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Jogesh C. Pati

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Abdus Salam

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WHY COLOUR FAILS TO BRIGHTEN FOR ELECTROPRODUCTION AND NEUTRINO SCATTERING EXPERIMENTS *

Jogesh C. Pati **
Department of Physics and Astronomy, University of Maryland,
College Park, Maryland, USA,
and
International Centre for Theoretical Physics, Trieste, Italy,

and

Abdus Salam
International Centre for Theoretical Physics, Trieste, Italy,
and
Imperial College, London, England.

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ABSTRACT

We show that the suppression of colour-brightening effects in lepto-production is a general property of a class of spontaneously broken colour-gauge theories, based on integer-charge quarks.

1. For integer charge quarks belonging, for example, to $(4,\bar{3},1)$ representation of a symmetry group [1),2)] $SU(4) \times SU(3)' \times U(1)'$, electric charge has two components: $Q = Q_{valency} + Q_{colour}$. $Q_{valency}$ for quarks equals $(2/3, -1/3, -1/3, 2/3)$ for the four valency quantum numbers $(p, n, \lambda, \chi = \text{charm})$. It is numerically the same for all three colours. Q_{colour} equals $(-2/3, 1/3, 1/3)$ for the three quark-colours (red, yellow and blue). The four leptons $(\nu_e, e^-, \nu^-, \nu_\mu)$ classified as $(4,1,1)$ are colour singlets; their charges $(0,-1,-1,0)$ are given by $Q_{valency}$ alone.

A major dilemma for integer-charge quark theories is the experimental result from electroproduction experiments [3)] that Q_{quark} is fractional and $\approx Q_{valency}$. There appears little brightening of Q_{colour}. Likewise, neutrino scattering seems not to exhibit colour quantum numbers. In this note we wish to show that the failure of colour to brighten is a perfectly general property of all (spontaneously broken) gauge theories where 1) leptons are colour singlets; 2) valency and colour are gauged independently, with weak interactions associated with valency gauging, strong with colour gauging, and 3) photon is generated via spontaneous symmetry breaking as a mixture of valency and colour gauge mesons.

The argument is simple; express the gauging pattern above symbolically in the form: $L_{int} = gW_\mu(J_\mu^{valency} + J_\mu^{leptons}) + fV_\mu J_\mu^{colour}$. Here W stands for the weak and V for the strong gauges, while the photon field A_μ is a mixture of W_3 and $\frac{1}{2}(\sqrt{3}V_3 + V_8)$. Notice now that before spontaneous symmetry breaking (when all fields are massless) leptons interact with the valency current through the intermediacy of W_μ's - but there is no interaction between $J^{leptons}$ and J^{colour}. Due to spontaneous symmetry breaking, valency and colour gauge mesons mix. This generates (through diagonalization of fields) the massless colour photon A_μ, but inevitably also the orthogonal colour gauge partner $\tilde{U}_\mu$ (with mass m_U), both of which contribute to lepton-colour interaction. This has the consequence that (barring logarithmic factors from renormalization group considerations) while lepton-valency interaction has the matrix element $(J^{lepton}\, 1/q^2\, J^{valency})$, the corresponding expression for lepton-colour interaction equals $J^{lepton}(1/q^2 - 1/(q^2 - m_U^2))\, J^{colour}$ when symmetry is spontaneously broken. It is the crucial negative sign between the two propagators in the second case, resulting from a diagonalization of fields, which suppresses colour-brightening effects. Colour is hard to excite for colour-singlet leptons.

2. Valency and colour gauging: Though the result stated in Sec.1 is general, we present its derivation within the gauge model [2] $SU_L(2) \times SU_R(2) \times SU(4)'$. This has the advantage that the effects of spontaneous symmetry breaking and the resulting mass matrix are available in detail. The basic fermionic multiplet is a 16-fold consisting of three quartets of red, yellow and blue quarks and four leptons. The six gauges W_L, W_R corresponding to $SU_L(2) \times SU_R(2)$ act on valency indices $[(p,n)+(\chi,\lambda)]_{L,R}$ while 15 gauges

$V(15) = V(8) + X(3) + X(3^*) + S^0(1)$ correspond to local colour symmetry [4] $SU(4)'$. The reduction of $V(15)$ shown above is under the $SU(3)'$ subgroup of $SU(4)'$, which acts on the red, yellow and blue quarks. (The leptons are singlets of this $SU(3)'$.) The gauge couplings associated with $W_{L,R}$ and V are $g_L^2/4\pi \approx g_R^2/4\pi \simeq 2\alpha$ and $f^2/4\pi \approx 1$ to 10. Spontaneous symmetry breaking is introduced through three Higgs-Kibble fields $A = (2+2, 2+2, 1)$, $B = (1, 2+2, \bar{4})$, $C = (2+2, 1, \bar{4})$ and gives ($m_{W_L^\pm} \approx 75$ GeV, $M_{W_R^\pm} \approx 200$ GeV), $m_X \sim m_S \approx 10^5$ GeV, while the $SU(3)'$ octet of colour gluons $V(8) = (V_\rho^+, V_{K^*}, V_3, V_8)$ acquire masses of the order of 3 to 5 GeV.

There are five neutral gauge fields $W_{3L,R}$, $V_{3,8}$ and S^0 coupled to the diagonal generators of the group. The physical gauge particles (defined through the mass matrix) are their linear combinations. We list below three of the "lighter" ones, the remaining two (Z^0 and $\tilde{S}$) being too heavy to be relevant:

$$A = e/fg[fW_{valency} + 2/\sqrt{3}\, g\, U^0]\ ,\quad V^0 = \frac{1}{2}[\sqrt{3}V_8 - V_3]$$
$$U = (3f^2 + 2g^2)^{-1/2}\,[\sqrt{3}\, f\, U^0 - g\, W_{valency} + O(\epsilon^2)]\ , \tag{1}$$

where $W_{valency} \equiv W_{3L} + W_{3R} - \sqrt{2/3}\,(g/f)S^0$ is the colour singlet piece of the photon; $U^0 \equiv 1/2(\sqrt{3}\, V_3 + V_8)$ is its colour octet piece; $e^2 = g^2f^2/2(f^2+g^2)$. The masses are: $m_A = 0$; $m_U = m_{V^0} \approx 3 \sim 5$ GeV. The $O(\epsilon^2)$ terms are corrections [5] of order $(m_U/m_{W_L})^2$. The coupling of the physical gauge particles [6] (i.e. A and $\tilde{U}$) (neglecting $(g^2/f^2) \lesssim (1/50)$, with $g^2 \approx 2e^2$) is:

$$L_I = eA_\mu[J_\mu^{val} + J_\mu^{col}] + U_\mu[(\sqrt{3}/2\, f\, J_\mu^{col} - (2/\sqrt{3})(e^2/f)\, J_\mu^{val}], \tag{2}$$

where the fermionic contents of the currents are:

$$J_\mu^{val} = J_\mu^{W_{3L}} + J_\mu^{W_{3R}} - \sqrt{2/3}\, J_\mu^{S^0}$$
$$= \sum_{\alpha = a,b,c}^{colour} \left(2/3\, \bar{p}_\alpha p_\alpha - 1/3\, \bar{n}_\alpha n_\alpha - 1/3\, \bar{\lambda}_\alpha \lambda_\alpha + 2/3\, \bar{\chi}_\alpha \chi_\alpha\right)_{L+R} - (\bar{e}e+\bar{\mu}\mu)_{L+R}$$
$$J_\mu^{col} = 2/\sqrt{3}\, J_\mu^{U^0} = \sum_{p\to n\to\lambda\to\chi} \left(-2/3\, \bar{p}_a p_a + 1/3\, \bar{p}_b p_b + 1/3\, \bar{p}_c p_c\right)_{L+R}\ . \tag{3}$$

Note that on account of the $W^{valency}$ component, the physical gauge particle $\tilde{U}_\mu$ becomes directly coupled to the electron. Such a coupling must be present within the model discussed above, if the photon must couple to J^{colour}. The coefficient determining the strength of such a coupling as well as its relative sign are fixed entirely by the composition of the massless photon and the renormalized effective gauge coupling parameters g^2 and f^2.

Note the crucial feature of Eq.(2); the strength factor for the current correlation $(J_\mu^{valency}(x)\ J_\nu^{colour}(x))$ arising due to photon interactions (in second order) is $+e^2$, while that arising due to $\tilde{U}$ interaction is $(-2/\sqrt{3}\ (e^2/f))\ ((\sqrt{3/2})/f) = -e^2$. The two contributions would exactly cancel if the propagators for the photon and $\tilde{U}$ were the same.

Consider now the matrix elements for electro- (or muon) production of colour singlet (X_{val}) versus colour octet (X_{col}) states, treating the leptons (but not hadrons) perturbatively. Colour singlet contributions arise primarily [7] through the familiar one-photon exchange; on the other hand, the colour octet production receives (comparable) contributions from two sources (the one-photon as well as $\tilde{U}$ exchange [8]). Thus,

$$M_{val} = M(eN \to e + X_{val}) = e^2(\bar{e}\gamma_\mu e)\ D_\gamma(q^2)(X_{val}|J^{val}|N)$$

$$M_{col} = M(eN \to e + X_{col}) = e^2(\bar{e}\gamma_\mu e)\ D_\gamma(q^2)\ \Delta(q^2)\ (X_{col}|J^{col}|N), \quad (4)$$

where $$\Delta(q^2) = \left(D_\gamma(q^2) - D_U(q^2)\right)/D_\gamma(q^2) = -m_U^2/(q^2 - m_U^2). \quad (5)$$

Here $D_\gamma(q^2)$ and $D_U(q^2)$ are the renormalized propagator functions for the photon and $\tilde{U}$, respectively, which, barring small logarithmic corrections (see later), are $(q^2)^{-1}$ and $(q^2 - m_U^2)^{-1}$, respectively. The ratio of matrix elements for production of colour versus valency in electroproduction (and also for time-like $q^2 > 0$ in $e^+ + e^-$) is the kinematic factor $\Delta(q^2)$. Note, a factor such as $\Delta(q^2)$ need never arise in a phenomenological approach to colour.

The kinematic distinction between colour and valency is not exclusive to this model. It is easy to convince oneself that it is a general property of all gauge models, which a) gauge valency and colour independently, b) generate the massless photon via spontaneous symmetry breaking as a mixture of valency and colour gauge mesons and c) treat leptons as singlets of colour SU(3)'.

3. Electro- (or muon) production of colour ($e + N \to e + X_{col}$): For integer charge quarks, the structure functions (representing cross-sections $F_{1,2}(q^2,\nu)$) are sums of two pieces:

$$F_i(q^2,\nu) = F^{val}(q^2,\nu) + \bar{F}_i^{col}(q^2,\nu), \text{ where } \bar{F}_i^{col} = \Delta^2(q^2)\ F_i^{col}(q^2,\nu), \quad (6)$$

$F_i^{val}(q^2,\nu)$ and $F_i^{col}(q^2,\nu)$ being defined in the usual manner by the Fourier transforms of the current correlation matrix elements $\langle N|J^{val}(x)\ J^{val}(0)|N\rangle$ and $\langle N|J^{col}(x)\ J^{col}(0)|N\rangle$. Writing $\rho^i_{dyn}(q^2,\nu) = F_i^{col}(q^2,\nu)/F_i^{val}(q^2,\nu)$, the ratio of colour octet versus colour singlet production is given by:

$$\bar{F}_i^{col}/F_i^{val} = \Delta^2(q^2)\ \rho^i_{dyn}(q^2,\nu), \quad (7)$$

where $\rho^i_{dyn}(q^2,\nu)$ depends upon the dynamics of colour versus valency transitions. Assuming a colour threshold $M_{col} \approx M_U + M_{baryon} \approx 4$ to 6 GeV and colour gluon masses $M_U \approx 3$ to 5 GeV (of the order of J/ψ masses), note that:

i) Below threshold for colour production, the function $\rho^i_{dyn}(q^2,\nu)$ is identically zero. Thus only J^{val} is operative, and quarks reveal only their fractional valency charges.

ii) Above threshold for colour-production $\rho_i(q^2,\nu)$ are non-vanishing. Noting that the characteristic masses (in the q^2-variable) for valency and colour transitions are of order m_ρ^2 and m_U^2 respectively [9], we might expect $F_i^{val}(q^2,\nu)$ to scale when q^2 and $M_N\nu \gg m_\rho^2$, and analogously $F_i^{col}(q^2,\nu)$ to scale when q^2 and $M_N\nu \gg m_U^2$. Using then the empirical fact [10] that $F_i^{val}(q^2,\nu)$ are damped for small q^2 and that they acquire their "full weight" (i.e. their scaling value) for $|q^2| > 2m_\rho^2$, we shall assume that $F_i^{col}(q^2,\nu)$

are damped analogously for $|q^2|$ and $M_N\nu < 2m_U^2$. In other words, one expects $F_i^{col}(q^2,\nu)$ to scale, but $\rho^i_{dyn}(q^2,\nu)$ to acquire its full scaling weight ($\approx$ of order unity) only when $|q^2|$ and $|M_N\nu|$ are $\gtrsim 2m_U^2$ with $M_X^2 \equiv M_N^2 + 2M_N\nu - |q^2|$ simultaneously above colour threshold.

These conditions are in fact met within the muon experiments at FNAL, where ρ^i_{dyn} may be expected to be of the order unity and colour expected to shine as brightly as valency. However, precisely in this region, the square of the kinematic gauge factor $\Delta(q^2)$ provides a rapid damping (e.g., $\Delta^2(q^2) = 1/9$ for $q^2 = -2m_U^2$; $\Delta^2 \approx 1/16$ for $q^2 = -3m_U^2$). In other words, exactly where $\rho^i(q^2,\nu)$ may be of order unity (i.e. $|q^2| > 2m_U^2$), the gauge factor $\Delta^2(q^2)$ is already tiny (< 1/10). On the other hand, when q^2 is small and one might have expected to escape the damping due to $\Delta^2(q^2)$ (for example, $\Delta^2(q^2) > 1/2$ for $|q^2| < m_U^2/4$; $\Delta^2 = 1$ for $q^2 = 0$), the dynamical factors $\rho^i(q^2,\nu)$ are likely to be damped compared with unity (possibly like $(q^2/2m_\rho^2)/(q^2/2m_U^2) = m_\rho^2/m_U^2$).

Thus the qualitative picture which emerges within the gauge theory approach is that the ratio of colour to valency production in electron-nucleon or muon-nucleon experiments is controlled by two factors, $\Delta^2(q^2)$ and $\rho^i(q^2,\nu)$, which appear to fight against each other, with the result that J^{col} does not receive its legitimate chance to contribute to colour production for any decent range of kinematic variables (q^2 and ν). Thus for all such experiments the bulk of the cross-section must come from J^{val} alone and quarks would behave as if they carried only their (fractional) valency charges.[11)]

4. Charged current neutrino production of colour $(\nu_\mu + N \to \mu^- + X_{col})$: If the mass matrix induces a mixing of (W_L^3, W_R^3) with the colour gluon U^0 to make the photon, the same mixing will inevitably induce either $W_L^\pm$ or $W_R^\pm$ to mix with the charged members of the colour gluon octet $V_\rho^\pm$ (or $V_{K^*}^\pm$, or both); the mixing angle being fully determined by the gauge masses and coupling parameters. Depending on the precise form of the mass matrix[2)] there are two cases.[12)]

Model A: V_ρ^+ mixes with W_L^+ (rather than W_R^+) ; the eigenstates are $\tilde{V}_\rho^+ = \cos\delta_L V_\rho^+ + \sin\delta_L W_L^+$ and $\tilde{W}_L^+ = \cos\delta_L W_L^+ - \sin\delta_L V_\rho^+$, with $\sin\delta_L = (m_{V_\rho}/m_{W_L})^2 (g/f)$. Exchanging both $\tilde{W}_L^+$ and $\tilde{V}_\rho^+$ and approximating the difference of their propagators by $(q^2 - m_{V_\rho}^2)^{-1}$ (justified, since $m_{W_L}^2 \gg m_{V_\rho}^2$ and q^2), we obtain $(g^2/2m_W^2 = G_F/\sqrt{2})$:

$$M(\nu_\mu + N \to \mu^- + X_{col}) = \frac{G_F}{\sqrt{2}} (\bar{\nu}_\mu\gamma_\mu(1+\gamma_5)\mu^-)\left(\frac{m_{V_\rho}}{q^2 - m_{V_\rho}^2}\right)\langle X_{col}|J^{col}|N\rangle . \qquad (8)$$

The mixing angle $\sin\delta_L = (m_{V_\rho}/m_{W_L})^2 (g/f)$. All other discussions and numerical estimates are based on the above correct expressions.

Note the appearance of the same suppression factor $\Delta(q^2) = m_{V_\rho}^2/(q^2 - m_{V_\rho}^2)$ for neutrino production of colour as for electroproduction $(m_{V_\rho} \approx m_U)$.

Model B: If V_ρ^+ mixes with W_R^+ (rather than with W_L^+), there is no colour production (barring the tiny $W_L^\pm - W_R^\pm$ mixing), since left-handed neutrinos ν_e and ν_μ do not couple to W_R.

5. Test of quark charges: The sum rule often used to determine quark charges involves the ratio of electro- and neutrino-production data:[3),10)]

$$r = \frac{4G_F^2 m_N E_\nu}{2\pi(\sigma_\nu + \sigma_{\bar{\nu}})} \int F_2^N \, dx . \qquad (9)$$

Including colour production and invoking an average value $\langle\epsilon\rangle$ for an estimate of the suppression effect for colour (in accordance with Sec.3), we obtain for Model A of Sec.4:

$$r = \frac{\langle Q^2_{val}\rangle_{eN} + \langle\epsilon\rangle\langle Q^2{}_{col}\rangle_{eN}}{\langle Q^2_{val}\rangle_{\nu N} + \langle\epsilon\rangle\langle Q^2{}_{col}\rangle_{\nu N}} . \qquad (10)$$

Substituting $\langle Q^2_{val}\rangle_{eN} = 5/18$, $\langle Q^2_{val}\rangle_{\nu N} = 1$, $\langle Q^2_{col}\rangle_{eN} = 4/18$ (or 0) and $\langle Q^2_{col}\rangle_{\nu N} = 1$ (or 0) for integer or fractionally charged quarks respectively, we obtain[13)] $r \approx (0.3$ to $0.29)$ (for $\langle\epsilon\rangle = 1/5$ to $1/10$) for integer charge quarks and ≈ 0.28 for fractional charges. Experimental data is consistent both with 0.28 and 0.3. This sum rule does not decide for or against either charge assignment, contrary to assertions in the literature.[3),10)]

Analogous to the modifications exhibited above, all parton-model-based sum rules[11),14)] (for eN and νN processes) need to be corrected, to take account of colour production, by factors which are no larger than 10% to at most 20%.

To conclude, in a gauge theory, colour-singlet leptons are simply not efficient for producing colour when they impinge upon colour-singlet protons.

6. Consider now colour production in $e^+ + e^-$ annihilation and the effect on $R = (e^+ + e^- \to \text{hadrons})/(e^+ + e^- \to \mu^+ + \mu^-)$ of the kinematic factor $\Delta(q^2)$ for $q^2 > 0$. For $q^2 > 2m_U^2$, $\Delta < 1$; so that assuming that for large q^2 there are no dynamical surprises for colour versus valency production (and barring renormalization group effects - see Sec.7) one may expect a net suppression of colour versus valency production for time-like $q^2 (> 0)$ (though at a rate much slower than for space-like $q^2 < 0$). Thus R approaches $R_{valency}$ for fairly large q^2.

In the region $q^2 \leqslant 2m_U^2$, $|\Delta| \geqslant 1$ and acts as an enhancement. We show that in spite of $\Delta^2 >> 1$ in the region $m_U < q^2 < \sqrt{2}m_U$, the net production of colour-continuum is not unduly enhanced due to the limited number of channels available as well as the meagre phase-space associated with them. For this purpose, assume that $\tilde{U}$ is the lowest 1^{--} colour-octet state with a relatively narrow width Γ. Consider production of colour with increasing q^2.

i) The state $\tilde{U}$ itself would be produced when $\sqrt{q^2} = m_U \pm \Gamma/2$ by its direct coupling to electrons (see Eq.(2)) without the intermediacy of the photon, which is orthogonal to $\tilde{U}$. There is no Δ factor for this production. (Possible candidates for identification[15] of $\tilde{U}$ are, for example, J(3.1) or a narrow component within $\psi(4.1)$.)

ii) Apart from the production of $\tilde{U}$, colour-production matrix element is non-zero only[16] for $\sqrt{q^2} \geqslant m_U + 2m_\pi$. As typical of the situations met with consider the $e^+ + e^-$ production of the $\tilde{U} + 2\pi$. The factor Δ^2 is as large as ≈ 45 at $(\tilde{U} + 2\pi)$-threshold with $m_U \approx 4$ GeV and is smaller if m_U is smaller. However, the production cross-section $\sigma(e^+e^- \to \tilde{U} + 2\pi)$ is $\approx$ $\approx [\alpha\Delta^2(q^2)h^2(q^2)]/12(2\pi)^4(q^2)^4$ times the three-body phase space $(\int \delta^4(\ ) (d^3p/E)^3)$. Even for $h^2/4\pi \approx 100$, where $h(q^2)$ is the effective strength of the matrix element $\langle \tilde{U}\pi\pi|J^{col}|0\rangle$, this is $\lessapprox \frac{1}{2}$ nb for[17] $\sqrt{q^2} \lessapprox m_U + 8m_\pi$. (For

comparison, remark that $e^+ + e^- \to$ hadrons (valency + colour) $\approx$ 10-20 nb at these energies.) This calculation is typical of the others (e.g. $\tilde{U} + 4\pi$, $\tilde{U} + 6\pi$, $\tilde{U} + \eta'$), which show that in spite of the enhancement due to $\Delta^2(q^2)$, colour production is no more than normal in the region $\sqrt{q^2} < \sqrt{2}m_U$. What exactly is the contribution of colour to R in a limited kinematic region and how it varies in this region is rather difficult to estimate, since it depends on the thresholds for colour pseudoscalars (and similar states) and on the dynamics of colour-resonant states. We shall not attempt this here.

7. For large q^2, renormalization group considerations would suggest that the propagators $1/q^2$ and $1/(q^2-m_U^2)$ (as well as $\rho_i(q^2,\nu)$) would be modified by logarithmic factors. ρ_i is presumably not affected unduly (since valency and colour may be expected to exhibit similar dynamics at high frequencies). Regarding the "kinematic" factor Δ, ignoring contributions from Higgs-Kibble mesons (in other words, assuming spontaneous symmetry breaking is dynamical) and ignoring g^2/f^2 terms, $D_\gamma = 1/q^2$ is expected to modify to $1/q^2\left[1 + (g^2/4\pi)(7/6\pi)\log -q^2/M^2\right]^{-5/28}$ and $D_U = (q^2 - m_U^2)^{-1}$ to $(q^2-m_U^2)^{-1}\left[1 + (f_c^2/4\pi)(3/\pi)\log -q^2/m_c^2\right]^{-1/4}$ for the symmetry $SU_L(2) \times SU_R(2) \times SU(4)'$. These modifications (even with Higgs-Kibble[2] A, B, C included, when the theory is still "temporarily" asymptotically free in the sense of Politzer) have little effect on the considerations of this paper for FNAL energies for the mass parameter m_c involved in the logarithmic factor for D_U being $\approx m_U$. However, for truly asymptotic q^2 the faster decrease of modified D_U relative to D_γ implies that for this gauge model colour and valency may shine forth equally brightly, and $R \to R_{val} + R_{col}$, when $q^2 \to \infty$. The nature of asymptotia and approach to it would, of course, depend on the gauge model chosen and the renormalization mass m_c.

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1) J.C. Pati and Abdus Salam, Phys. Rev. D8, 1240 (1973).

2) J.C. Pati and Abdus Salam, Phys. Rev. D10, 275 (1974).

3) See, for example, F. Gilman, in Proceedings of the XVII International Conference on High Energy Physics, London, 1974.

4) The gauging of $SU(3)' \times U(1)'$, as stated in Sec.1, would be sufficient (see Ref.1). We retain $SU(4)'$ gauging for ease of comparison with Ref.2.

5) Exact expression for $\tilde{U}$ may be found in Ref.2.

6) V^0 interaction is not exhibited, since it is coupled only to hadrons, but not to leptons. If $\tilde{U}$ and V^0 are allowed to mix, the conclusions are not affected.

7) $\tilde{U}$ exchange contribution to colour singlet production is smaller than the one-photon contribution by a factor $= (2/\sqrt{3})^2\ (e^2/f^2)$.

8) Note that multiple $\tilde{U}$ exchange may be neglected here to the same extent as multiple photon exchange.

9) This is suggested, for example, by a dispersion theoretic approach to $F_i^{col}(q^2,\nu)$, where the integral in q^2 starts from $(m_U + 2m_\pi)^2$, while for $F_i^{val}(q^2,\nu)$ it starts from the threshold $4m_\pi^2$. This is also suggested by a generalized vector meson dominance approach, where the scaling mass m_ρ^2 for valency would be replaced by m_U^2 .

10) See, for example, B.C. Barish, Invited talk at the American Physical Society (Division of Particles and Fields), September 1974.

11) In general, by the same token, the contributions of the spin-1 colour gluons (possessing charge only through J^{col}) to the ratio (σ_L/σ_T) is also suppressed. However, in the intermediate regions of $|q^2|$ and $M_N\nu$ ($\lesssim m_U^2$) the contribution may be significant and may even fluctuate depending upon the variation of Δ^2 versus $\rho_i(q^2,\nu)$.

12) The two cases correspond to the interchange of the patterns for $\langle B \rangle$ and $\langle C \rangle$. See Ref.2 for details (see also footnote 21 of Ref.2).

13) Note that if the suppression factor $\langle \epsilon \rangle$ is set equal to 1, r equals (1/4) = 0.25 for model A (Sec.3) with integer charge quarks. The often quoted value (Ref.3) $r = \frac{1}{2}$ for integer charges is based upon setting not only $\langle \epsilon \rangle = 1$ but also $\langle Q^2_{col} \rangle_{\nu N} = 0$ (i.e. assuming that neutrinos can not produce colour). In our notation this would be prejudging in favour of model B versus model A (Sec.3).

14) C.H. Llewellyn Smith, Physics Reports 3C (1972).

15) J.C. Pati and Abdus Salam, Phys. Rev. Letters 34, 613 (1975); Phys. Letters (to be published).

16) This assumes that (colour-pseudoscalar + three-pion) threshold lies higher and that no resonant state exists between m_U and $(m_U + 2m_\pi)$. Note if $J(3.1) = \tilde{U}$, the $\psi_2(3.7)$ particle would be interpreted as a colour-resonance with an enhancement $\Delta^2 \approx 5.5$ for its production.

17) Note for $\sqrt{q^2} \gtrsim m_U + m_\eta'$, the Δ factor begins to be unimportant (in providing enhancement), taking $m_U \approx 3$ or 4 GeV.