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ABSTRACT

We investigate the possibility of transition to a symmetry-preserving phase in K^0 phenomena, in strong external magnetic fields.

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I. ESTIMATES OF CRITICAL FIELDS

Most current theories of particle symmetries assume that violations of these symmetries come about through a spontaneous breaking-mechanism which produces non-zero expectation values for certain scalar (elementary or composite) fields. As is well known [1] these expectation values may make a phase transition to a zero value for certain critical temperatures and possibly also for certain critical external magnetic field strengths H_c, H_{c_1}, H_{c_2}, ... It is the purpose of this note to point out that it is conceivable that the charge asymmetry (associated with CP violation) in $K_L \to \pi^\pm + \ell^\mp + \bar{\nu}(\nu)$ decays may disappear for fields $\approx 8 \times 10^{10}$ G(auss) if CP violation is milli-weak in character, while the Cabibbo angle may be reduced to zero - leading to suppression of certain hyperon decays - in fields of the order of 10^{16} G. These estimates are so strongly model-dependent that it may be worthwhile in any case to make a systematic phenomenological search for effects on particle asymmetries of strong magnetic fields of 10^6 G upwards.

The notion of critical fields (above which spontaneously broken symmetries are restored) is well known from the Ginzburg-Landau theory of superconductivity, which is also the prototype of spontaneous symmetry-breaking mechanisms employed in particle physics. An order of magnitude estimate of the mean critical field H_c in this theory is provided by considering the free energy:

$$F = F_{normal} + \alpha(T)\,|\phi|^2 + \frac{\lambda(T)}{2}\,|\phi|^4 + \cdots \qquad (1)$$

For temperature $T < T_c$, F has a minimum when

$$|\langle\phi\rangle|^2 = -\frac{\alpha}{\lambda} \; . \qquad (2)$$

The "thermodynamic" critical field close to T_c is then given by [2]:

$$\frac{H_c^2(T)}{8\pi} = F_{normal} - F_{superconducting} = \frac{1}{2}\,\frac{\alpha^2}{\lambda} = \frac{1}{2}\,\lambda\,|\langle\phi\rangle|^4 \; . \qquad (3)$$

Now in superconductivity theory the (Cooper-pair) field ϕ is itself charged and the magnetic field interacts directly with it. For K^0 fields (which we wish to investigate in this note) this is not the case. However (in an

SU(3)-symmetric theory) the magnetic field does interact in higher order loop graphs. To see the qualitative emergence of a formula like (3), we consider (for T = 0) in Sec.II an O(3)-symmetric gauge model, where the electromagnetic field is part of a gauge-triplet. Even though we are not dealing with a thermodynamic system, a critical field H_c appears to exist in a one-loop approximation above which the expectation value of the neutral field $\langle\phi_3\rangle$ vanishes – the relation between H_c and $\langle\phi_3\rangle$ having the general form of formula (3). We shall employ this formula for order of magnitude estimates in this note. Clearly H_c will be small for those situations where $\langle\phi\rangle$ and λ are small.

Now for strong interaction symmetries like SU(2) or SU(3), $\langle\phi\rangle$'s are typically $\approx$ BeV (or somewhat less), with $\lambda \approx 1$. However, for weak interaction symmetries – and these provide the more spectacular physical situations – with universal gauge coupling of order unity ($g^2 \approx \frac{4\pi}{137}$, $g \approx \frac{1}{3}$), $\langle\phi\rangle$'s are typically tens or hundreds of BeV in magnitude, except for "off-diagonal" situations. To illustrate what we mean, consider the $\langle\phi\rangle$'s associated with real rotations of the n-Λ system (Cabibbo angle) and $\langle\phi\rangle$'s associated with complex rotations, which signal CP violation in a <u>milli-weak</u> manner.

To estimate the magnitude of H_c for these cases – assuming that it exists – consider the ideas of Lee, Pais, Primack [3] or Mohapatra and Pati [4]. In their milli-weak theory, the latter authors write the mass matrix for the n,Λ quark system in the form:

$$f(\bar{n}\ \bar{\Lambda})_L\ U_L \begin{pmatrix} m_n & 0 \\ 0 & m_\Lambda \end{pmatrix} V_R^{-1} \begin{pmatrix} n \\ \Lambda \end{pmatrix}_R \quad , \tag{4}$$

where U_L and V_R are matrices of the type:

$$\begin{vmatrix} \cos\theta_{L,R} & -\sin\theta_{L,R}\, e^{i\delta_{L,R}} \\ \sin\theta_{L,R}\, e^{i\delta_{L,R}} & \cos\theta_{L,R} \end{vmatrix} \tag{5}$$

with θ and δ real. The real part of the off-diagonal elements of the expectation-value matrix $U_L \begin{vmatrix} m_n & 0 \\ 0 & m_\Lambda \end{vmatrix} V_R^{-1}$ (assuming for simplicity $\theta_L = \theta_R$) is given by:

$$\frac{1}{2}\ (m_\Lambda - m_N)\ \sin 2\theta_c\ \cos\frac{\varepsilon}{2} \quad , \tag{6}$$

where $\delta_L = -\delta_R = \varepsilon/2$. Now, from CP violation, ε in this model can be

as small as 2×10^{-3}. With $m_\Lambda - m_N \approx 175$ MeV, $\theta_c \sim 15^\circ$ and assuming f in (4) to be typically ≈ 1, we obtain:

$$\mathrm{Re}\ \langle\phi\rangle\big|_{\text{off-diagonal}} \approx 45\ \mathrm{MeV} \quad . \tag{7}$$

Using formula (3) and making the <u>ad hoc</u> assumption that the numerical constant (λ) appearing there is of order unity, we obtain

$$H_c \approx 3 \times 10^{16}\ \mathrm{G} \quad , \tag{8}$$

(using the conversion formula $\langle\phi\rangle$ (MeV) = 0.3 H (kilogauss) cm).

If fields of this magnitude are applied, the real part of the "off-diagonal" expectation value may vanish and, with it, the Cabibbo angle, suppressing certain hyperon and strange-particle decays.

Also, the imaginary part of the off-diagonal matrix element of the mass matrix is $\frac{f}{2}(m_\Lambda + m_N) \sin 2\theta_c \sin\varepsilon$. For a heavy quark model ($m_\Lambda \sim 5$ GeV) this expectation value may make a transition to a zero value for fields $H_c \approx 10^{14}$ G – and with it the (milli-weak) CP asymmetry. *) For a light quark model ($m_\Lambda \sim 300$ MeV) this may reduce to 8×10^{10} G.

Since in these estimates we have assumed, for simplicity, that all model-dependent constants like f and λ are $\approx$ unity and, further, since formula (3) itself is a very crude estimate, it is clear that H_c may change by orders of magnitude, and also not one but a hierarchy of critical fields may be discovered when a detailed investigation is carried through. This situation is not unfamiliar in superconductivity theory, where, depending on the ratio of the parameters λ and g (the gauge coupling), superconductors are divided into Type I and Type II, with Type II displaying a variety of critical fields [5] H_c, H_{c_1}, H_{c_2}, H_{c_3} and a corresponding variety of physical (vorticity) characteristics. (For V_3 Ga, for example, there are three widely differing critical fields, $H_{c_1}(T = 0) \approx 200$ G, $H_c(T = 0) \approx 6000$ G, and $H_{c_2}(T = 0)$ which is as large as 300,000 G.) Detailed models, where we shall also investigate the analogues of fields H_{c_1}, H_{c_2}, in particle physics, will be considered elsewhere.

Fields of strength $\approx 10^5$ G are possibly technically feasible for use in conjunction with K-beam experiments. Clearly, in order to test the ideas expressed here, stronger fields will be needed. There is no doubt, however, that if the basic ideas of spontaneous symmetry-breaking are correct and if we believe in the validity of extrapolating from the one-loop calculation

*) A preliminary model of spontaneous superweak CP breaking appears to give a value $\approx 10^9$ G for the critical field. This is likely to be a gross – though morale-building – underestimate from the experimental point of view.

presented in Sec.II, then there would exist critical fields for which the broken symmetries are likely to be restored. The future task of the theory is to explore such situations where the field strength required is not excessively intense and within reach of forseeable technology. Since it is commonly assumed that the mean magnetic fields in pulsars are $\approx 10^{12} \sim 10^{14}$ G , it would appear – for what it is worth – that CP-violating phenomena do not take place in pulsars, though the Cabibbo angle is still non-zero.

II. A MODEL CALCULATION

To illustrate the effect of a uniform magnetic field on symmetry breaking, we give a simple model calculation. A Lagrangian which exhibits local O(3) symmetry is given by:

$$\mathcal{L} = -\frac{1}{4}\,\underset{\sim}{F}^2_{\mu\nu} + \frac{1}{2}\,(\nabla_\mu \underset{\sim}{\phi})^2 + \frac{\mu^2}{2}\,\underset{\sim}{\phi}^2 - \frac{\lambda}{4}\,(\underset{\sim}{\phi}^2)^2 \quad ,$$

where the scalar and vector fields, $\underset{\sim}{\phi}$ and $\underset{\sim}{A}_\mu$, are triplets with respect to the O(3) symmetry and the covariant derivatives which appear are

$$\nabla_\mu \underset{\sim}{\phi} = \partial_\mu \underset{\sim}{\phi} + e\,\underset{\sim}{A}_\mu \times \underset{\sim}{\phi}$$

$$\underset{\sim}{F}_{\mu\nu} = \partial_\mu \underset{\sim}{A}_\nu - \partial_\nu \underset{\sim}{A}_\mu + 2e\,\underset{\sim}{A}_\mu \times \underset{\sim}{A}_\nu \quad .$$

The parameters μ^2 and λ are both positive: this implies a symmetry breakdown, $O(3) \to O(2)$, in the tree approximation. The vacuum expectation value of $\underset{\sim}{\phi}$ is non-vanishing and, indeed, may be used to define a direction in iso-space

$$\langle \phi^i \rangle = \sqrt{\frac{\mu^2}{\lambda}}\;\delta^{i3} \quad .$$

The corresponding component of the gauge potential, A^3_μ , does not acquire a mass and so may be identified with the electromagnetic potential. In the tree approximation the presence of an external background electromagnetic field, $\langle A^3_\mu \rangle \neq 0$, has no influence on the expected value of the neutral component $\langle \phi^3 \rangle$ of the matter triplet.

When quantum corrections are taken into account this is no longer true. Already, the one-loop corrections to the effective action, given formally by:

$$\Gamma_{(1)}(\phi,A) = \frac{\hbar}{2i}\,\ln \mathrm{Det}\, G_{(0)}(\phi,A) \quad ,$$

(where $G_{(0)}$ denotes the set of classical propagation functions for small disturbances on a given background) imply a coupling between ϕ^3 and A^3_μ even when the charged components $\phi^{1,2}$ and $A^{1,2}_\mu$ are set equal to zero in the modified ground state.

In the uniform magnetic background, where all fields vanish except for

$$\phi^3(x) = \varphi \quad , \quad A^3_1(x) = \frac{1}{2}\,H\,x_2 \quad , \quad A^3_2(x) = -\frac{1}{2}\,H\,x_1$$

with φ and H constant, the one-loop contribution to the effective potential, $V_{(1)}(\varphi,H)$, is determined approximately by the differential formula *)

$$2\,\frac{\partial V_{(1)}}{\partial\varphi^2} = \frac{\lambda\hbar}{8\pi}\int_0^\infty \frac{ds}{s}\left[\left\{\frac{eH}{\sinh(seH)}\exp[-s(\lambda\varphi^2-\mu^2)] - \frac{\exp[-s(\lambda M^2-\mu^2)]}{s}\left(1 - s\lambda(\varphi^2 - M^2)\right)\right\} + \frac{3}{2\pi s}\left\{\exp[-s(3\lambda\varphi^2-\mu^2)] - \exp[-s(3\lambda M^2-\mu^2)]\left(1-3s\lambda(\varphi^2-M^2)\right)\right\}\right]$$

This formula is approximate because we have not included the contributions due to intermediate vectors. The term with the H-dependent factor here is the contribution to $V_{(1)}$ of the intermediate charge pair $\phi^{1,2}$, while the H-independent factor is due to the neutral ϕ^3.

To determine the critical field, we should set $\varphi = 0$ and require that $2\partial V_{(1)}/\partial\varphi^2|_{\varphi=0}$ should compensate the tree contribution $2\partial V_{(0)}/\partial\varphi^2|_{\varphi=0} = -\mu^2$. Unfortunately, the neutral particle contribution to $\partial V^{(1)}/\partial\varphi^2$ is complex if $\varphi^2 < \mu^2/3\lambda$. This means that our approximation is inadequate. The source of this complexity is easily traced.

*) This expression is renormalized at $H = 0$, $\varphi = M$, i.e. $\partial V_{(1)}/\partial\varphi^2$ and $\partial^2 V_{(1)}/\partial(\varphi^2)^2$ both vanish at this point.

In restricting ourselves to one-loop effects we are not allowing the magnetic field to act on the neutral intermediate particles. Hence when $\varphi = 0$ these appear to carry imaginary mass, $i\mu$. However, the very object of the computation – $\partial V/\partial\varphi^2 = 0$ – is the setting to zero of the neutral particle mass. Clearly, we should be setting up a self-consistent (Dyson) equation: which means bringing in the contributions of higher loops. *) Short of this, we can obtain an order of magnitude idea of the critical field by setting both μ and φ to zero specifically in the above expression for $\partial V_{(1)}/\partial\varphi^2$. That is, we solve the restricted problem

$$0 = -\mu^2 + \frac{\lambda\hbar}{8\pi}\int_0^\infty \frac{ds}{s}\left[\left\{\frac{eH_c}{\sinh(seH_c)} - \frac{\exp[-s\lambda M^2]}{s}(1 + s\lambda M^2)\right\} + \frac{3}{2\pi s}\left\{1 - \exp[-3s\lambda M^2](1 + 3s\lambda M^2)\right\}\right]$$

or

$$\frac{\mu^2}{M^2} = \frac{\lambda^2\hbar}{8\pi}\int_0^\infty \frac{du}{u}\left[\frac{eH_c/\lambda M^2}{\sinh(ueH_c/\lambda M^2)} - e^{-u}\left(\frac{1}{u}+1\right) + \frac{3}{2\pi}\left\{\frac{1}{u} - e^{-3u}\left(\frac{1}{u}+3\right)\right\}\right] = \frac{\lambda^2\hbar}{8\pi}\left[a + b\frac{eH_c}{\lambda M^2}\right],$$

where

$$a = 1 + \frac{9}{2\pi} \approx 2.5 \quad \text{and} \quad b = \int_0^\infty \frac{dv}{v}\frac{d}{dv}\left(\frac{v}{\operatorname{sh} v}\right) \approx -1 .$$

On choosing the reference mass equal to μ we obtain

$$H_c = \frac{1}{b}\left(\frac{8\pi}{\lambda^2\hbar} - a\right)\frac{\lambda\mu^2}{e} = \frac{1}{b}\left(\frac{8\pi}{\lambda^2\hbar} - a\right)\frac{\lambda^2\langle\phi\rangle^2}{e} .$$

Since b is negative, we must have $\lambda^2\hbar > \frac{8\pi}{a}$.

*) For a similar pathology in a calculation of critical temperature see the paper of Dolan and Jackiw, Ref.1.

This estimate of H_c should not be taken as anything more than a rough indication.*) The form of this expression for H_c is basically similar to formula (3) in Sec.I, apart from the numerical factor multiplying $\langle\phi\rangle^2$. It is factors of this type which in a realistic calculation may drastically alter the order of magnitude estimates given in Sec.I. Also, for a situation involving a number of non-zero $\langle\phi\rangle$'s, one may conjecture that H_c would be an expression involving a sum of terms like $\sum f_i\langle\phi_i\rangle^2$ with sequences of opposing signs among the coefficients f_i. This, in turn, may lead to the desired diminution of the absolute magnitudes of the critical field strengths. It is perhaps worth remarking that the calculation presented above for the critical field does not constitute a general proof that such a field always does exist. This problem clearly needs further study.

The effects described in this note could also be achieved by pumping electromagnetic energy into the K_0 system by means of lasers [6] with power output $\sim 10^{18} - 10^{22}$ watts/cm^2. This mechanism will be explored in a future publication.

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*) Notice, however, that the implied critical field would be considerably depressed if $\lambda^2\hbar \approx 8\pi/a$. Our estimates in Sec.I may turn out to be very conservative.