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SUPERSYMMETRY, PARITY AND FERMION-NUMBER CONSERVATION

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ABSTRACT

A general rule is formulated for the construction of renormalizable supersymmetric Lagrangians which admit a conserved quantum number (baryon or lepton number) associated with fermions. There are two unusual features. Firstly, there is a class of supermultiplets in which the spinor particle with unit fermion number is accompanied by a scalar particle which carries two units of fermion number. Only in a very restricted class of theories can the appearance of such difermions be avoided. Secondly, the fermion number content of a supermultiplet is governed by its chiral type. Thus, with the conventions adopted here, it is shown that, in scalar supermultiplets, right-handed spinor fermions are associated with ordinary scalar bosons while left-handed spinor fermions are associated with di-fermionic bosons. In gauge supermultiplets ordinary vector bosons are associated with left-handed spinor fermions. In consequence of this peculiar multiplet structure, the problem of defining a conserved parity is non-trivial. It is shown that parity doubling provides no solution unless either supersymmetry or fermion number is violated explicitly. In fact, the solution to the parity problem makes essential use of local symmetries. A class of renormalizable supersymmetric Lagrangians which conserve both fermion number and parity is given. For this class the interactions are governed by a gauge principle.

I. INTRODUCTION

A persistent difficulty with supersymmetric schemes [1] has, from the beginning, concerned the definition of fermion number.[2] (This quantity, i.e. baryon or lepton or baryon + lepton number, will be denoted by F in the following.) Basically this problem is a reflection of the fact that, once having put fermions and bosons together in a supermultiplet, one is hard put to distinguish them. Any quantum number carried by a fermion tends to be carried also by its boson partners. Notwithstanding this, there is a solution to the problem. Our purpose in this paper is to define the exact nature of the problem and to discuss the general features of supersymmetric and renormalizable models in which a fermion number is defined.

In Sec.II we exhibit the central role played by chirality in defining a fermion number for supersymmetric multiplets. The supersymmetry generators resolve into chiral components which anticommute among themselves and are related by hermitian conjugation. This leads to an association of chirality with fermion number. Indeed, the association turns out to be extremely rigid. We find, for example, that if positive chirality scalar supermultiplets contain $F = 1$ right-handed spinors and $F = 0$ spin-zero Bose particles, then negative chirality supermultiplets which contain $F = 1$ left-handed spinors must contain $F = 2$ spin-zero difermions.

A second consequence of this association is the difficulty of constructing supersymmetric Lagrangians which conserve parity as well as fermion number. In particular we show in Secs.III and IV that renormalizable and supersymmetric models in which both fermion number and parity are conserved are necessarily based on a local gauge symmetry. It is further shown that the maximal local symmetry realizable on a set of $2n$ two-component spinors consistently with these requirements is $Sp(2n)$. We conclude with a simple example based on a spontaneously broken U(1) internal symmetry which preserves parity as well as fermion number. An appendix is devoted to some speculations concerning the unification of weak, electromagnetic and strong interactions.

II. FERMION NUMBER AND THE STRUCTURE OF SUPERMULTIPLETS

Supersymmetry is a kinematical extension of Poincaré symmetry which is obtained by adjoining to the generators of space-time translations, P_μ, and homogeneous Lorentz transformations, $J_{\mu\nu}$, a set of four operators, S_α. These operators are defined by the algebraic relations

$$[S_\alpha, P_\mu] = 0$$
$$[S_\alpha, J_{\mu\nu}] = \frac{1}{2}(\sigma_{\mu\nu}S)_\alpha$$
$$\{S_\alpha, S_\beta\} = -(\gamma_\mu C)_{\alpha\beta} P_\mu \tag{2.1}$$

and by the reality condition

$$S = C\,\bar{S}^T, \tag{2.2}$$

where C is the usual charge conjugation matrix. The operators S_α, which generate "supertranslations", thus transform as a Dirac spinor of the Majorana type. They give a preferred role to Majorana fields in the supermultiplet structure.

It is clear that if the members of a supermultiplet are to be distinguished by a quantum number - which we shall call fermion number F - then this number must be carried by some of the generators. The only way to introduce such a quantum number is by means of the transformations

$$S \to e^{-\alpha\gamma_5} S \tag{2.3}$$

with α real. (Ordinary phase transformations on S would be compatible with neither the Majorana constraint (2.2) nor the anticommutator rule in (2.1).) The chiral components

$$S_\pm = \frac{1 \pm i\gamma_5}{2} S$$

transform irreducibly and contragrediently

$$S_\pm \to e^{\pm i\alpha} S_\pm \, . \tag{2.4}$$

We may say that the positive chirality*) component S_+ carries $F = 1$ while its hermitian conjugate S_- carries $F = -1$.

*) In earlier papers we carelessly referred to positive-chirality spinors as "left-handed". With our choice of $\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$, the matrix element $(0|\psi_+|p,\lambda)$ is non-vanishing only if the light-like state $|p,\lambda\rangle$ carries helicity $\lambda = +\frac{1}{2}$, i.e. is right-handed. Here $\psi_+ = \frac{1 + i\lambda_5}{2}\psi$ is the positive chirality projection of ψ. For the conventions used here see Salam and Strathdee, Ref.1.

This rigid association of fermion number with chirality is a fundamental property of supersymmetry. It provides very strict limits on the possible forms of supersymmetric and fermion-number conserving Lagrangian models. Indeed, it has the immediate consequence that only in exceptional cases do such models admit a conserved parity [*]. Examples of such will be given in following sections.

It might be thought that the strict association between fermion number and chirality could be diluted by means of parity doubling. Thus, one might suppose that the supertranslation generators are comprised of two independent pieces.

$$S = S^1 + S^2 , \tag{2.5}$$

where

$$\left.\begin{aligned} S^1 &\to e^{-\alpha\gamma_5} S^1 \qquad (2.6a) \\ \text{and} \quad S^2 &\to e^{+\alpha\gamma_5} S^2 \end{aligned}\right\} . \qquad (2.6b)$$

If these two pieces are indeed independent - so that we can, for example, envision multiplets of S^1 on which $S^2 = 0$, and vice versa - then we must have

$$\begin{aligned} \{S^1 , S^1\} &= -(\gamma C)\, P^1 \\ \{S^1 , S^2\} &= 0 \\ \{S^2 , S^2\} &= -(\gamma C)\, P^2 , \end{aligned} \tag{2.7}$$

[*] In earlier work, "parity" was assumed to act on the supertranslation generators according to the formula

$$S \to i\gamma_0 S .$$

Clearly, however, this transformation reverses chirality and therefore fermion number as well. It should be interpreted as CP rather than P if fermion number is to have any significance. Invariance with respect to this transformation is the rule rather than the exception.

i.e. the total 4-momentum is partitioned into independent pieces, P^1 and P^2. Moreover, these two pieces are separately conserved. Thus, the conservation of $S^1 + S^2$ and the fermion number defined by (2.6) implies the conservation of both S^1 and S^2 and therefore of both P^1 and P^2. It follows that Green's functions made from products of fields belonging to the two subgroups must factorize. This means, in turn, that there can be no communication between the two worlds in which S^1 and S^2 operate: they must be truly independent and in each of them a strict association of fermion number with chirality is maintained. [*] Again, because of the Green's function factorization it is not possible to envisage any spontaneous mechanism for breaking this independence. Only an explicit violation of either fermion number or supersymmetry could lead to communication between the two worlds.

It is conceivable that a basic modification (i.e. enlarging) of the supersymmetry group might admit a less restrictive definition of fermion number. We have not been able to do this, however, and shall therefore proceed to analyse supermultiplets in the scheme defined by (2.1)-(2.4).

Supersymmetric Lagrangian models receive their most natural expression in terms of superfields. Of these, only two varieties have so far proved useful in the construction of renormalizable models. Firstly there are the chiral scalar superfields $\Phi_\pm(x,\theta)$, whose component structure is conveniently expressed by

$$\Phi_\pm(x,\theta) = e^{\mp \frac{1}{4}\bar\theta \not\partial \gamma_5 \theta} \left[A_\pm(x) + \bar\theta\psi_\pm(x) + \tfrac{1}{4}\bar\theta(1 \pm i\gamma_5)\theta\, F_\pm(x)\right], \tag{2.8}$$

where $A_\pm$ and $F_\pm$ are Bose fields and $\psi_\pm$ are chiral spinors, i.e.

$$i\gamma_5\psi_\pm = \pm\psi_\pm .$$

The variable θ is an anticommuting Majorana spinor. Assigning fermion number to the component fields by means of γ_5 transformations in such a way that both ψ_+ and ψ_- carry unit fermion number, i.e.

[*] By explicit computation we have verified that the three- and four-point functions made up from products of superfields, some of which transform according to (2.6a) and others according to (2.6b), do factorize into two non-communicating factors.

$$\Phi_+(x,\theta) \rightarrow \Phi_+(x,e^{-\alpha\gamma_5}\theta)$$

$$\Phi_-(x,\theta) \rightarrow e^{2i\alpha}\,\Phi_-(x,e^{-\alpha\gamma_5}\theta) \ , \tag{2.9}$$

one finds for the components

$$\begin{pmatrix} A_+ \\ \psi_+ \\ F_+ \end{pmatrix} \rightarrow \begin{pmatrix} A_+ \\ e^{i\alpha}\,\psi_+ \\ e^{2i\alpha}\,F_+ \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} A_- \\ \psi_- \\ F_- \end{pmatrix} \rightarrow \begin{pmatrix} e^{2i\alpha}\,A_- \\ e^{i\alpha}\,\psi_- \\ F_- \end{pmatrix} \ . \tag{2.10}$$

This means that the spin-zero fields A_+ and F_- are ordinary bosons with $\mathbb{F} = 0$ while A_- and F_+ are difermions with $\mathbb{F} = 2$. The presence of such difermions is inescapable in supersymmetric theories.

The fields $F_\pm$ play an auxiliary role and can always be eliminated from the Lagrangian since their derivatives never appear. The fields $A_\pm$, on the other hand, are true dynamical variables and have particles associated with them. What has emerged from the above assignments, therefore, is the curious rule:

In scalar supermultiplets positive chirality spin-half fermions are associated with ordinary ($\mathbb{F} = 0$) spin-zero bosons while negative chirality fermions are associated with difermionic ($\mathbb{F} = 2$) spin-zero bosons.*)

The second kind of superfield used in the construction of renormalizable models is the gauge superfield $\Psi(x,\theta)$, which plays the role of a potential when local internal symmetries are involved. Such fields belong to the adjoint representation of the internal symmetry and in general they have the full complement of component fields when expanded in powers of θ. However, the gauge freedom can be exploited to eliminate half of these components from $\Psi(x,\theta)$, which can then be expressed in the restricted form

$$\Psi(x,\theta) = \frac{1}{4}\,\bar\theta i\gamma_\nu\gamma_5\theta\; U_\nu(x) + \frac{1}{2\sqrt{2}}\,\bar\theta\theta\;\bar\theta\gamma_5\lambda(x) + \frac{1}{16}\,(\bar\theta\theta)^2\,D(x) \ . \tag{2.11}$$

*) Other, more exotic, assignments are possible if the fermion numbers of ψ_+ and ψ_- are allowed to differ from unity. A generalization of this sort in which spinors with $\mathbb{F} = 3$ are involved will be considered in Sec. IV.

The Bose fields U_ν and D belong to the adjoint representation and may be taken as hermitian matrices. The spinors λ, also in the adjoint representation, may be taken as hermitian in the sense

$$(\bar\theta\lambda)^\dagger = \bar\theta\lambda \ .$$

They are essentially a set of Majorana spinors. Fermion number is assigned to the components of the gauge superfield by means of the γ_5 transformation

$$\Psi(x,\theta) \rightarrow \Psi(x, e^{-\alpha\gamma_5}\,\theta) \tag{2.12}$$

exactly as for Φ_+. One finds

$$\begin{pmatrix} U_\nu \\ \lambda \\ D \end{pmatrix} \rightarrow \begin{pmatrix} U_\nu \\ e^{\alpha\gamma_5}\,\lambda \\ D \end{pmatrix} \ . \tag{2.13}$$

The chiral spinors λ_+ and λ_- carry fermion number $\mathbb{F} = -1$ and $\mathbb{F} = 1$, respectively, while U_ν and D carry $\mathbb{F} = 0$. Notice that the chiral spinors are related by hermitian conjugation,

$$\bar\theta\lambda_+ = (\bar\theta\lambda_-)^\dagger \ . \tag{2.14}$$

It is particularly important to remark that the positive chirality spinor λ_+ is an antifermion ($\mathbb{F} = -1$) while its conjugate, the negative chirality spinor λ_-, is a fermion ($\mathbb{F} = 1$) according to the conventions adopted above. This fact is highly relevant to the construction of parity-conserving models. Indeed, we shall prove that renormalizable fermion-number and parity-conserving supersymmetric models are necessarily based on local symmetries.

III. FERMION-NUMBER CONSERVING LAGRANGIANS

Consider now the general form of supersymmetric Lagrangians. Given two sets of "matter" fields, Φ_+ and Φ_-, expressed as columns and which are acted upon by a local symmetry

$$\Phi_+ \rightarrow e^{i\Lambda_1^{+}}\,\Phi_+ \ ,$$

$$\Phi_- \rightarrow e^{i\Lambda_2^{+}}\,\Phi_- \ , \tag{3.1}$$

where Λ_{1+} and Λ_{2+} are (in general *)) independent complex matrix structures of positive chirality, and given the corresponding gauge superfields Ψ_1 and Ψ_2 which transform according to

$$e^{2\Psi_1} \to e^{i\Lambda_{1+}^\dagger} e^{2\Psi_1} e^{-i\Lambda_{1+}}$$

$$e^{2\Psi_2} \to e^{i\Lambda_{2+}^\dagger} e^{2\Psi_2} e^{-i\Lambda_{2+}} , \tag{3.2}$$

then the Lagrangian is constructed as follows. Firstly, there is the matter field kinetic term,

$$\mathcal{L}_k = \frac{1}{8} (\bar{D}D)^2 \left[\phi_+^\dagger e^{2\Psi_1} \phi_+ + \phi_-^\dagger e^{-2\Psi_2} \phi_-\right] , \tag{3.3}$$

which contains the gauge couplings. Secondly, there is a term independent of the gauge fields which includes mass and self-interaction terms involving the matter fields

$$\mathcal{L}_m = -\frac{1}{2} \bar{D}D \left[\phi_-^\dagger m(\phi_+) + \text{h.c.}\right] , \tag{3.4}$$

where $m(\phi_+)$ is restricted by renormalizability to be a polynomial which contains terms of no higher than the second degree, and by gauge invariance to satisfy the identity

$$e^{-i\Lambda_{2+}} m(e^{i\Lambda_{1+}} \phi_+) = m(\phi_+) . \tag{3.5}$$

The linearity of $\mathcal{L}_m$ in the negative chirality fields ϕ_- results from the requirement of fermion-number conservation.

Thirdly, there is the gauge field kinetic term. Its expression in terms of superfields is rather cumbersome because of the need to define the superfield analogues of the field strength tensors. These take the form of chiral spinor superfields

$$\Psi_{\alpha\pm} = -\frac{i}{2\sqrt{2}} \bar{D} \frac{1 \mp i\gamma_5}{2} D \left[e^{\mp 2\Psi} \left(\frac{1 \pm i\gamma_5}{2} D\right)_\alpha e^{\pm 2\Psi}\right] . \tag{3.6}$$

one set for Ψ_1 and an analogous one for Ψ_2. The field strengths $\Psi_{1\pm}$, for example, transform according to

*) The groups which act on ϕ_+ and ϕ_- may be identical or they may contain a common subgroup or they may be completely independent.

$$\Psi_{1+} \to e^{i\Lambda_{1+}} \Psi_{1+} e^{-i\Lambda_{1+}} ,$$

$$\Psi_{1-} \to e^{i\Lambda_{1+}^\dagger} \Psi_{1-} e^{-i\Lambda_{1+}^\dagger} . \tag{3.7}$$

In the special gauge defined by (2.11) they take the form

$$\Psi_{1\pm} = e^{\mp \frac{1}{4}\bar{\theta}\not{\partial}\gamma_5\theta} \left[\lambda_{1\pm} + \frac{1}{\sqrt{2}} \left(-\gamma_5 D_1 \pm \frac{i}{2} \sigma_{\mu\nu} U_{1\mu\nu}\right) \frac{1 \pm i\gamma_5}{2} \theta + \frac{1}{4}\bar{\theta}(1 \pm i\gamma_5)\theta \, (-i\not{\nabla}\lambda_{1\mp})\right] , \tag{3.8}$$

where the usual Yang-Mills covariant forms appear,

$$U_{1\mu\nu} = \partial_\mu U_{1\nu} - \partial_\nu U_{1\mu} - i[U_{1\mu}, U_{1\nu}] ,$$

$$\nabla_\mu \lambda_{1\pm} = \partial_\mu \lambda_{1\pm} - i[U_{1\mu}, \lambda_{1\pm}] . \tag{3.9}$$

Notice that the field strengths (3.6) are chiral in two senses: with respect to their component structure, $(1 \mp i\gamma_5)D\Psi_\pm = 0$, and with respect to their spinor index, $(1 \mp i\gamma_5)\Psi_\pm = 0$. They are related by complex conjugation in exactly the same sense as are the spinors λ_+ and λ_-, viz.

$$\bar{\theta}\Psi_- = (\bar{\theta}\Psi_+)^\dagger . \tag{3.10}$$

The field strengths resolve into a set of independent blocks, one for each simple factor in the local symmetry group, just as for Yang-Mills theories. Correspondingly, the gauge field kinetic term takes the form of a sum of terms, one for each block. With each block a coupling constant is associated. One can write

$$\Psi_\pm = \sum_k \Psi_\pm^k T^k , \tag{3.11}$$

where the hermitian matrices T^k span the adjoint representation and are normalized by

$$\frac{1}{2} \text{Tr}(T^k T^\ell) = \delta^{k\ell} . \tag{3.12}$$

In this basis the gauge field kinetic term is given by

$$\mathcal{L}_g = \frac{1}{8} \bar{D}D \sum_k \left(\frac{1}{g_k^2} \bar{\Psi}_-^k \Psi_+^k + \text{h.c.}\right) \tag{3.13}$$

and, corresponding to any (central) U(1) factor *), the additional terms

$$\mathcal{L}_\xi = \frac{1}{8}(\bar{D}D)^2 \, (\xi_k V^k) \, , \qquad (3.14)$$

which can be shown to be supersymmetric. The coupling constants g_k are dimensionless while the parameters ξ_k have the dimensions of $(\text{mass})^2$.

We now exhibit the component structure of the terms (3.3), (3.4), (3.13) and (3.14) in the special gauge. Firstly, the matter field kinetic term reads:

$$\begin{aligned}
\mathcal{L}_k = \; & \nabla_\mu A_+^\dagger \nabla_\mu A_+ + \bar{\psi}_+ i\not{\partial}\psi_+ + F_+^\dagger F_+ \\
& + i\sqrt{2}\,(A_+^\dagger \bar{\lambda}_{1-}\psi_+ - \bar{\psi}_+\lambda_{1-}A_+) + A_+^\dagger D_1 A_+ \\
& + \nabla_\mu A_-^\dagger \nabla_\mu A_- + \bar{\psi}_- i\not{\partial}\psi_- + F_-^\dagger F_- \\
& + i\sqrt{2}\,(A_-^\dagger \bar{\lambda}_{2+}\psi_- - \bar{\psi}_-\lambda_{2+}A_-) - A_-^\dagger D_2 A_- \, . \qquad (3.15)
\end{aligned}$$

where the covariant derivatives are given by:

$$\begin{aligned}
\nabla_\mu A_+ &= \partial_\mu A_+ - iU_{1\mu}A_+ \\
\nabla_\mu A_- &= \partial_\mu A_- - iU_{2\mu}A_- \, , \qquad (3.16)
\end{aligned}$$

and similarly for $\nabla_\mu\psi_+$ and $\nabla_\mu\psi_-$. Secondly, the purely matter term (3.4), with $m(\Phi_+)$ given by

$$m(\Phi_+) = m_0 + m_1\Phi_+ + m_2\Phi_+\Phi_+ \, , \qquad (3.17)$$

takes the form

$$\begin{aligned}
\mathcal{L}_m = \; & F_-^\dagger \left[m_0 + m_1A_+ + m_2A_+A_+\right] \\
& + \bar{\psi}_- \left[m_1\psi_+ + m_2\,(\psi_+A_+ + A_+\psi_+)\right] \\
& + A_-^\dagger \left[m_1F_+ + m_2\,(F_+A_+ + A_+F_+ + \psi_+ C^{-1}\psi_+)\right] \\
& + \text{h.c.} \qquad (3.18)
\end{aligned}$$

*) The matrix $\xi_k T^k$ must commute with all matrices T^ℓ .

The coefficients m_0 , m_1 and m_2 must of course have a tensorial structure appropriate to the internal symmetry which is expressed through the identity (3.5). The number of independent parameters among them is generally very limited. Finally, the gauge field kinetic terms (3.13) and (3.14) take the respective forms

$$\mathcal{L}_g = \sum \frac{1}{g_k^2}\left[-\frac{1}{4}(U^k_{\mu\nu})^2 + \bar{\lambda}^k_- \, i\not{\partial}\lambda^k_- + \frac{1}{2}(D^k)^2\right] \qquad (3.19)$$

$$\mathcal{L}_\xi = \xi_k D^k \, . \qquad (3.20)$$

To conclude, we remark that the dependence of the classical potential V on the fields $A_\pm$ takes a very particular form in supersymmetric models. Since the auxiliary fields $F_\pm$ and D satisfy algebraic equations they can be eliminated at once. Solving these equations to express the auxiliary fields in terms of A_+ and A_- one finds that V is given by

$$V(A_+,A_-) = F_+^\dagger F_+ + F_-^\dagger F_- + \sum \frac{1}{2g_k^2}(D^k)^2 \, . \qquad (3.21)$$

This potential is manifestly non-negative. Moreover, if its minimum value is found to be greater than zero, then at least one of the auxiliary fields must have a non-vanishing vacuum expectation value. Such non-vanishing auxiliary field values are a signal of spontaneous supersymmetry breakdown.[3] Indeed, one can show quite generally that the corresponding Goldstone spinor is given by

$$\nu_- \sim \sum \frac{1}{g_k^2}\langle D^k\rangle \lambda^k_- + \langle F_-^\dagger\rangle \psi_- + C\,(\bar{\psi}_+ \langle F_+\rangle)^T \, . \qquad (3.22)$$

If fermion number is not spontaneously violated [4] then $\langle F_+\rangle = 0$ and the "neutrino" is a pure (left-handed) fermion.

In a realistic theory it is necessary to distinguish baryon and lepton number. There are two possibilities. Firstly, one can assign lepton numbers $L = \pm 1$ and baryon numbers $B = 0$ to the chiral components $S_\pm$ of the supersymmetry generators. In this scheme all members of a supermultiplet would carry the same baryon number while lepton number is distributed in the manner discussed above. Thus, the chiral supermultiplets $\Phi_\pm = (A_\pm, \psi_\pm, F_\pm)$ would have the content

$$\begin{aligned}
\Phi_+ &\sim (B, L-1) \;, \quad (B, L) \;, \quad (B, L+1) \\
\Phi_- &\sim (B, L+1) \;, \quad (B, L) \;, \quad (B, L-1)
\end{aligned}$$

and the gauge fields $\Psi = (U_\mu, \lambda_-, D)$ would have the content distribution $(B,L-1)$, (B,L), $(B,L-1)$. An alternative scheme would be to assign $L = 0$ and $B = \pm 1$ to the chiral components $S_\pm$. In this scheme all members of a supermultiplet would carry the same lepton number. Indeed,

$$\Phi_+ \sim (B-1,L) \quad , \quad (B,L) \quad , \quad (B+1,L)$$

$$\Phi_- \sim (B+1,L) \quad , \quad (B,L) \quad , \quad (B-1,L) \quad .$$

If B and L are both conserved, and if the neutrino is to be a lepton, then only the first scheme is feasible. On the other hand, if neither B nor L is conserved but only their sum, $F = B + L$, then the second scheme could be used if there is in the theory a (spontaneous-breaking) mechanism which forces baryon-lepton admixtures to be small [5].

IV. PARITY-CONSERVING MODELS

It was remarked above that for renormalizable theories (which are the only ones we investigate here) parity conservation is impossible in the absence of a local symmetry. The truth of this observation becomes apparent upon examination of the only feasible (i.e. supersymmetric, fermion-number conserving and renormalizable) interaction in such a case, which is given explicitly by the formula (3.18). For a parity operation to be defined, the supermultiplets Φ_+ and Φ_- must be put into correspondence. Firstly, the spinor components can transform only according to a rule of the form:

$$\psi_+ \rightarrow \omega \gamma_0 \psi_- \quad . \tag{4.1}$$

where ω is some unitary matrix subject to the constraint $\omega^2 = \pm 1$. Secondly, the $F = 2$ spin-zero components, A_-, must transform among themselves,

$$A_- \rightarrow \omega' A_- \quad , \tag{4.2}$$

where ω' is another matrix like ω. Now consider the particular interaction term in (3.18),

$$A_-^\dagger \alpha_2 \psi_+ C^{-1} \psi_+ \quad .$$

Application of the transformations (4.1) and (4.2) to this term changes it to a form which is clearly not present in (3.18). This shows that interactions of the form (3.18) cannot support space reflections and justifies our remark. We shall prove now that if a local symmetry is present it is possible to set up a parity-conserving interaction.

For definiteness consider the case of local SU(n) symmetry. Since the gauge field Ψ — an $n \times n$ hermitian traceless matrix — contains a set of negative chirality fermions, λ_-, in the adjoint representation, it is necessary that the matter system should contain at least a set of positive chirality fermions, ζ_+, also in the adjoint representation. These fermions must belong to a (traceless) supermultiplet S_+ which we shall call the supplementary gauge fields. Under the action of local SU(n) the augmented gauge field system transforms according to

$$\begin{aligned} e^{2g\Psi} &\rightarrow e^{i\Lambda_+^\dagger} e^{2g\Psi} e^{-i\Lambda_+} \\ S_+ &\rightarrow e^{i\Lambda_+} S_+ e^{-i\Lambda_+} \quad , \end{aligned} \tag{4.3}$$

where Λ_+ is a traceless $n \times n$ matrix of positive chirality and g is a dimensionless coupling constant.

Other matter fields may also be present in the form of supermultiplets Φ_+ and Φ_-. It is necessary that Φ_+ and Φ_-^* should transform contragrediently. For simplicity we take just a pair of n-component columns transforming according to

$$\begin{aligned} \Phi_+ &\rightarrow e^{i\Lambda_+} \Phi_+ \\ \Phi_- &\rightarrow e^{i\Lambda_+^\dagger} \Phi_- \quad . \end{aligned} \tag{4.4}$$

The most general supersymmetric, fermion-number conserving and renormalizable Lagrangian for this system is given by

$$\begin{aligned} \mathcal{L} = & \frac{1}{8} \bar{D}D \left[\frac{1}{2} \mathrm{Tr} \left(\bar{\Psi}_- \Psi_+ + \bar{\Psi}_+ \Psi_- \right) \right] \\ & + \frac{1}{8} (\bar{D}D)^2 \left[\frac{1}{2} \mathrm{Tr} \left(S_+^\dagger e^{2g\Psi} S_+ e^{-2g\Psi} \right) \right] \\ & + \frac{1}{8} (\bar{D}D)^2 \left[\Phi_+^\dagger e^{2g\Psi} \Phi_+ + \Phi_-^\dagger e^{-2g\Psi} \Phi_- \right] \\ & - \frac{1}{2} \bar{D}D \left[\Phi_-^T (M + h S_+) \Phi_+ + \mathrm{h.c.} \right] \quad , \end{aligned} \tag{4.5}$$

where h is a dimensionless coupling constant and M is a mass.

In order to deal with the parity question it is necessary to express (4.5) in terms of the component fields. For $\Phi_\pm$ and Ψ we adopt the expansions (2.8) and (2.11), respectively, and for S_+ the expansion

$$S_+ = e^{-\frac{1}{4}\bar\theta\not\partial\gamma_5\theta}\left[a_+(x) + \bar\theta\zeta_+(x) + \frac{1}{4}\bar\theta(1+i\gamma_5)\theta\, f_+(x)\right] . \qquad (4.6)$$

For convenience of writing we introduce the negative chirality antifermion ζ_- defined as the conjugate of ζ_+, i.e.

$$\bar\theta\zeta_- = (\bar\theta\zeta_+)^\dagger = \bar\zeta_+\theta \qquad . \qquad (4.7)$$

This means that the sum, $\zeta = \zeta_+ + \zeta_-$, is a Majorana spinor like λ .

Applying the formulae (3.15), (3.18), (3.19) to the Lagrangian (4.5) one finds

$$\begin{aligned}
\mathcal{L} = \frac{1}{2}\,\mathrm{Tr}\Big[&-\frac{1}{4}U_{\mu\nu}^2 + \bar\lambda_-\, i\not\nabla\lambda_- + \frac{1}{2}D^2 \\
&+ \nabla_\mu a_+^\dagger\, \nabla_\mu a_+ + \bar\zeta_+ i\not\nabla\zeta_+ + f_+^\dagger f_+ \\
&+ i\sqrt{2}\,g\left([a_+^\dagger,\bar\lambda_-]\zeta_+ - \bar\zeta_+\,[\lambda_-,a_+]\right) + g\,a_+^\dagger\,[D,a_+]\Big] \\
&+ \nabla_\mu A_+^\dagger\,\nabla_\mu A_+ + \bar\psi_+ i\not\nabla\psi_+ + F_+^\dagger F_+ \\
&\quad + i\sqrt{2}\,g\left\{A_+^\dagger\bar\lambda_-\psi_+ - \bar\psi_+\lambda_- A_+\right\} + g\,A_+^\dagger D A_+ \\
&+ \nabla_\mu A_-^\dagger\,\nabla_\mu A_- + \bar\psi_- i\not\nabla\psi_- + F_-^\dagger F_- \\
&\quad + i\sqrt{2}\,g\left\{A_-^\dagger\bar\lambda_+\psi_- - \bar\psi_-\lambda_+ A_-\right\} - g\,A_-^\dagger D A_- \\
&+ M\left\{A_-^\dagger F_+ + F_-^\dagger A_+ - \bar\psi_-\psi_+ + \text{h.c.}\right\} \\
&+ h\,\Big\{F_-^\dagger a_+ A_+ + A_-^\dagger f_+ A_+ + A_-^\dagger a_+ F_+ \\
&\qquad - A_-^\dagger\bar\zeta_-\psi_+ - \bar\psi_- a_+\psi_+ - \bar\psi_-\zeta_+ A_+ + \text{h.c.}\Big\} \quad . \qquad (4.8)
\end{aligned}$$

where the various covariant derivatives are given by

$$\begin{aligned}
U_{\mu\nu} &= \partial_\mu U_\nu - \partial_\nu U_\mu - ig[U_\mu,U_\nu] \\
\nabla_\mu\lambda_- &= \partial_\mu\lambda_- - ig[U_\mu,\lambda_-] \\
\nabla_\mu a_+ &= \partial_\mu a_+ - ig[U_\mu,a_+] \\
\nabla_\mu\zeta_+ &= \partial_\mu\zeta_+ - ig[U_\mu,\zeta_+] \\
\nabla_\mu A_\pm &= \partial_\mu A_\pm - ig\,U_\mu A_\pm \\
\nabla_\mu\psi_\pm &= \partial_\mu\psi_\pm - ig\,U_\mu\psi_\pm \quad .
\end{aligned} \qquad (4.9)$$

The appearance of (4.8) can be greatly simplified if the coupling parameter h is given the specific value

$$h = g\sqrt{2} \quad . \qquad (4.10)$$

Indeed, with this restriction the system is parity conserving. This can be seen more easily by eliminating the auxiliary fields, $F_\pm$, f_+ and D , resolving a_+ into hermitian and antihermitian parts,

$$a_+ = \frac{1}{\sqrt{2}}\,(a + ib) \quad , \qquad (4.11)$$

and introducing 4-component spinors,

$$\begin{aligned}
\psi &= \psi_+ + \psi_- \quad , \\
\chi &= \zeta_+ + i\lambda_- \quad .
\end{aligned} \qquad (4.12)$$

The Lagrangian (4.8) then reduces to

$$\begin{aligned}
\mathcal{L} = \frac{1}{2}\,\mathrm{Tr}\Big[&-\frac{1}{4}U_{\mu\nu}^2 + \bar\chi i\not\nabla\chi + \frac{1}{2}(\nabla_\mu a)^2 + \frac{1}{2}(\nabla_\mu b)^2 \\
&\qquad + g\bar\chi\Big([a,\chi] + [b,\gamma_5\chi]\Big)\Big] \\
&+ \nabla_\mu A_+^\dagger\,\nabla_\mu A_+ + \nabla_\mu A_-^\dagger\,\nabla_\mu A_- + \bar\psi(i\not\nabla - M)\psi \\
&- g\,\bar\psi(a-\gamma_5 b)\psi - g\sqrt{2}\left\{\bar\psi\chi A_+ - i\bar\psi\gamma_5\chi^c A_- + \text{h.c.}\right\} \\
&- V \quad .
\end{aligned} \qquad (4.13)$$

where the covariant derivatives are easily deducible from (4.9), (4.11) and (4.12). The conjugate spinor χ^c is defined in the usual fashion

$$\bar{\theta}\chi^c = (\bar{\theta}\chi)^\dagger = \bar{\theta}(\zeta_- - i\lambda_+) \quad . \tag{4.14}$$

The potential V has the standard form (3.21),

$$V = F_+^\dagger F_+ + F_-^\dagger F_- + \frac{1}{2}\,\mathrm{Tr}\left(f_+^\dagger f_+ + \frac{1}{2}D^2\right) , \tag{4.15}$$

where the auxiliary fields are given by

$$\begin{aligned} F_+ &= -(M + g(a-ib))A_- \\ F_- &= -(M + g(a+ib))A_+ \\ f_+ &= -g\,2\sqrt{2}\left(A_-A_+^\dagger - \frac{1}{n}A_+^\dagger A_-\right) \\ D &= -2g\left(A_+A_+^\dagger - A_-A_-^\dagger + \frac{1}{2}[a,b]\right) + \frac{2g}{n}\left(A_+^\dagger A_+ - A_-^\dagger A_-\right) \quad . \end{aligned} \tag{4.16}$$

(Recall that f_+ and D are traceless $n \times n$ matrices.) On substituting the expressions (4.16) into (4.15) one finds

$$\begin{aligned} V = &-\frac{g^2}{4}\,\mathrm{Tr}[a,b]^2 \\ &+ A_+^\dagger\left[(M + ga)^2 + g^2b^2\right]A_+ + A_-^\dagger\left[(M + ga)^2 + g^2b^2\right]A_- \\ &+ g^2\left(1 - \frac{1}{n}\right)\left[(A_+^\dagger A_+)^2 + (A_-^\dagger A_-)^2\right] \\ &- 2g^2\left(1 + \frac{2}{n}\right)(A_+^\dagger A_-)(A_-^\dagger A_+) \\ &+ 2g^2\left(2 + \frac{1}{n}\right)(A_+^\dagger A_+)(A_-^\dagger A_-) \quad . \end{aligned} \tag{4.17}$$

Parity conservation in the Lagrangian (4.13) with the potential (4.17) is now manifest. For the spinors we take

$$\begin{aligned} \psi &\to \gamma_0\psi \\ \chi &\to \gamma_0\chi \quad \text{and} \quad \chi^c \to -\gamma_0\chi^c \quad . \end{aligned} \tag{4.18}$$

Among the bosons: U_μ is a vector; a, A_+ and A_- are scalars; b is a pseudoscalar. (The odd relative parity of χ and χ^c results from the definition (4.14).) Other parity assignments are equally feasible. Thus, instead of (4.18) one could take

$$\begin{aligned} \psi &\to \omega\gamma_0\psi \\ \chi &\to \omega\gamma_0\chi\omega^{-1} \quad \text{and} \quad \chi^c \to -\omega\gamma_0\chi^c\omega^{-1} , \end{aligned} \tag{4.19}$$

where ω is a unitary matrix satisfying $\omega^2 = 1$. The corresponding rules for the bosons are then:

$$\begin{aligned} U_0 &\to \omega\, U_0\, \omega^{-1} , \quad U_i \to -\omega\, U_i\, \omega^{-1} , \\ a &\to \omega\, a\, \omega^{-1} , \quad b \to -\omega\, b\, \omega^{-1} , \\ A_\pm &\to \omega\, A_\pm \quad . \end{aligned} \tag{4.20}$$

This completes the discussion of the parity-conserving local SU(n) model. Other local symmetries are treated in exactly the same way and any number of matter supermultiplets may be introduced in the form of contragredient pairs Φ_+ , Φ_-^* . In every case the couplings are determined entirely by the gauge principle.

It may eventually be of some interest to set up renormalizable, supersymmetric and parity-conserving models in which some of the vector particles carry two units of fermion number. We shall conclude this section by showing how such "exotic" models are constructed in general and how the fermion-number assignments are to be made.

Suppose we are given a set of matter fields Φ_+ and Φ_- in the form of n-component columns. What is the maximal parity-conserving local symmetry which can act upon this set? To answer this question it is convenient to combine Φ_+ with $\Phi_-^* = \Phi_+'$ into a 2n-component column - which we shall again denote simply Φ_+ . The maximal local symmetry which can act on this quantity is U(2n) but this will not be compatible with parity conservation. The problem is to find the largest possible subgroup of U(2n) which does admit a conserved parity operation.

In addition to the matter multiplet Φ_+ , the system must include the gauge potential Ψ and its supplementary part Σ_+ . The action of the group is given by

$$\Phi_+ \to e^{i\Lambda_+}\Phi_+$$
$$e^{2g\Psi} \to e^{i\Lambda_+^\dagger} e^{2g\Psi} e^{-i\Lambda_+}$$
$$S_+ \to e^{i\Lambda_+} S_+ e^{-i\Lambda_+} \quad . \tag{4.21}$$

Parity conservation requires that S_+ should couple to the matter multiplet. Renormalizability then requires that this coupling should be to the product $\Phi_+ \times \Phi_+$. The only such coupling would have the form

$$-\frac{1}{2}\bar{D}D\left[h\,\Phi_+^T\,\eta\,S_+\,\Phi_+ + \text{h.c.}\right] , \tag{4.22}$$

where Φ_+^T denotes the transpose of Φ_+ and η is a $2n \times 2n$ numerical matrix. Gauge invariance requires that η have the property

$$e^{i\Lambda_+^T}\,\eta\, e^{i\Lambda_+} = \eta \, . \tag{4.23}$$

Furthermore, since the product $\Phi_+ \times \Phi_+$ is necessarily symmetric, only the symmetric part of ηS_+ can couple. Any antisymmetric part of ηS_+ would be of no service to the parity question so we must impose the constraint that ηS_+ be symmetric

$$(\eta S_+)^T = \eta S_+ \quad . \tag{4.24}$$

Since S_+ belongs to the adjoint representation, this constitutes a constraint on the group structure. Indeed, the conditions (4.23) and (4.24) are sufficient to determine the maximal local symmetry. Let x denote an arbitrary element of the infinitesimal algebra. Then (4.23) and (4.24) imply, respectively,

$$x^T\eta + \eta x = 0$$
$$x^T\,\eta^T - \eta x = 0 \quad .$$

Since for the maximal case we can assume that x is a non-singular matrix, it follows that η must be antisymmetric. This means that the maximal parity-conserving symmetry is certainly contained in Sp(2n). To show that in fact it coincides with Sp(2n) we have only to write out in terms of component fields the Lagrangian,

$$\mathcal{L} = \frac{1}{8}\bar{D}D\left[\frac{1}{2}\mathrm{Tr}\left(\bar{\Psi}_-\Psi_+ + \bar{\Psi}_+\Psi_-\right)\right]$$
$$+ \frac{1}{8}(\bar{D}D)^2\left[\frac{1}{2}\mathrm{Tr}\left(S_+^\dagger\, e^{2g\Psi}\, S_+ e^{-2g\Psi}\right) + \Phi_+^\dagger\, e^{2g\Psi}\,\Phi_+\right]$$
$$-\frac{1}{2}\bar{D}D\left[\frac{g}{\sqrt{2}}\,\Phi_+^T\,\eta\,S_+\,\Phi_+ + \text{h.c.}\right] \quad . \tag{4.25}$$

Before doing this, however, we shall discuss the fermion-number assignments.

The Lagrangian (4.25) can be shown to be invariant under the γ_5 transformation, $\theta \to e^{\alpha\gamma_5}\theta$, acting as follows:

$$\Phi_+ \to e^{i\alpha(Q-1)}\,\Phi_+$$
$$S_+ \to e^{i\alpha Q}\, S_+\, e^{-i\alpha Q}$$
$$\Psi \to e^{i\alpha Q}\,\Psi\, e^{-i\alpha Q} \quad , \tag{4.26}$$

where Q belongs to the symplectic algebra. These formulae generalize the transformations (2.9), (2.12) and lead to exotic fermion-number assignments. One possible choice for the matrix Q , expressed in a canonical basis where

$$\eta = \mathrm{diag}\left(\tau_2^{(1)},\ldots,\tau_2^{(n)}\right) \tag{4.27}$$

is given by the diagonal matrix with alternating eigenvalues, ± 1 ,

$$Q = \mathrm{diag}\left(\tau_3^{(1)},\ldots,\tau_3^{(n)}\right) \quad . \tag{4.28}$$

It is a simple matter to show that in each of the two-dimensional sectors on which the τ matrices act the F-content is given by

$$A_+ = \begin{pmatrix} 0 \\ -2 \end{pmatrix} \qquad \psi_+ = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \qquad F_+ = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$
$$a_+ = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} \qquad \zeta_+ = \begin{pmatrix} 1 & 3 \\ -1 & 1 \end{pmatrix} \qquad f_+ = \begin{pmatrix} 2 & 4 \\ 0 & 2 \end{pmatrix}$$
$$U_\mu = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} \qquad \lambda_- = \begin{pmatrix} 1 & 3 \\ -1 & 1 \end{pmatrix} \qquad D = \begin{pmatrix} 0 & 2 \\ -2 & 0 \end{pmatrix} \quad . \tag{4.29}$$

Thus we have not only vectors with $F = 2$ but spinors with $F = 3$.

To express the Lagrangian (4.25) in a manifestly parity-conserving fashion it is necessary as before to resolve the matrix a_+ into hermitian and antihermitian parts,

$$a_+ = \frac{1}{\sqrt{2}} (a + ib)$$

and to introduce the 4-spinor

$$\chi = \zeta_+ + i\lambda_- \quad .$$

The matter spinor ψ_+ has no independent negative chirality relative, however. In this case we must define ψ_- by complex conjugation,

$$\bar{\theta}\psi_- = \eta(\bar{\psi}_+\theta)^T \quad . \tag{4.30}$$

The 4-spinor $\psi = \psi_+ + \psi_-$ is therefore subject to a Majorana-like constraint

$$\bar{\theta}\psi = -i\eta(\bar{\psi}\theta)^T \quad . \tag{4.31}$$

This constraint is of course consistent with the fermion-number assignments since Q is a symplectic generator. With these notational conventions the Lagrangian (4.25) assumes the form

$$\begin{aligned}\mathcal{L} = \frac{1}{2}\,\mathrm{Tr}\Big[&-\frac{1}{4}U_{\mu\nu}^2 + \bar{\chi}i\not{\nabla}\chi + \frac{1}{2}(\nabla_\mu a)^2 + \frac{1}{2}(\nabla_\mu b)^2 \\ &+ g\,\bar{\chi}\Big([a,\chi] + [b,\gamma_5\chi]\Big)\Big] \\ &+ \nabla_\mu A_+^\dagger \nabla_\mu A_+ + \frac{1}{2}\bar{\psi}i\not{\nabla}\psi - \frac{g}{2}\bar{\psi}(a - \gamma_5 b)\psi \\ &- g\sqrt{2}\,\Big(\bar{\psi}\chi A_+ + \text{h.c.}\Big) - V\,,\end{aligned} \tag{4.32}$$

where the covariant derivatives take the usual form and the potential V is given by

$$V = F_+^\dagger F_+ + \frac{1}{2}\,\mathrm{Tr}\Big(f_+^\dagger f_+ + \frac{1}{2}D^2\Big)\,, \tag{4.33}$$

where the auxiliary fields are expressed in terms of A_+ and a_+. Here it is convenient to introduce a complete set of $2n \times 2n$ symplectic generators, T^k, which satisfy the commutation rules,

$$[T^k, T^\ell] = i\, f^{k\ell m}\, T^m \quad ,$$

and are normalized as in (3.12). The auxiliary fields are given by

$$\begin{aligned}F_+ &= -g\eta(a - ib)A_+^* \\ f_+^{k*} &= -\frac{g}{\sqrt{2}} A_+^T\, \eta\, T^k A_+ \\ D^k &= -g\, A_+^\dagger T^k A_+ + \frac{i}{2} g\, \mathrm{Tr}(T^k[a,b]) \quad .\end{aligned} \tag{4.34}$$

On substituting these expressions into (4.33) one finds that the terms linear in b cancel and the potential reduces to the form

$$V = \frac{g^2}{2}\Big[(A_+^\dagger T^k A_+)^2 + |A_+^T \eta\, T^k A_+|^2 + \Big(\frac{i}{2}\mathrm{Tr}\, T^k[a,b]\Big)^2 + 2A_+^\dagger(a^2 + b^2)A_+\Big]\,. \tag{4.35}$$

The Lagrangian (4.32) with the potential given by (4.35) is clearly parity conserving.

V. A MODEL WITH LOCAL U(1) SYMMETRY

The simplest possible illustration of the type of model discussed in Sec.IV is provided by a system with local U(1) symmetry. It consists of a single pair of matter supermultiplets Φ_+ , Φ_- and the gauge pair Ψ and S_+ . The Lagrangian for this system takes the form (4.11)

$$\mathcal{L} = -\frac{1}{4} U_{\mu\nu}^2 + \bar{\chi} i \not\partial \chi + \frac{1}{2} (\partial_\mu a)^2 + \frac{1}{2} (\partial_\mu b)^2$$
$$+ \ |\nabla_\mu A_+|^2 + |\nabla_\mu A_-|^2 + \bar{\psi} i \not\nabla \psi$$
$$- g\bar{\psi}(a-\gamma_5 b)\psi - g\sqrt{2}\left[\bar{\psi}(\chi A_+ - i\gamma_5 \chi^c A_-) + \text{h.c.}\right] - V \ , \qquad (5.1)$$

where the potential is given by

$$V = |F_+|^2 + |F_-|^2 + |f_+|^2 + \frac{1}{2} D^2 \qquad (5.2)$$

with

$$F_\pm = -g(a \mp ib)A_\mp$$
$$f_+ = -g\sqrt{2}\, A_- A_+^*$$
$$D = \xi - g(|A_+|^2 - |A_-|^2) \ . \qquad (5.3)$$

On substitution of these expressions into (5.2) the potential reduces to

$$V = g^2(a^2+b^2)\ (|A_+|^2 + |A_-|^2) + 2g^2|A_+|^2\ |A_-|^2 + \frac{g^2}{2}\ \left\{\frac{\xi}{g} - |A_+|^2 + |A_-|^2\right\}^2 . \qquad (5.4)$$

In (5.1) the covariant derivatives take the form appropriate to an abelian symmetry, viz.

$$U_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu \ , \quad \nabla_\mu \psi = (\partial_\mu - igU_\mu)\psi \ , \quad \nabla_\mu A_\pm = (\partial_\mu - igU_\mu)A_\pm \ . \qquad (5.5)$$

Minimization of the classical potential (5.2) is trivial. Indeed one can see that all the auxiliary fields (5.3) vanish at the point

$$\langle a \rangle = \langle b \rangle = \langle A_- \rangle = 0 \ , \ \langle A_+ \rangle = \sqrt{\xi/g} \qquad (5.6)$$

provided $\xi/g > 0$. This point is non-degenerate and, since $\langle V \rangle = 0$, it is clearly an absolute minimum. The vacuum respects supersymmetry as well as parity and fermion number. The local symmetry is broken, however.

One can show that all particles in this model have a common mass. The free Lagrangian is easily obtained by substituting into (5.1) and (5.4) the shifted field

$$A_+ = \sqrt{\frac{\xi}{g}} + \frac{1}{\sqrt{2}}(A_1 + iA_2) \qquad (5.7)$$

and retaining only bilinear terms. One finds

$$\mathcal{L}_{(2)} = -\frac{1}{4} U_{\mu\nu}^2 + \frac{M^2}{2}\left[U_\mu - \frac{1}{M}\partial_\mu A_2\right]^2$$
$$+ \frac{1}{2}(\partial_\mu a)^2 + \frac{1}{2}(\partial_\mu b)^2 + \frac{1}{2}(\partial_\mu A_1)^2 - \frac{M^2}{2}(a^2 + b^2 + A_1^2)$$
$$+ \ |\partial_\mu A_-|^2 - M^2|A_-|^2$$
$$+ \ \bar{\chi} i \not\partial \chi + \bar{\psi} i \not\partial \psi - M(\bar{\psi}\chi + \bar{\chi}\psi) \ , \qquad (5.8)$$

where the common mass is given by

$$M = \sqrt{2g\xi} \ . \qquad (5.9)$$

The scalar field A_2 can be removed by a gauge transformation. One is left with two real scalars, A_1 and a ; a real pseudoscalar, b ; a scalar di-fermion, A_- ; a real vector, U_μ ; two Dirac spinors, $(\psi+\chi)/\sqrt{2}$ and $\gamma_5(\psi-\chi)/\sqrt{2}$, of opposite parity.

Interactions are characterized by a single dimensionless coupling constant g . The model is presumably renormalizable but we have not examined the quantum corrections. (Conceivably some of the symmetries which are present in the classical approximation may be lost in the higher orders.) Although this model can have no broader interest we have included it here as the simplest representative of a supersymmetric system in which both parity and fermion number are conserved.

APPENDIX

In this appendix we indicate briefly how a unified supersymmetric gauge theory of strong, weak and electromagnetic interactions may eventually be obtained using the ideas of this paper. What we wish to emphasise particularly is that, once the basic set of fermions is given, the structure of the Higgs-Kibble system - its representation content and the form of the potential - is largely determined.

To be definite consider the model of Pati and Salam[5] with its two basic fermion multiplets,

$$\psi_{1\pm} = \begin{pmatrix} p_a & p_b & p_c & \nu \\ n_a & n_b & n_c & e \end{pmatrix}_\pm$$
$$\psi_{2\pm} = \begin{pmatrix} \chi_a & \chi_b & \chi_c & \nu' \\ \lambda_a & \lambda_b & \lambda_c & \mu \end{pmatrix}_\pm \qquad \text{(A.1)}$$

which transform irreducibly under the action of $SU(2) \times SU(2) \times SU(4)$. The left-handed fields ψ_{1-} and ψ_{2-} belong to the representation $(2,1,\bar{4})$ while the right-handed fields ψ_{1+} and ψ_{2+} belong to $(1,2,\bar{4})$. (In practice, the right-handed $SU(2)$ may be replaced by $U(1)$, in which case the fields ψ_{1+} and ψ_{2+} transform reducibly. Likewise, the colour group $SU(4)$ may be restricted to $SU(3) \times U(1)$. For the present discussion we shall retain the full group structure.) These fermions are embedded in chiral supermultiplets $\phi_{1\cdot}$ and $\phi_{2\pm}$.

If there were no weak interactions it would be a simple matter to set up a parity-conserving system of coloured gluons. The gluons are accommodated in a gauge superfield Ψ belonging to the adjoint representation (1,1,15) and this would be accompanied by a supplementary supermultiplet S_+, also in (1,1,15). Parity would be conserved. However, when the weak interactions are taken into account, the product $\phi_{1+}\ \phi_{1-}^{+}$, for example, belongs to (2,2,1+15) and so cannot couple to the supplementary S_+. It is therefore essential to introduce an S_+ in the representation (2,2,15). Here the strong interaction 15-fold occurs four times. Our idea is that one combination of these can serve to restore parity conservation when weak interactions are neglected. The other three 15-folds in S_+ will inevitably cause parity violation and their effects at low energy must therefore be suppressed by ensuring that the particles described by these three 15-folds are sufficiently massive.

Now the extra three 15-folds in S_+ represent a surplus of right-handed fermions. Such a surplus means either that the system contains many massless fermions or else that fermion number is spontaneously violated. To evade this situation and to ensure that these particles are indeed massive, we propose to introduce a comparable number of left-handed fermions in a supermultiplet S_-, belonging to (1,3,15). (The three 15-fold right-handed fermions in S_+ and left-handed fermions in S_- will be capable of combining into Dirac 4-component spinors if suitable interaction terms between them exist in the theory.) There is yet a further balancing to perform. Contained in the gauge fields Ψ_1 and Ψ_2 associated with $SU(2) \times SU(2)$ are six left-handed fermions. To compensate these we propose to introduce right-handed fermions in the multiplets

$$S_+' \sim (2,2,1) \quad \text{and} \quad S_+'' \sim (1,3,1) \quad .$$

There is now a surplus of one right-handed fermion and so, finally, we introduce a left-handed singlet

$$S_{0-} \sim (1,1,1) \quad .$$

This singlet is of particular importance since it is the only field in the entire system which can involve a dimensional coupling in the Lagrangian and thus set the scale of masses.

The importance of the multiplet S_+' stems from the fact that it possesses a trilinear interaction with S_+ and S_-. It is from this interaction ($\approx \langle S_+' \rangle\ S_+ S_-^{+}$) that the heavy masses of the three 15-folds in S_+ and S_- are expected to emerge.

This scheme is of course highly speculative. Our hope would be that all the fields associated with parity violation will acquire large masses in zeroth order while those in the parity-conserving sector acquire their smaller masses through a radiative mechanism. Other mass scales could be introduced by restricting the colour group to $SU(3) \times U(1)$ or extending it to $SU(4) \times U(1)$ or by restricting the right-handed $SU(2)$ to $U(1)$. (The leptonic model of Fayet[2] is a theory of local $SU(2) \times U(1)$ in which $S_+' = (2,1+1)$ and $S_{0-} = (1,1)$. There are two mass scales and a surplus of one left-handed fermion. In this simple model there is no need to rely on radiative mechanisms to fix the structure of the theory. The one surplus left-handed fermion is bound to remain massless and provides the neutrino of

Fayet's model. What would be even more interesting to investigate is the question if in the model described in this appendix (where the number of left- and right-handed spinors is equal) a spontaneous breakdown of supersymmetry gives rise to a 4-component massless particle, and if this 4-component object appropriately splits into the two observed chiral neutrinos.)

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