^0\sin\delta + A_{2+}^0\cos\delta \;, \qquad m_{\phi_1^0} = h\sqrt{a_1^2 + a_2^2}$$

$$\phi_2^0 = \sqrt{2}\,\mathrm{Re}\left\{A^0_{1+}\cos\delta - A^0_{2+}\sin\delta\right\} \quad , \quad m_{\phi_2^0} = \sqrt{2(g_1^2 + g_0^2)(a_1^2 + a_2^2)} \;.$$

$$\phi_2^{0'} = \sqrt{2}\,\mathrm{Im}\left\{A^0_{1+}\cos\delta - A^0_{2+}\sin\delta\right\} \quad , \quad m_{\phi_2^{0'}} = 0 \;,$$

$$\phi_3^0 = A^0_{3-} \quad , \quad m_{\phi_3^0} = \sqrt{k_1^2 a_1^2 + k_2^2 a_2^2} \;.$$

$$\phi_4^0 = A^0_{4+}\sin\delta' + A^0_{5+}\cos\delta' \quad , \quad m_{\phi_4^0} = \sqrt{k_1^2 a_1^2 + k_2^2 a_2^2} \;,$$

$$\phi_5^0 = A^0_{4+}\cos\delta' - A^0_{5+}\sin\delta' \quad , \quad m_{\phi_5^0} = 0 \;.$$

Here the numbers a_1 and a_2 which represent vacuum expectation values of A^0_{1+} and A^0_{2+} may be chosen real and are fixed in terms of the parameters in the Lagrangian by the equations

$$g_0\xi = (g_0^2 + g_1^2)\,(a_1^2 - a_1^2) \;,$$

$$0 = s + h\,a_1 a_2 \;.$$

The angles θ, δ and δ' are then given by

$$\tan\theta = \frac{g_0}{g_1} \quad , \quad \tan\delta = \frac{a_2}{a_1} \quad , \quad \tan\delta' = \frac{k_2 a_2}{k_1 a_1} \;.$$

Notice the mass relations

$$m(\phi_5^+)^2 = 2k_1^2 a_1^2 + 2g_1^2(a_1^2 - a_2^2) = m(e^+)^2 + \frac{1}{2}m(\mu^-)^2 - \frac{1}{2}m(M^+)^2 \;,$$

$$m(W^+)^2 = 2g_1^2\,(a_1^2 + a_2^2) = \frac{1}{2}\left[m(\mu^-)^2 + m(M^+)^2\right] \;.$$

Quite clearly the model is unrealistic so far as masses are concerned, since electron and muon masses ($\sqrt{2}\,k_1 a_1$ and $2g_1 a_1$) cannot both be small relative to M^+ and W^+ masses, without ϕ_5^+ becoming tachyonic. Thus, in this model, e^+, μ^- , ϕ_5^+ , M^+ and $W^\pm$ must have similar masses *). Notwithstanding this restriction, we have retained the suggestive nomenclature e^+ , μ^- , E^- , M^+ for leptons in order that one may fix pictorially on familiar decay modes.

*) Of course if the parameters are chosen so as to force $a_2 \gg a_1$ then we are dealing with an extremum which is not a minimum and the system will choose a different phase in which other fields acquire a non-zero value. We have not attempted this more difficult analysis.

Note that at this stage the massless neutrinos, left and right, are given by

$$\nu_L = \lambda^3_- \sin\theta + \lambda^0_- \cos\theta \;,$$

$$\nu_R = \psi^0_{4+}\cos\delta' - \psi^0_{5+}\sin\delta' \;.$$

ν_L is purely "gauge"[2], while ν_R is purely "matter". The minimization of the effective potential, up to the tree approximation as above, exhibits degeneracy, signalled by the appearance of a pseudo-Goldstone boson ϕ_5^0 . Notice that ϕ_5^0 is a linear combination of fields A_{4+} and A_{5+} and that the same linear combination of spinors ψ_{4+} and ψ_{5+} appears in the composition of the second (massless) neutrino ν_R . This phenomenon is a relic of supersymmetry.

Now, to remove this degeneracy of the tree approximtion, one must invoke radiative corrections and compute the effective potential at least to one-loop level. This has been done [3] and indeed the degeneracy is lifted with the pseudo-Goldstone boson acquiring a real (non-tachyonic) mass for a certain range of the coupling parameters. However, to this approximation, the global symmetry U(1) typifying conservation of electron-number remains unbroken. Both neutrinos ν_L and ν_R (with the compositions given above) remain massless.

Further, one can show that (since $\langle D^3\rangle \neq 0$, $\langle D^0\rangle \neq 0$) ν_L is the genuine Goldstone spinor, subject to low-energy theorems, while the second neutrino, ν_R , is an ordinary fermion dragged along to a state of masslessness simply because the numbers of left-handed and right-handed spinors in the theory balance. Note that in the charged weak current e^+_R is coupled to ν_R while μ^-_L is coupled to ν_L . In other words, ν_L represents the muonic neutrino while ν_R is the electronic antineutrino.

In the next section we evaluate a decay amplitude, in which the non-Goldstone neutrino ν_R is emitted and verify the absence of a low-energy theorem.

III. TWO DECAY PROCESSES

We consider the decay of μ^- into $\overline{e^+}$ with the emission of a pair of neutral fermions

$$\mu^- \to \overline{e^+} + \nu_R + M^0 \;.$$

We shall show in this section that this process displays no low-energy theorem.

The relevant terms in the interaction Lagrangian we shall need are:

$$\mathcal{L}_{int} \approx -i2\sqrt{2}\, g_1 \sin\theta \sin\delta\, \phi_1^+(\bar\nu_L \mu_R^-) - 2g_1(\cos\delta)^2\, \phi_1^+(\bar M_R^0 \mu_L^-)$$

$$-i2\, g_1 \sin\theta\, W_\rho^+(\bar\nu_L\gamma_\rho\mu_L^-) - h\cos\delta\,\phi_1^+(\bar M_L^0\mu_R^-)$$

$$-\sqrt{2}\, g_1 \sin\delta'\, W_\rho^-(\bar\nu_R\gamma_\rho e_R^+) + \sqrt{2}\, g_1 \sin\delta\, W_\rho^+(\bar M_R^0\gamma_\rho\mu_R^-)$$

$$-\sqrt{2}\, k_2\cos\delta'\cos\delta\, \phi_1^{+*}(\bar\nu_R e_L^+) - \sqrt{2}\, k_1 \sin\delta\, \phi_5^{+*}(\bar M_R^0 e_L^+)$$

$$+ i2\sqrt{2}\, g_1 \sin\theta\, \phi_5^{+*}(\bar\nu_L e_R^+) + 2g_1\sin\delta'\,\phi_5^+(\bar\nu_R\mu_L^-)$$

$$+(k_2\cos\delta\cos\delta' - k_1\sin\delta\sin\delta')\,\phi_3^0(\bar\nu_R\, C\, \bar M_R^{0T})$$

$$+(2\sqrt{2}\, g_1 - k_1)\,\phi_3^{0*}\,\mu_L^{-T}\, C^{-1} e_L^+ \qquad (3.1)$$

(in the unitary gauge with $\phi_2^+ = 0$). The matrix element is obtained by contracting the boson lines in the second-order expression $i^2T(\mathcal{L}_{int}(1)\,\mathcal{L}_{int}(2))$. We find:

$$T_1 = :-\sqrt{2}\, k_2 \cos\delta' \cos\delta\, \bar\nu_R e_L^+ \langle T\phi_1^{+*}\phi_1^+\rangle \left[h\cos\delta\, \bar M_L^0\mu_R^- + 2g_1(\cos\delta)^2\, \bar M_R^0\mu_L^-\right]$$

$$+ 2g_1^2 \sin\delta' \sin\delta\, \bar\nu_R\gamma_\rho e_R^+ \langle TW_\rho^- W_\lambda^+\rangle\, \bar M_R^0\gamma_\lambda\mu_R^-$$

$$+ 2\sqrt{2}\, k_1 g_1 \sin\delta' \sin\delta\, \bar\nu_R\mu_L^- \langle T\phi_5^+\phi_5^{+*}\rangle\, \bar M_R^0 e_L^+$$

$$+ (k_1 - 2\sqrt{2} g_1)(k_2\cos\delta\cos\delta' - k_1\sin\delta\sin\delta')\,\bar\nu_R C \bar M_R^{0T}\langle T\phi_3^0\phi_3^{0*}\rangle\, \mu_L^{-T} C^{-1} e_L^+ : \qquad (3.2)$$

Thus

$$\langle e^+\nu_R M^0|T_1|\mu^-\rangle = \bar u_R(\nu)\, v_L(e^+)\, \bar u_L(M^0)\, u_R(\mu^-) \left[\frac{-i\sqrt{2}\, k_2 h \cos\delta' \cos(2\delta)}{(p_\mu - p_M)^2 - m(W)^2}\right]$$

$$+ \bar u_R(\nu)\, v_L(e^+)\, \bar u_R(M^0)\, u_L(\mu^-) \left[\frac{-i\sqrt{2}\, k_2 g_1 \cos\delta' \cos\delta}{(p_\mu - p_M)^2 - m(W)^2} - \frac{i\sqrt{2}\, k_1 g_1 \sin\delta' \sin\delta}{(p_\mu - p_\nu)^2 - m(\phi_5^+)^2} + \frac{1}{2}\,\frac{(k_1 - 2\sqrt{2}\, g_1)(k_2\cos\delta\cos\delta' - k_1\sin\delta\sin\delta')}{(p_\mu - p_e)^2 - m(\phi_3^0)^2}\right]$$

$$- \frac{1}{4}\, \bar u_R(\nu)\, \sigma_{\lambda\rho}\, v_L(e^+)\, \bar u_R(M^0)\, \sigma_{\lambda\rho}\, u_L(\mu^-) \left[\frac{i\sqrt{2}\, k_1 g_1 \sin\delta' \sin\delta}{(p_\mu - p_\nu)^2 - m(\phi_5^+)^2} + \frac{1}{2}\,\frac{(k_1 - 2\sqrt{2}\, g_1)(k_2\cos\delta\cos\delta' - k_1\sin\delta\sin\delta')}{(p_\mu + p_e)^2 - m(\phi_3^0)^2}\right]$$

$$+ \bar u_R(\nu)\, \gamma_\rho\, v_R(e^+)\, \bar u_R(M^0)\, \gamma_\rho\, u_R(\mu^-) \left[\frac{-i\, 2 g_1^2 \sin\delta' \sin\delta}{(p_\mu - p_M)^2 - m(W)^2}\right] . \qquad (3.3)$$

Since ν_R is not the Goldstone neutrino, there is no reason for (3.3) to vanish for $p_{\nu_R} \to 0$, and indeed it does not.

It is instructive to consider the process

$$\mu^- \to e^+ + \nu_R + \nu_L$$

in the limit $p_{\nu_L} \to 0$ and to verify that the amplitude does vanish in accordance with Goldstone's theorem. The lowest-order matrix element is given by the effective terms:

$$-T_2 = :i\, 4k_2\, g_1 \sin\theta \sin\delta \cos\delta \cos\delta'\, \bar\nu_L e_L^+ \langle T\phi_1^{*+}\phi_1^+\rangle\, \bar\nu_L\mu_R^-$$

$$+ i\, 2\sqrt{2}\, g_1^2 \sin\theta \sin\delta'\, (\bar\nu_R\gamma_\rho e_R^+) \langle T W_\rho^- W_\lambda^+\rangle\, \bar\nu_L\gamma_\lambda\mu_L^-$$

$$+ i\, 4\sqrt{2}\, g_1^2 \sin\theta \sin\delta'\, \bar\nu_R\mu_L^- \langle T\phi_5^+\phi_5^{*+}\rangle\, \bar\nu_L e_R^+ :$$

and is proportional to $\frac{1}{(p_\mu - p_{\nu_R})^2 - m^2(\phi_5)} + \frac{1}{(p_\mu - p_{\nu_L})^2 - m^2(W)}$. In the limit $p_{\nu_L} \to 0$, this duly vanishes on account of the mass relation $m(e)^2 - m(\phi_5)^2 + m(\mu)^2 - m(W)^2 = 0$. Notice that the amplitudes vanishes also for $p_{\nu_R} \to 0$. This fortuitous zero is a consequence of the accidental symmetry of T_2 (and the mass relation above) for $\nu_L \leftrightarrow \nu_R$, $e \leftrightarrow \mu$, $p_\mu \leftrightarrow -p_e$ (ϕ_5 and W unchanged). To see that this accidental symmetry will not survive if the next order is computed, note that the Lagrangian (3.1) is not symmetric for this discrete transformation.

To conclude, in the present state of experimental uncertainty about how many massless (or near massless $m \approx m_{\nu_e}$) neutrinos there are in nature, and the consequent theoretical uncertainty with regard to the construction of a realistic supersymmetric model of unified weak and electromagnetic interaction, all that (at best) one may infer from low-energy theorems is that the massless electronic neutrino is not a Goldstone spinor. Spontaneously broken supersymmetric theories, however, appear fully capable of describing non-Goldstone massless neutrinos in addition to Goldstone neutrinos and for these there are no inhibiting low-energy theorems.

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