A MASSLESS NON-GOLDSTONE NEUTRINO
IN A SUPERSYMMETRIC MODEL

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A massless non-Goldstone neutrino in a supersymmetric model

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ABSTRACT

A supersymmetric model (with supersymmetry spontaneously broken) is exhibited which contains two massless neutrinos and two conserved lepton numbers. Only one of the two neutrinos is a Goldstone fermion with amplitudes which exhibit low-energy theorems.

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b) **Surplus of one**

If there is one extra left-handed spinor field, then the conservation of lepton-number would imply at least one massless lepton even when supersymmetry is preserved. If supersymmetry breaks, then either this lepton becomes itself the Goldstone spinor or another massless pair must appear.

c) **Surplus of two**

With lepton-number conservation there will be (at least) two massless particles, one of which may be the Goldstone spinor and therefore subject to low-energy theorems. (It is not always possible to distinguish models of this type from those of type a): a re-assignment of lepton-number can sometimes point up their equivalence.

If lepton-number is spontaneously broken, for cases a) and c), there must exist a Majorana-Goldstone spinor plus a second Majorana spinor which may or may not be massless.

II. **THE MODEL**

The Lagrangian admits the usual local symmetry of weak and electromagnetic interactions, SU($2_L$ $\times$ U(1)$_Y$). In addition to the gauge fields with $(I, Y)$ content,

\[ \psi_0 \sim (0, 0), \quad \psi_1 \sim (1, 0), \]

we introduce six chiral matter fields:

\[ \phi_{0-} \sim (0, 0), \quad \phi_{1+} \sim (1, 1), \quad \phi_{2+} \sim (1, 1), \quad \phi_{3+} \sim (1, 0), \]

\[ \phi_{4+} \sim (1, -1), \quad \phi_{5+} \sim (1, 1). \]

The Lagrangian is given by the expression

\[ \mathcal{L} = \frac{1}{8} (\bar{A}_0^+)^2 \left[ \psi_{0-}^2 + \phi_{1+} \psi_{1-}, \exp \left( 2g \psi_{1-}^2 - 2g \psi_{0-} \right) \phi_{1+}^+ \right] + \]

\[ + \phi_{2+}^+ \exp \left( 2g \psi_{1-}^2 - 2g \psi_{0-} \right) \phi_{2+}^0 \psi_{2+}^0 \psi_{3+}^0 \exp \left( 2g \psi_{1-}^2 + 2g \psi_{0-} \right) \phi_{3+}^0 \]

where \( \psi_0, \phi_{1+}, \phi_{2+}, \phi_{3+}, \phi_{4+}, \phi_{5+} \) are the Goldstone fields and \( \phi_{1+}, \phi_{2+}, \phi_{3+}, \phi_{4+}, \phi_{5+} \) are the massless states. The "Yang-Mills" field strengths \( \psi_{1+}, \psi_{2+}, \psi_{3+} \) are defined in Ref. 1. Together with the four gauge spinors (which are all left-handed), there are altogether eight left-handed and eight right-handed spinors in the theory. As will be seen later, after spontaneous symmetry breakdown, they will represent four massive charged and three massive neutral leptons plus two massless (one left-handed and one right-handed) neutrinos.

In order to exhibit the detailed structure of this Lagrangian, it is convenient to use the Wee-Zumino gauge where the Lagrangian takes a polynomial form. We can write it as a sum of distinct kinetic terms \( \mathcal{L}_k \), which include the gauge interactions, and self-interaction terms \( \mathcal{L}_m \), among the matter fields.

\[ \mathcal{L} = \mathcal{L}_k(\phi_{0-}) + \mathcal{L}_k(\phi_{1+}) + \mathcal{L}_k(\phi_{2+}) + \mathcal{L}_k(\phi_{3+}) + \mathcal{L}_k(\phi_{4+}) + \]

\[ + \mathcal{L}_k(\phi_{5+}) + \mathcal{L}_k(\psi_0) + \mathcal{L}_k(\psi_1) + \mathcal{L}_k(\psi_2). \]

The singlet \( \phi_{0-} \) has no gauge interactions so that

\[ \mathcal{L}_k(\phi_{0-}) = |A_{\phi_0}|^2 + |\psi_{0-}|^2 + |\psi_{1-}|^2. \]

The matter doublets \( \phi_{1+}, \phi_{2+}, \phi_{3+} \) have similar kinetic terms, typically,

\[ \mathcal{L}_k(\phi_{1+}) = \psi_{1+}^0 \psi_{1+}^0 + \psi_{1-}^0 \psi_{1-}^0 + \psi_{2+}^0 \psi_{2+}^0 + \psi_{3+}^0 \psi_{3+}^0. \]

The covariant derivatives are defined by

\[ \mathcal{V}_0 \mathcal{A}_{\psi_0} = \left[ \mathcal{V}_0 + ig \mathcal{A}_0 \right] \mathcal{A}_{\psi_0}, \]

and similarly for \( \mathcal{V}_1 \).

\[ \text{To make this paper self-contained, some of the results of Ref. 3 are recapitulated.} \]
The gauge field kinetic terms are of the usual Yang-Mills form,
\[ \mathcal{L}_k(v_i) = -\frac{1}{4} \epsilon_{\mu
u} \hat{v}_i \cdot \hat{v}_i, \]
where
\[ \hat{v}_i = \frac{1}{2} (\partial_\mu v_i^\mu + \lambda_\mu v_i^\mu - \frac{1}{2} \rho_\mu v_i^\mu + \xi_\mu v_i^\mu). \]

Finally, the matter field interaction term is given by
\[ \mathcal{L}_m = \sum_{\Phi} \left( \lambda_\mu \Delta^\mu_\nu \mathcal{L}_{\Phi} + \mathcal{L}_{\Phi} \right), \]
where
\[ \mathcal{L}_{\Phi} = \sum_{\Phi} \left( \frac{1}{2} \left( \partial_\mu \Phi^\dagger \partial^\mu \Phi + m^2 \Phi^\dagger \Phi \right) - m^2 \Phi^\dagger \Phi \right). \]

In the tree approximation, spontaneous symmetry breakdown occurs, with the spectrum of masses being given as follows:

1. Vectors

The details of notation are explained in Ref.4. The theory has been constructed in accordance with the prescriptions given in that paper so as to conserve an overall lepton-number. However, in addition to this number, so far as the Lagrangian is concerned, the superfields $\Phi_{1-}$ and $\Phi_{2-}$ admit of an extra global $U(1)$ gauge transformation which we associate with the conservation of an "electron-number".

In the tree approximation, spontaneous symmetry breakdown occurs, with the spectrum of masses being given as follows:

\[ m^{(\pm)} = \sqrt{2} \frac{(a_1^2 + a_2^2)}{2}, \]

\[ m^{(0)} = \sqrt{2} \frac{(a_1^2 + a_2^2)}{2}, \]

\[ m^{(1)} = \sqrt{2} \frac{(a_1^2 + a_2^2)}{2}, \]

\[ m^{(2)} = \sqrt{2} \frac{(a_1^2 + a_2^2)}{2}, \]

\[ m^{(3)} = \sqrt{2} \frac{(a_1^2 + a_2^2)}{2}, \]
Here the numbers $a_1$ and $a_2$ which represent vacuum expectation values of $A^0_{1+}$ and $A^0_{2-}$ may be chosen real and are fixed in terms of the parameters in the Lagrangian by the equations

$$\phi_0^0 = \sqrt{2} \Re \left( A^0_{1+} \cos \theta - A^0_{2-} \sin \theta \right),$$

$$\phi_2^0 = \sqrt{2} \Im \left( A^0_{1+} \cos \theta - A^0_{2-} \sin \theta \right),$$

$$\phi_3^0 - A^0_{1+} \sin \theta + A^0_{2-} \cos \theta, \quad \phi_3^0 = 0,$$

$$\phi_4^0 = A^0_{1+} \cos \theta - A^0_{2-} \sin \theta, \quad \phi_4^0 = 0.$$

The angles $\theta, \delta$ and $\delta'$ are then given by

$$\tan \theta = \frac{\phi_3^0}{\phi_1^0}, \quad \tan \theta = \frac{\phi_4^0}{\phi_1^0}, \quad \tan \delta' = \frac{\phi_2^0}{\phi_1^0}.$$

Notice the mass relations

$$m(\phi_1^0)^2 = 2a_1^2 + 2a_2^2 - s_2^2 = m(e^+)^2 - \frac{1}{2} m(\nu)^2 - \frac{1}{2} m(D^0)^2,$$

$$m(\nu)^2 = 2a_2^2 (s_2^1 + a_1^2) = \frac{1}{2} \left[ m(e^+)^2 + m(\nu)^2 \right].$$

Quite clearly the model is unrealistic so far as masses are concerned, since electron and muon masses ($\nu^2_{1L} + 2\nu_{2L}^2$ and $2\nu_{1L}^2$) cannot both be small relative to $m$ and $W^+$. In this model, $\nu^2$, $\nu^+$, $\nu^0$ and $\nu^0$ must have similar masses. Notwithstanding this restriction, we have retained the suggestive nomenclature $e^+$, $e^+$, $\nu^0$ for leptons in order that one may fix pictorially on familiar decay modes.

Note that at this stage the massless neutrinos, left and right, are given by

$$\nu_L = \lambda e \sin \theta + \lambda^0 \cos \theta,$$

$$\nu_R = \psi_{1+} \cos \theta - \psi_{1+} \sin \theta.$$

$\nu_L$ is purely "gauge" while $\nu_R$ is purely "matter". The minimisation of the effective potential, up to the tree approximation as above, exhibits degeneracy, signalled by the appearance of a pseudo-Goldstone boson $\phi_3^0$. Notice that $\phi_3^0$ is a linear combination of fields $A_{1+}$ and $A_{2-}$ and that the same linear combination of spinors $\psi_{1+}$ and $\psi_{1+}$ appears in the composition of the second (massless) neutrino $\nu_R$. This phenomenon is a relic of supersymmetry.

Now, to remove this degeneracy of the tree approximation, one must invoke radiative corrections and compute the effective potential at least to one-loop level. This has been done and indeed the degeneracy is lifted with the pseudo-Goldstone boson acquiring a real (non-tachyonic) mass for a certain range of the coupling parameters. However, to this approximation, the global symmetry $U(1)$ typifying conservation of electron-number remains unbroken. Both neutrinos $\nu_L$ and $\nu_R$ (with the compositions given above) remain massless.

Further, one can show that (since $\langle \nu^3 \rangle = 0, \langle \nu^0 \rangle = 0$) $\nu_L$ is the genuine Goldstone spinor, subject to low-energy theorems, while the second neutrino, $\nu_R$, is an ordinary fermion dragged along to a state of masslessness simply because the numbers of left-handed and right-handed spinors in the theory balance. Note that in the charged weak current $e^L$ is coupled to $\nu_R$ while $\nu_R$ is coupled to $\nu_L$. In other words, $\nu_R$ represents the muonic neutrino while $\nu_R$ is the electronic antineutrino.

In the next section we evaluate a decay amplitude, in which the non-Goldstone neutrino $\nu_R$ is emitted and verify the absence of a low-energy theorem.

III. TWO DECAY PROCESSES

We consider the decay of $\nu^+ \rightarrow \nu^+ + \nu^0$, with the emission of a pair of neutral fermions

$$\nu^+ + \nu^0 + \nu^0.$$

We shall show in this section that this process displays no low-energy theorem.
The relevant terms in the interaction Lagrangian we shall need are:

\[ \mathcal{L}_{\text{int}} = -i \sqrt{2} g_1 \sinb \sin \phi \phi^0_1 \phi^+_{\mu R} - 2 g_1 (\cos \delta)^2 \phi^0_1 \phi^+_{\mu R} \]

\[ -i \sqrt{2} g_1 \sin b \phi^+ \phi^0_1 \phi^+_{\mu R} - b \cos \delta \phi^0_1 \phi^+_{\mu R} \]

\[ - \sqrt{2} g_1 \sin b \phi^+ \phi^0_1 \phi^+_{\mu R} + \sqrt{2} g_1 \sin b \phi^0_1 \phi^+_{\mu R} \]

\[ - \sqrt{2} g_1 \cos \delta \phi^+ \phi^0_1 \phi^+_{\mu R} - \sqrt{2} g_1 \sin \phi \phi^0_1 \phi^+_{\mu R} \]

\[ + i \sqrt{2} g_1 \sin b \phi^0_1 \phi^+_{\mu R} + i g_1 \sin b \phi^0_1 \phi^+_{\mu R} \]

\[ + (k_2 \cos \delta \cos \phi' - k_1 \sin \phi \sin \phi') \psi^0_2 \nu^0_2 \]

\[ + (2 \sqrt{2} g_1 - k_1) \phi^0_1 \phi^+_{\mu R} \]

\[ (3.1) \]

(in the unitary gauge with \( \phi^0_2 = 0 \)). The matrix element is obtained by contracting the boson lines in the second-order expression \( \mathcal{L}_{\text{int}}(1) \). We find:

\[ T_1 = -i \sqrt{2} g_1 \cos \delta \cos \phi' \nu_\mu \epsilon_\mu \phi^0_1 \phi^+_{\mu R} \]

\[ + 2 g_1 \sin b \phi^+ \epsilon_\mu \phi^0_1 \phi^+_{\mu R} \]

\[ + 2 \sqrt{2} g_1 \sin b \phi^+ \epsilon_\mu \phi^0_1 \phi^+_{\mu R} \]

\[ + (k_2 - 2 \sqrt{2} g_1) (k_2 \cos \delta \cos \phi' - k_1 \sin \phi \sin \phi') \psi^0_2 \nu^0_2 \]

\[ (3.2) \]

Thus

\[ \langle \bar{\nu}_R(u) \nu_L(e^+) \rangle = \bar{\nu}_R(v) \nu_L \epsilon^+ \]

\[ \left[ -i \sqrt{2} g_1 \cos \delta \cos (2 \delta) \right] \]

\[ + \bar{v}_R(v) v_L \epsilon^+ \]

\[ + \bar{v}_R(v) v_L \epsilon^+ \]

\[ (3.3) \]

Since \( \nu_R \) is not the Goldstone neutrino, there is no reason for \( 3.3 \) to vanish for \( R_R \rightarrow 0 \), and indeed it does not.

It is instructive to consider the process

\[ \nu^- \rightarrow e^+ + \bar{\nu}_R + \nu_L \]

in the limit \( R_R \rightarrow 0 \) and to verify that the amplitude does vanish in accordance with Goldstone's theorem. The lowest-order matrix element is given by the effective terms:

\[ - \bar{v}_R = \bar{v}_R(v) v_L \epsilon^+ \nu_L \epsilon^+ \]

\[ \left[ -i \sqrt{2} g_1 \cos \delta \cos (2 \delta) \right] \]

\[ + \bar{v}_R(v) v_L \epsilon^+ \]

\[ (3.3) \]
also for $p \rightarrow 0$. This fortuitous zero is a consequence of the accidental symmetry $m(e) - m(u) + m(W) = 0$. Notice that the amplitudes vanish also for $p \rightarrow 0$. This fortuitous zero is a consequence of the accidental symmetry of $T_2$ (and the mass relation above) for $p \rightarrow p_0$, $e \rightarrow u$, $p_0 \rightarrow p$, and $W$ unchanged. To see that this accidental symmetry will not survive if the next order is computed, note that the Lagrangian (3.1) is not symmetric of $T$ (and the mass relation shove) for $v \rightarrow v_\perp$, $e \rightarrow u$, $p \rightarrow p_\perp$.

To conclude, in the present state of experimental uncertainty about how many massless (or near massless $m \approx 0$) neutrinos there are in nature, and the consequent theoretical uncertainty with regard to the construction of a realistic supersymmetric model of unified weak and electromagnetic interaction, all that (at best) one may infer from low-energy theorems is that the massless electronic neutrino is not a Goldstone spinor. Spontaneously broken supersymmetric theories, however, appear fully capable of describing non-Goldstone massless neutrinos in addition to Goldstone neutrinos and for these there are no inhibiting low-energy theorems.

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