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A SUPERSYMMETRIC GLUON MODEL *

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ABSTRACT

It is pointed out that parity doubling does not provide a satisfactory resolution of the conflict between parity and fermion-number conservation in supersymmetric gauge theories. A new generalized gauge principle is proposed which overcomes this difficulty for both abelian and non-abelian local symmetries.

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I. INTRODUCTION

The purpose of this note is to exhibit a supersymmetric model with global $SU(n) \times SU(n)$ and local $U(1)$ invariance which conserves both parity and fermion number without introducing parity doubling.*) In the last section we generalize the new method to non-abelian local symmetries.

For the construction of supersymmetric and renormalizable Lagrangian models there are available essentially two types of field**). Firstly, there is the chiral superfield, $\Phi_+(x,\theta)$, which includes two complex scalar fields, $A_+(x)$ and $F_+(x)$, and a positive-chirality spinor $\psi_+(x)$. (Closely related to this is the superfield $\Phi_-(x,\theta)$ containing two scalars, $A_-(x)$ and $F_-(x)$, and a negative-chirality spinor $\psi_-(x)$.) The fields A_+ , ψ_+ and F_+ (or A_- , ψ_- and F_-) must belong to the same representation of any given internal symmetry group. Secondly, there is the gauge superfield $\Psi(x,\theta)$, which (in a suitable gauge) includes a vector, U_μ , a negative-chirality spinor, λ_- , and a scalar, D , all belonging to the adjoint representation of the internal local symmetry. (The fields $F_\pm$ and D play an auxiliary role and are not associated directly with particles.)

To our knowledge there is only one way to assign a conserved fermion (baryon or lepton) number to these fields ***). This is by means of the γ_5 transformations on the Majorana spinor co-ordinate θ , viz. $\theta \to e^{\alpha\gamma_5}\theta$. On defining the negative-chirality gauge spinor λ_- to be a "fermion", one finds that the components (A_+ , ψ_+ , F_+) must carry the respective fermion numbers (0,1,2). Similarly, the components (A_- , ψ_- , F_-) must carry (2,1,0). That is, a negative-chirality fermion (or positive-chirality anti-fermion) is associated through supersymmetry with an ordinary boson A_+ ; a positive-chirality fermion (or negative-chirality antifermion), however, is associated with a "di-fermion" A_- (or antidi-fermion A_-^*). It is in this supersymmetric correspondence of fermion-number zero bosons A_+ with fermion-number two bosons A_- that all the conflicts of parity versus fermion-number conservation lie.

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*) In previous work it has not been realized that parity doubling does not provide a way out of conflicting demands of parity and fermion-number conservation in supersymmetric theories. This will be discussed in a forthcoming paper "Supersymmetry and unified gauge theories".

**) Here we shall follow the notation and techniques set out in Ref.1.

***) The use of γ_5 transformations for this purpose was suggested by Salam and Strathdee and, independently, by Fayet in Ref.2. (Special instances of this application (not recognized at the time as such) were given by Salam and Strathdee, by Ferrara and Zumino and by Delbourgo, Salam and Strathdee, in Ref.2.

This rigid linking of fermion number to chirality appears to be a fundamental feature of supersymmetry. On the face of it one cannot expect to have both parity and fermion number conserved in such theories. Of course it would be possible to restore reflection symmetry by doubling the set of fields and assigning fermion number in the mirror world thus created by means of the reflected γ_5 transformations $\theta \to e^{-\alpha\gamma_5}\theta$. This parity doubling does not yield any fundamental improvement, however, since there can be no communication between the world and its mirror image without violating either fermion number of supersymmetry explicitly.

The model to be discussed here is designed so as to avoid the dilemmas of parity doubling. We shall in fact show that by equating two coupling constants - which are independent insofar as supersymmetry and fermion number are concerned - one is enabled to define a conserved parity without violating fermion number.

II. THE MODEL

The model contains two matter superfields, Φ_+ and Φ_-, which belong to the representation $(n,\bar{n})$ of global $SU(n) \times SU(n)$ and transform according to

$$\Phi_+ \to e^{i\Lambda_+}\Phi_+ \quad \text{and} \quad \Phi_- \to e^{i\Lambda_+^*}\Phi_- \tag{1}$$

under local U(1). In addition to the gauge potential Ψ, which is a global singlet and transforms like a local gauge field,

$$\Psi \to \Psi + \frac{i}{2g}(\Lambda_+^* - \Lambda_+) , \tag{2}$$

there is the local and global scalar S_+. The most general supersymmetric, γ_5-invariant and renormalizable Lagrangian is given by

$$\begin{aligned} &\frac{1}{8}(\bar{D}D)^2 \left[e^{2g\Psi}\,\mathrm{Tr}(\Phi_+^\dagger\Phi_+) + e^{-2g\Psi}\,\mathrm{Tr}(\Phi_-^\dagger\Phi_-) + S_+^* S_+\right] \\ &-\frac{1}{2}(\bar{D}D)\left[(m + h\,S_+)\,\mathrm{Tr}(\Phi_-^\dagger\Phi_+) + \text{h.c.}\right] \end{aligned} \tag{3}$$

plus the gauge field terms. Introducing components for the various superfields by

$$\begin{aligned} \Phi_\pm &= e^{\mp\frac{1}{4}\bar{\theta}\partial\!\!\!/\gamma_5\theta}\left(A_\pm + \tfrac{1}{2}\bar{\theta}(1 \pm i\gamma_5)\psi_\pm + \tfrac{1}{4}\bar{\theta}(1 \pm i\gamma_5)\theta F_\pm\right) \\ S_+ &= e^{-\frac{1}{4}\bar{\theta}\partial\!\!\!/\gamma_5\theta}\left(a_+ + \tfrac{1}{2}\bar{\theta}(1+i\gamma_5)\zeta_+ + \tfrac{1}{4}\bar{\theta}(1+i\gamma_5)\theta f_+\right) \\ \Psi &= \tfrac{1}{4}\bar{\theta}i\gamma_\nu\gamma_5\theta\, V_\nu + \frac{1}{2\sqrt{2}}\bar{\theta}\theta\bar{\theta}\lambda + \frac{1}{16}(\bar{\theta}\theta)^2 D , \end{aligned} \tag{4}$$

where $\lambda = \lambda_+ + \lambda_-$ is a Majorana spinor, and the gauge bosons V_μ and D are real, one can write the Lagrangian in the form

$$\begin{aligned} \mathcal{L} = &-\tfrac{1}{4}V_{\mu\nu}^2 + \bar{\lambda}_- i\partial\!\!\!/\lambda_- + \tfrac{1}{2}D^2 - \xi D \\ &+ \mathrm{Tr}\Big[\nabla_\mu A_+^\dagger \nabla_\mu A_+ + \bar{\psi}_+ i\nabla\!\!\!\!/\,\psi_+ + F_+^\dagger F_+ \\ &\quad + ig\sqrt{2}\left(A_+^\dagger\bar{\lambda}_-\psi_+ - \bar{\psi}_+\lambda_- A_+\right) + gD\,A_+^\dagger A_+ \\ &\quad + \nabla_\mu A_-^\dagger \nabla_\mu A_- + \bar{\psi}_- i\nabla\!\!\!\!/\,\psi_- + F_-^\dagger F_- \\ &\quad + ig\sqrt{2}\left(A_-^\dagger\bar{\lambda}_+\psi_- - \bar{\psi}_-\lambda_+ A_-\right) - gD\,A_-^\dagger A_-\Big] \\ &+ |\partial_\mu a_+|^2 + \bar{\zeta}_+ i\partial\!\!\!/\zeta_+ + |f_+|^2 \\ &+ m\,\mathrm{Tr}\left[A_-^\dagger F_+ + F_-^\dagger A_+ - \bar{\psi}_-\psi_+ + \text{h.c.}\right] \\ &+ h\,\mathrm{Tr}\Big[a_+(A_-^\dagger F_+ + F_-^\dagger A_+) + f_+ A_-^\dagger A_+ \\ &\qquad - a_+\bar{\psi}_-\psi_+ - \bar{\psi}_- A_+\zeta_+ - \bar{\zeta}_- A_-^\dagger\psi_+ + \text{h.c.}\Big] , \end{aligned} \tag{5}$$

where $\zeta_- = C\,\overline{\zeta}_+^T$, a negative-chirality antifermion, is defined such that the sum $\zeta = \zeta_+ + \zeta_-$ is a Majorana spinor of the same type *) as λ .

The Lagrangian (5) respects fermion number but not parity. However, parity conservation is restored if we make the identification

$$b = g\sqrt{2} \tag{6}$$

and define the Dirac fermions

$$\psi = \psi_+ + \psi_-$$

$$\chi = \zeta_+ + i\lambda_-$$

and the antifermion **)

$$\chi^c = \zeta_- + i\lambda_+ \quad . \tag{7}$$

It is useful also to resolve the singlet a_+ into real and imaginary parts:

$$a_+ = \frac{1}{\sqrt{2}}\,(a + ib) \quad . \tag{8}$$

Making these substitutions in (5) and eliminating the auxiliary fields $F_\pm$ and D , the Lagrangian takes the form:

$$\begin{aligned} \mathcal{L} = & -\tfrac{1}{4}V_{\mu\nu}^2 + \overline{\chi}\, i\not{\partial}\chi + \tfrac{1}{2}(\partial_\mu a)^2 + \tfrac{1}{2}(\partial_\mu b)^2 \\ & + \mathrm{Tr}\Big[\nabla_\mu A_+^\dagger \nabla_\mu A_+ + \nabla_\mu A_-^\dagger \nabla_\mu A_- + \overline{\psi}(i\not{\nabla} - m)\psi \\ & \quad - g\,\overline{\psi}(a - \gamma_5 b)\psi - g\sqrt{2}\big(\overline{\psi}A_+\chi + \overline{\psi}A_-\chi^c + h.c.\big)\Big] \\ & - V(A_+, A_-, a, b) \,, \end{aligned} \tag{9}$$

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*) That is, $\zeta = +C\overline{\zeta}^T$ and $\lambda = +C\overline{\lambda}^T$.

**) Notice that $\chi_+^c = -C\overline{\chi}_-^T$, $\chi_-^c = C\overline{\chi}_+^T$, i.e. $\chi^c = -i\gamma_5\, C\overline{\chi}^T$. The factor $-i\gamma_5$ serves to give χ^c even parity relative to χ and leads to the designation of A_- as a scalar. It could be left out, in which event A_- becomes a pseudoscalar (see Eq.(9) and footnote on p.6.)

where the potential is given by

$$V = \tfrac{1}{2}D^2 + \mathrm{Tr}(F_+^\dagger F_+ + F_-^\dagger F_-) \tag{10}$$

with

$$D = \xi - g\,\mathrm{Tr}(A_+^\dagger A_+ - A_-^\dagger A_-)$$

$$F_+ = -\big[m + g(a-ib)\big]\, A_-$$

$$F_- = -\big[m + g(a+ib)\big]\, A_+ \quad , \tag{11}$$

i.e.

$$V = \frac{1}{2}\Big[\xi - g\,\mathrm{Tr}(A_+^\dagger A_+ - A_-^\dagger A_-)\Big]^2 + \Big[(m + ga)^2 + g^2b^2\Big]\,\mathrm{Tr}(A_+^\dagger A_+ + A_-^\dagger A_-) \;. \tag{12}$$

The covariant derivatives which appear in (9) are all of the same type,

$$\nabla_\mu A_+ = \partial_\mu A_+ - ig\, V_\mu A_+ \quad ,$$

etc. It follows that the Lagrangian (9) is invariant under space reflections with V_μ a vector, a and b scalar and pseudoscalar, respectively, A_+ and A_- scalars, and the spinors ψ, χ and χ^c all transforming like *) $\psi \to \gamma_0\psi$.

To summarize, the system contains the scalars A_+ and A_- in $(n,\overline{n})$; the singlet scalar, a , and pseudoscalar, b ; the singlet vector V_μ ; the Dirac spinors ψ and χ in $(n,\overline{n})$ and $(1,1)$, respectively. The bosons A_- carry fermion number $F = 2$. The fields A_+ , A_- and ψ are coupled minimally to V_μ . There is one dimensionless coupling constant, g , and two parameters, m^2 and ξ , with the dimension $(\mathrm{mass})^2$.

It is easily seen that the potential (12) has an absolute minimum $\langle V \rangle = 0$ given by

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*) In terms of the chiral components: $\psi_+ \to \gamma_0\psi_-$, $\psi_- \to \gamma_0\psi_+$, $\lambda_- \to -i\gamma_0\zeta_+$, $\zeta_+ \to i\gamma_0\lambda_-$, $\lambda_+ \to -i\gamma_0\zeta_-$ and $\zeta_- \to i\gamma_0\lambda_+$. These rules are of course compatible with the Majorana constraints $\lambda_+ = C\overline{\lambda}_-^T$ and $\zeta_- = C\overline{\zeta}_+^T$.

$$\langle a \rangle = -\frac{m}{g}$$

$$\langle b \rangle = 0$$

$$\langle A_- \rangle = 0$$

$$\mathrm{Tr}\left(\langle A_+^\dagger \rangle \langle A_+ \rangle \right) = \frac{\xi}{g} \tag{13}$$

provided ξ/g is positive. The vacuum therefore respects supersymmetry, parity and fermion number but violates the global $SU(n) \times SU(n)$. The minimum is degenerate, however, and one cannot discover the residual symmetry without taking into consideration at least the one-loop corrections to V. The local symmetry is of course broken and the gluon acquires a mass.

To conclude we indicate briefly how the system is generalized for non-abelian local symmetries. Suppose we have a pair of matter multiplets Φ_+ and Φ_- (which transform like the fundamental representation of SU(n)),

$$\Phi_+ \to e^{i\Lambda_+} \Phi_+ \quad , \quad \Phi_- \to e^{i\Lambda_+^\dagger} \Phi_- \quad ,$$

so that $\Phi_-^\dagger \Phi_+$ is invariant. In addition to the usual gauge potential, Ψ - a hermitian traceless $n \times n$ matrix - which transforms according to

$$e^{2g\Psi} \to e^{i\Lambda_+^\dagger} e^{2g\Psi} e^{-i\Lambda_+} \quad ,$$

we introduce a subsidiary traceless matrix of "gauge" fields, S_+, in the adjoint representation,

$$S_+ \to e^{i\Lambda_+} S_+ e^{-i\Lambda_+} \quad .$$

The invariant, supersymmetric and fermion-number-conserving Lagrangian for this system takes the form

$$\frac{1}{8}(\bar{D}D)^2 \left[\Phi_+^\dagger e^{2g\Psi} \Phi_+ + \Phi_-^\dagger e^{-2g\Psi} \Phi_- + \mathrm{Tr}\left(S_+^\dagger e^{2g\Psi} S_+ e^{-2g\Psi} \right) \right]$$

$$- \frac{1}{2} \bar{D}D \left[M \Phi_-^\dagger \Phi_+ + h \Phi_-^\dagger S_+ \Phi_+ + h.c. \right]$$

plus the kinetic terms for Ψ. It can be shown, after a certain amount of labour, that the identification $h = g\sqrt{2}$ suffices to admit a conserved parity. What happens is that the $n^2 - 1$ fermions λ_- from Ψ and ζ_+ from S_+ combine to make $n^2 - 1$ Dirac spinors $\chi = \zeta_+ + i\lambda_-$. These Dirac spinors transform according to $\chi \to \gamma_0 \chi$ under space reflections.

What we appear to have here is a generalized gauge principle in which the basic gauge fields are comprised in two seemingly independent supermultiplets Ψ and S_+. Clearly they are not independent, however, and we expect that a deeper understanding of the supersymmetry formalism will allow them to be treated in a manifestly unified manner.

REFERENCES

1) Abdus Salam and J. Strathdee, ICTP, Trieste, preprint IC/74/42, to be published in Phys. Rev. D.

2) Abdus Salam and J. Strathdee, ICTP, Trieste, preprint IC/74/126, to be published in Nucl. Phys. B;
P. Fayet, preprint PTENS 74/7, to be published in Nucl. Phys. B;
Abdus Salam and J. Strathdee, Phys. Letters 51B, 363 (1974);
S. Ferrara and B. Zumino, Nucl. Phys. B79, 413 (1974);
R. Delbourgo, Abdus Salam and J. Strathdee, Phys.Letters 51B, 475 (1974).

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