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# A THEOREM CONCERNING GOLDSTONE FERMIONS *

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ABSTRACT

For any given model Lagrangian with spontaneous supersymmetry breaking we give the composition of the expected Goldstone fermion in terms of fields in the model.

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The concept of a Goldstone fermion which must accompany a spontaneously broken supersymmetry was formulated in Ref.1. Subsequently, explicit models were constructed which exhibit the emergence of such fermions [2]. It is our purpose in this note to prove a general theorem, for any given model, concerning the composition of such fermions in terms of the fields and their expectation values. We wish to consider in particular models which contain a number of matter and gauge supermultiplets corresponding to internal symmetry groups of the usual kind.

To begin with, we summarize the analogous and well-understood case of the Goldstone bosons [3] which accompany spontaneous breakdown of a global internal symmetry. Given a set of spin-zero fields $\phi^i(x)$ belonging to some representation of a global symmetry, with

$$\delta\phi^i = \omega^\alpha (T_\alpha)^i_j \, \phi^j ,$$

the invariance of the effective action $V(\phi)$ implies:

$$0 = \frac{\partial V}{\partial \phi^j} (T_\alpha)^j_k \, \phi^k . \tag{1}$$

One can now deduce the following theorem.

If for some $\alpha$ we have

$$(T_\alpha)^i_j \langle \phi^j \rangle \neq 0 ,$$

where $\langle \phi^i \rangle$ denotes the expectation value of $\phi^i$ in the physical vacuum, then there exists a zero-mass particle associated with this value of $\alpha$, whose composition is given by

$$G_\alpha(x) = \frac{1}{N} \sum_i \phi^i(x) \, (T_\alpha)_{ij} \langle \phi^j \rangle .$$

Here $N$ is a normalization factor. On differentiating (1) with respect to $\phi^\ell$, the theorem is easily proved [3] by considering the boson mass matrix $\left( \frac{\partial^2 V}{\partial \phi^i \partial \phi^j} \right)_{\phi^i = \langle \phi^i \rangle}$ and showing that $G_\alpha(x)$ is an eigenvector corresponding to a zero eigenvalue.

Consider supersymmetry now. The action of a supertranslation on a set of real supermultiplets, in the absence of gauge interactions, is given by

$$\begin{aligned}
\delta A_j &= \bar{\epsilon}\psi_j \\
\delta B_j &= \bar{\epsilon}\gamma_5\psi_j \\
\delta\psi_j &= \left(F_j - \gamma_5 B_j - i\not{\partial}(A_j + \gamma_5 B_j)\right)\epsilon \\
\delta F_j &= \bar{\epsilon}\left(-i\not{\partial}\psi_j\right) \\
\delta G_j &= \bar{\epsilon}\left(-i\not{\partial}\gamma_5\psi_j\right) ,
\end{aligned}$$

where $\epsilon$ is an infinitesimal Majorana spinor [4]. The components $A_j$, $B_j$, $F_j$ and $G_j$ are real while $\psi_j$ is a Majorana spinor (the index $j$ referring to the internal symmetry group, which for illustrative purposes we have taken as orthogonal $O(n)$). In the presence of gauge interactions these rules must be modified to read [*]:

$$\begin{aligned}
\delta A_j &= \bar{\epsilon}\psi_j \\
\delta B_j &= \bar{\epsilon}\gamma_5\psi_j \\
\delta\psi_j &= \left(F_j - \gamma_5 B_j - i\not{\nabla}(A_j + \gamma_5 B_j)\right)\epsilon \\
\delta F_j &= \bar{\epsilon}\left(-i\not{\nabla}\psi_j + ig(\lambda A)_j + ig\gamma_5(\lambda B)_j\right) \\
\delta G_j &= \bar{\epsilon}\left(-i\not{\nabla}\gamma_5\psi_j + ig(\lambda B)_j - ig\gamma_5(\lambda A)_j\right) ,
\end{aligned}$$

where the Yang-Mills covariant derivatives are defined by

$$\nabla_\mu A_j = \partial_\mu A_j - ig\,(W_\mu A)_j \quad ,$$

[*] These formulae are valid in the special gauge of Wess and Zumino who gave the Abelian formulation in Ref.5.



etc. Here $W_\mu$ is a real antisymmetric matrix as is also $\lambda$. These rules must be supplemented by the transformation formulae for the gauge "potentials" $W_\mu$, $\lambda$ and $D_5$:

$$\begin{aligned}
\delta W_\mu &= -i\bar{\epsilon}\gamma_\mu\lambda \\
\delta\lambda &= \left(-\gamma_5 D_5 + \tfrac{1}{2}\sigma_{\mu\nu}W_{\mu\nu}\right)\epsilon \\
\delta D_5 &= -i\bar{\epsilon}\not{\nabla}\gamma_5\lambda \quad ,
\end{aligned}$$

where

$$\begin{aligned}
W_{\mu\nu} &= \partial_\mu W_\nu - \partial_\nu W_\mu - ig\,[W_\mu, W_\nu] \\
\nabla_\mu\lambda &= \partial_\mu\lambda - ig\,[W_\mu, \lambda] \quad .
\end{aligned}$$

These transformation rules can be used to deduce Ward identities for the current $S_{\mu\alpha}$ (a Rarita-Schwinger spinor) which generates supertranslations. In particular, we find [*]

$$\begin{aligned}
\partial_\mu \langle T^* S_\mu(x)\ \bar{\psi}_j(0)\rangle &= i\delta_4(x)\langle F_j - \gamma_5 G_j\rangle \\
\partial_\mu \langle T^* S_\mu(x)\ \bar{\lambda}_{jk}(0)\rangle &= i\delta_4(x)\langle -\gamma_5 D_{5jk}\rangle \quad .
\end{aligned}$$

Now spontaneous supersymmetry breakdown can be effected by non-vanishing $\langle F\rangle$, $\langle G\rangle$ or $\langle D\rangle$'s. From the form of these identities, clearly the Goldstone fermion $G(x)$ must have the composition

$$G(x) = \frac{1}{N}\left[\sum_j \langle F_j - \gamma_5 G_j\rangle\ \psi_j(x) + \frac{1}{2}\sum_{jk}\langle -\gamma_5 D_{5jk}\rangle\ \lambda_{jk}\right] ,$$

where $N$ is a normalization factor. The result can be verified (analogous to the Goldstone boson case) by considering the Fermi mass matrix, with matrix elements

$$\frac{\partial^2 V}{\partial\psi_i\,\partial\bar{\psi}_j} \ , \quad \frac{\partial^2 V}{\partial\lambda_{ij}\,\partial\bar{\psi}_k} \ , \quad \frac{\partial^2 V}{\partial\lambda_{ij}\,\partial\bar{\lambda}_{k\ell}} \quad .$$

[*] The first of these identities was given by Abdus Salam and J. Strathdee in Ref.1.

Although the above result has been stated for an orthogonal internal symmetry group, it is in fact quite general. Its significance emerges if we identify the Goldstone spinor with the neutrino. In a unified supersymmetric theory of weak, strong and electromagnetic interactions, where the leptonic, baryonic as well as the electric charge content of matter and gauge fields $\psi_i$ and $\lambda_{ij}$ is pre-specified, the identification of $G(x)$ with one of the neutrinos will uniquely determine which of the expectation values $\langle F_i \rangle$, $\langle G_i \rangle$ or $\langle D_{ij} \rangle$ must be non-zero; thereby specifying the manner in which supersymmetry will have to break spontaneously. As noted by Fayet [6], the basic supersymmetric group (with one current $S_{\mu\alpha}(x)$) can give rise to just one Goldstone fermion and therefore (barring parity-doubling) just one massless neutrino. To admit of more than one massless neutrino through the mechanism of this note, the supersymmetry group will need to be extended to contain more currents $S^i_{\mu\alpha}(x)$. The resulting models will probably be non-renormalizable.



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