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# SUPERSYMMETRY AND FERMION-NUMBER CONSERVATION

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# SUPERSYMMETRY AND FERMION-NUMBER CONSERVATION *

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## ABSTRACT

The appearance of Majorana fermions in supersymmetric theories makes the conservation of fermion-(baryon-or lepton) number - particularly in those models which are renormalizable — something of a problem. To solve this problem, it appears necessary to tolerate some bosons with fermion-number two, and to generate masses through a radiative spontaneous symmetry-breaking mechanism.

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## I. INTRODUCTION

A deep symmetry between bosons and Majorana fermions is the supposition which underlies the recently proposed supersymmetric models [1]. However, in nature there are no Majorana fermions. It is necessary to define a conserved fermion-number (F) such as baryon-(B) or lepton-number (L) . This quantum number must distinguish the fermion members of a supermultiplet from the bosons. It is clear that simply complexifying the spinor components will not do, since the supersymmetry will impose a corresponding complexification on the boson components as well: the associated quantum number is shared equally by all members of the supermultiplet and cannot represent fermion number F .

There have been two distinct proposals for defining a conserved fermion-number. One of them [2] makes use of $\gamma_5$ transformations on the space of co-ordinates $\theta_\alpha$ and is applicable to a fairly broad class of models: those in which the chiral components $\Phi_+$ and $\Phi_-$ have no mutual interactions. This class includes also the interactions with gauge fields provided the left-and right-handed types are treated as independent dynamical variables. A draw-back of this kind of scheme is the difficulty one generally finds in writing a mass term for the particles. In addition, it is possible to have renormal-izable couplings among the system of matter (non-gauge) fields only at the price of introducing bosons with fermion-number F = 2 ("deuterons").

The second proposal [3] is of more limited applicability. It is based on the judicious combination of a set of gauge potentials with a supermultiplet of matter fields in the adjoint representation of some local symmetry. In this kind of model no mass terms and no self-couplings of the matter multiplet are permitted.

The purpose of this note is to review these proposals and indicate some problems associated with them.

A quite different approach would be to enlarge the supersymmetry group, replacing the Majorana generator of supertranslations, $S_\alpha$ , by a pair of Dirac spinors $S_\alpha$ and $\bar{S}^\alpha$ . In this case the fundamental represent-ation will contain Dirac spinors (F = 1) in addition to real scalars. Unfortunately, it will contain deuterons (bosons with F = 2) and, worse, it will most likely not lend itself to the building of renormalizable models. We shall not consider this approach here.

## II. THE $\gamma_5$ METHOD

In the superfield notation the fundamental supermultiplet is represented by

$$\Phi_\pm(x,\theta) = \exp\left[\mp \frac{1}{4}\bar{\theta}\not{\partial}\gamma_5\theta\right]\left[A_\pm(x) + \bar{\theta}\,\frac{1 \pm i\gamma_5}{2}\,\psi(x) + \frac{1}{2}\,\bar{\theta}\,\frac{1 \pm i\gamma_5}{2}\,\theta\, F_\pm(x)\right] , \qquad (2.1)$$

where $A_\pm$ and $F_\pm$ are spin-zero parity mixtures and $\psi$ is a Dirac spinor. The superfields $\Phi_+$ and $\Phi_-$ transform independently under supertransformations and proper Poincaré transformations but are interchanged by space reflections. It is possible to impose a reality condition, $\Phi_- = \Phi_+^*$, or

$$A_- = A_+^* \quad , \quad \psi = C\,\bar{\psi}^T \quad , \quad F_- = F_+^* \quad ,$$

which halves the number of independent components. However, if this is not done one may contemplate the effects of a new kind of transformation,

$$\Phi_\pm(x,\theta) \rightarrow e^{in\alpha}\;\Phi_\pm\big(x,\,\exp[\mp\alpha\gamma_5]\theta\big) \quad , \qquad (2.2)$$

or

$$A_\pm \rightarrow e^{in\alpha} A_\pm \quad , \quad \psi \rightarrow \exp[i(n+1)\alpha]\,\psi \quad , \quad F_\pm \rightarrow \exp[i(n+2)\alpha]\,F_\pm \quad . \qquad (2.2')$$

The quantum number associated with these phase transformations is clearly a scalar which takes the values, n, n+1, n+2 in the supermultiplet. We propose to identify it with "fermion-number".

A simple model in which this number is conserved is given by the Lagrangian [*)]

$$\mathcal{L} = \frac{1}{8}\,(\bar{D}D)^2\left(\Phi_+^*\Phi_+ + \Phi_-^*\Phi_-\right) - \frac{g}{2}\,\bar{D}D\left[\Phi_+^3 + \Phi_-^3 + \Phi_+^{*3} + \Phi_-^{*3}\right] \quad , \qquad (2.3)$$

where the weight n must take the value -2/3 . The invariance of (2.3) is readily proved with the help of the transformation rules for the covariant derivatives,

*) This is a direct generalization of the Wess-Zumino model [4)] to complex fields with zero mass.



$$D'\Phi_\pm(x,\theta') = e^{in\alpha}\exp[\mp\alpha\gamma_5]\,D\,\Phi_\pm(x,\theta)$$
$$= \exp[i(n-1)\alpha]\,D\,\Phi_\pm(x,\theta) \quad ,$$

where $\theta' = \exp[\pm\alpha\gamma_5]\,\theta$ . Notice that the choice $n = -2/3$ precludes the possibility of a mass term in (2.3). The fermion-number assignments -2/3 , 1/3 and 4/3, respectively, to A, $\psi$ and F are viable only for massless particles. As we shall see, this problem is a persistent one.

Note that the above assignment of fermion-numbers can be rescaled, so that the spin-$\frac{1}{2}$ particle carries F = 1 while the bosons $A_\pm$ carry F = -2 . The auxiliary fields $F_\pm$ carry F = 4 but are not associated with particles. This is in fact a model of "baryons and anti-deuterons".

Gauge interactions can be treated in the same way provided the local symmetry governing them acts independently on chiral superfields $\Phi_+$ and $\Phi_-$ . For example, if these fields belong to the fundamental representation of a local symmetry,

$$\Phi_\pm \rightarrow e^{i\Lambda_\pm}\;\Phi_\pm \quad , \qquad (2.4)$$

then the gauge potentials $\Psi$ and $\Psi'$ are required to transform according to

$$e^{2g\Psi} \rightarrow e^{i\Lambda_+^+}\;e^{2g\Psi}\;e^{-i\Lambda_+} \quad ,$$
$$e^{-2g\Psi'} \rightarrow e^{i\Lambda_-^+}\;e^{-2g\Psi'}\;e^{-i\Lambda_-} \quad . \qquad (2.5)$$

The gauge-invariant kinetic term for the matter fields is given by

$$\mathcal{L}_{matter} = \frac{1}{8}\,(\bar{D}D)^2\left(\Phi_+^+\,e^{2g\Psi}\,\Phi_+ + \Phi_-^+\;e^{-2g\Psi'}\,\Phi_-\right) \quad . \qquad (2.6)$$

The gauge-invariat kinetic term for the gauge potentials is given by

$$\mathcal{L}_{gauge} = \frac{1}{8}\,\bar{D}D\left\{\bar{\Psi}_\alpha\Psi_\alpha + \bar{\Psi}'_\alpha\Psi'_\alpha\right\} \quad , \qquad (2.7)$$

where the (spinor) superfield strengths $\Psi_\alpha$ and $\Psi'_\alpha$ are defined in terms of the potentials by $\Psi_\alpha = \Psi_{\alpha+} + \Psi_{\alpha-}$ , $\Psi'_\alpha = \Psi'_{\alpha+} + \Psi'_{\alpha-}$ with

$$\Psi_{\alpha\pm} = -\frac{i}{\sqrt{2}}\frac{1}{2g}\bar{D}\,\frac{1 \mp i\gamma_5}{2}\,D\left(e^{\mp 2g\Psi}\left(\frac{1 \pm i\gamma_5}{2}\,D\right)_\alpha e^{\pm 2g\Psi}\right)$$

$$\Psi'_{\alpha\pm} = -\frac{i}{\sqrt{2}}\frac{1}{2g'}\bar{D}\,\frac{1 \mp i\gamma_5}{2}\,D\left(e^{\pm 2g'\Psi'}\left(\frac{1 \pm i\gamma_5}{2}\,D\right)_\alpha e^{\mp 2g'\Psi'}\right) . \tag{2.8}$$

Introduction of gauge potentials in a Lagrangian which respects the $\gamma_5$ transformations (2.2) will not disturb this invariance provided the potentials transform according to

$$\Psi(x,\theta) \to \Psi\left(x, e^{-\alpha\gamma_5}\theta\right)$$

$$\Psi'(x,\theta) \to \Psi'\left(x, e^{\alpha\gamma_5}\theta\right) . \tag{2.9}$$

This means that the components defined by

$$\Psi = \frac{1}{4}\,\bar\theta i\gamma_\nu\gamma_5\theta\, W_\nu + \frac{1}{2\sqrt{2}}\,\bar\theta\theta\bar\theta\gamma_5\lambda + \frac{1}{16}\,(\bar\theta\theta)^2\, D \tag{2.10}$$

(and likewise for $\Psi'$) should transform according to

$$W_\nu\,,\,W'_\nu \to W_\nu\,,\,W'_\nu \quad ; \quad D,D' \to D,D'$$

and

$$\lambda \to e^{\alpha\gamma_5}\lambda \quad , \quad \lambda' \to e^{-\alpha\gamma_5}\lambda' . \tag{2.11}$$

These field components are essentially real. That is, in a suitable basis for the local symmetry, the boson components $W_\nu$ and $D$ are real and the fermion components $\lambda$ are 4-component Majorana spinors. The Majorana pair $\lambda$ , $\lambda'$ may now be replaced by a Dirac spinor

$$\chi = \frac{1 - i\gamma_5}{2}\,\lambda + \frac{1 + i\gamma_5}{2}\,\lambda' \tag{2.12}$$

and its conjugate $\bar\chi$ . These fields transform according to

$$\chi \to e^{i\alpha}\,\chi \;,\; \bar\chi \to e^{-i\alpha}\,\bar\chi \tag{2.13}$$

and thus carry unit (±1) fermion-number. The invariance of $\mathcal{L}_{gauge}$ in (2.7) for the transformations (2.11) may be verified by noting that under (2.9) $\Psi_{\alpha\pm} \to e^{\mp i\alpha}\,\Psi_{\alpha\pm}$ , $\Psi'_{\alpha\pm} \to e^{\pm i\alpha}\,\Psi'_{\alpha\pm}$. In detail,



$$\Psi_{\alpha\pm} = \left(\frac{1 \pm i\gamma_5}{2}\right)^\beta_\alpha \exp\left[\mp\frac{1}{4}\,\bar\theta\partial\!\!\!/\gamma_5\theta\right]\left[\lambda_\beta + \frac{1}{\sqrt{2}}\left(\left(\frac{1}{2}\,\sigma_{\mu\nu}\,\theta\right)_\beta W_{\mu\nu} \pm \theta_\beta D\right)\right.$$

$$\left. + \frac{1}{4}\,\bar\theta(1 \pm i\gamma_5)\theta\,\left(\frac{1}{i}\nabla\!\!\!\!/\,\lambda\right)_\beta\right] , \tag{2.14}$$

where

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - ig\,[W_\mu, W_\nu]$$

$$\nabla_\mu\lambda = \partial_\mu\lambda - ig\,[W_\mu,\lambda] ,$$

and likewise for $\Psi'_{\alpha\pm}$ . In terms of these components and specializing to the case of $g = g'$, one can write $\mathcal{L}_{gauge}$ in (2.7) in the form:

$$\mathcal{L}_{gauge} = \mathrm{Tr}\left[-\frac{1}{4}\,V^2_{\mu\nu} - \frac{1}{4}\,A^2_{\mu\nu} + \bar\chi\, i\nabla\!\!\!\!/\,\chi + \frac{1}{2}\,D^2 + \frac{1}{2}\,D^2_5\right] , \tag{2.15}$$

where

$$V_\mu = \frac{1}{\sqrt{2}}\,(W_\mu + W'_\mu) \quad , \quad D = \frac{1}{\sqrt{2}}\,(D-D')$$

$$A_\mu = \frac{1}{\sqrt{2}}\,(W_\mu - W'_\mu) \quad , \quad D_5 = \frac{1}{\sqrt{2}}\,(D+D')$$

and

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - \frac{ig}{\sqrt{2}}\left([V_\mu,V_\nu] + [A_\mu,A_\nu]\right)$$

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - \frac{ig}{\sqrt{2}}\left([V_\mu,A_\nu] + [A_\mu,V_\nu]\right)$$

$$\nabla_\mu\chi = \partial_\mu\chi - i\,\frac{g}{\sqrt{2}}\left([V_\mu,\chi] - i\,[A_\mu,\gamma_5\chi]\right) . \tag{2.16}$$

For comparison we give the component structure of the matter field kinetic term (2.6):

$$\mathcal{L}_{matter} = \mathrm{Tr}\Big[\nabla_\mu A^\dagger\nabla_\mu A + \nabla_\mu B^\dagger\nabla_\mu B + \bar\psi i\nabla\!\!\!\!/\,\psi + F^\dagger F + G^\dagger G$$

$$- \frac{ig}{\sqrt{2}}\left(A^\dagger[D,A] + B^\dagger[D,B]\right) + \frac{g}{\sqrt{2}}\left(A^\dagger[D_5,B] - B^\dagger(D_5,A]\right)$$

$$+ g\;[\bar\chi, A^\dagger + \gamma B^\dagger]\psi - g\;\bar\psi[A + \gamma_5 B,\chi]\Big] , \tag{2.17}$$

where

$$\nabla_\mu A = \partial_\mu A - \frac{g}{\sqrt{2}}[V_\mu, A] - \frac{ig}{\sqrt{2}}[A_\mu, B]$$

$$\nabla_\mu B = \partial_\mu B - \frac{g}{\sqrt{2}}[V_\mu, B] + \frac{ig}{\sqrt{2}}[A_\mu, A]$$

$$\nabla_\mu \psi = \partial_\mu \psi - \frac{ig}{\sqrt{2}}[V_\mu + i\gamma_5 A, \psi] \quad . \tag{2.18}$$

One sees that the $\gamma_5$ transformations (2.2) and (2.9) can be used to generate a conserved fermion-number in a gauge Lagrangian, provided the system of superfields resolves into two independent chiral sets $\{\Phi_+, \Phi_+^*, \Psi\}$ and $\{\Phi_-, \Phi_-^* \Psi'\}$ .

## III. TOWARDS A REALISTIC MODEL

Given a set of N chiral superfields $\Phi_{+p}$ , $p = 1,2,\ldots,N$, and their conjugates $\Phi_{-p} \equiv \Phi_{+p}^*$, the most general renormalizable interaction is given by

$$\mathcal{L} = \frac{1}{8}(\bar{D}D)^2 (\Phi_{-p}\Phi_{+p}) - \frac{1}{2}\bar{D}D \left\{M_{pq}\Phi_{+p}\Phi_{+q} + g_{pqr}\Phi_{+p}\Phi_{+q}\Phi_{+r} + \text{h.c.}\right\} , \tag{3.1}$$

where $M_{pq}$ and $g_{pqr}$ are symmetrical . (Summations over repeated indices are implied.) As a rule one is interested in systems which display a global symmetry. (Later we include local symmetries as well.) This means there will be many identities among the parameters M and g . For example, the largest conceivable global symmetry would be SU(N), which obtains if

$$M_{pq} = M\,\delta_{pq} \quad \text{and} \quad g_{pqr} = g\,d_{pqr} \tag{3.2}$$

(for $N \geq 3$ ; for $N = 2$, g vanishes).

We shall not attempt to categorize those subgroups of SU(N) which would be compatible with a fermion-number symmetry of the type discussed in the last section. A number of general remarks can however be made. If a typical boson contribution is extracted from the mass term, say

$$M_{12}\,A_1\,F_2 \ ,$$



or from the interaction term, say

$$g_{123}\,A_1\,A_2\,F_3 \ ,$$

then, if fermion-number is to be conserved, we must have

$$F(A_1) + F(F_2) = 0 \quad \text{if} \quad M_{12} \neq 0 \ ,$$

$$F(A_1) + F(A_2) + F(F_3) = 0 \quad \text{if} \quad g_{123} \neq 0 \ ,$$

where F(A), etc. signify the fermion-number of the field in the parenthesis. From the rule (2.2') it follows that

$$F(A_1) + F(A_2) = -2 \quad \text{if} \quad M_{12} \neq 0$$

$$F(A_1) + F(A_2) + F(A_3) = -2 \quad \text{if} \quad g_{123} \neq 0 \ .$$

This means simply that even if some of the bosons carry F = 0 there must be others in the theory which carry F = 2. Deuterons are thus a necessary feature of any fermion-number conserving theory in which either mass or interaction terms are present. (This does not apply to interactions with gauge potentials which enter through the kinetic term.) Two examples follow.

### A. $SU(n)_+ \times SU(n)_-$

Let $\Phi_{+p}$ and $\Phi_{-p}$ , $p = 1,\ldots,n^2 - 1$, belong to the adjoint representation so that $d_{pqr}$ exists for $n \geq 3$ . The only way to obtain F conservation is by imposing the rule

$$\Phi_{\pm p} \rightarrow \exp[-\tfrac{2}{3}i\alpha]\ \Phi_{\pm p}\left(x, \exp[\mp\alpha\gamma_5]\theta\right) ,$$

so that all bosons $A_{\pm p}$ will carry the fractional number $F = -2/3$ (or, after rescaling $F = -2$ , assuming that these are the only fields in the theory). To obtain some fields with zero fermion-number, it is necessary to introduce a second pair $\Phi'_{+p}$ and $\Phi'_{-p}$ and to postulate that:

$$\left.\begin{aligned} \Phi_\pm &\rightarrow \Phi_\pm\left(x, \exp[\mp\alpha\gamma_5]\theta\right) , \\ \Phi'_\pm &\rightarrow e^{-2i\alpha}\ \Phi'_\pm\left(x, \exp[\mp\alpha\gamma_5]\theta\right) . \end{aligned}\right\} \tag{3.3}$$

In this case, fields $A_\pm$ carry fermion-number $F = 0$, $\psi$ carries $F = +1$, while $A'_\pm$ are anti-deuterons ($F = -2$) and $\psi'$ carries $F = -1$. The non-diagonal "mass term" $M \Phi_{+p} \Phi'_{+p}$ and the interaction term

$$f\, d_{pqr}\, \Phi_{+p}\, \Phi_{+q}\, \Phi'_{+r} + \text{h.c.}$$

are both F-conserving though the diagonal "mass terms" $M \Phi_{+p} \Phi_{+p}$ and $M \Phi'_{+p} \Phi'_{+p}$ does not conserve $F$. One could go on to deal with a triplet of octets:

$$\left(f\, d_{pqr} + f'\, f_{pqr}\right) \Phi^1_{+p}\, \Phi^2_{+q}\, \Phi^3_{+r} + \text{h.c.}, \qquad (3.4)$$

and again have integer F values if one of the multiplets ($\Phi^3$ say) contains an anti-deuteron. (In this example if we keep only $f' \neq 0$ then a new "colour" symmetry SU(3') between $\Phi^1$, $\Phi^2$ and $\Phi^3$ emerges.)

## B. $(SU(3) \times SU(3))_+ \times (SU(3) \times SU(3))$

A special circumstance for $n = 3$ allows the construction of a more economical model. Let $\Phi_+$ and $\Phi_-$ be independent $3 \times 3$ matrices. The symmetry $(SU(3) \times SU(3))_+ \times (SU(3) \times SU(3))_-$ is respected by the interaction

$$f\left(\det\Phi_+ + \det\Phi_- + \text{h.c.}\right). \qquad (3.5)$$

The F values are integers if we let the fields $A_\pm$ in the first two columns of $\det\Phi_\pm$ carry $F = 0$ while the third column carries $F = -2$. In this case the fermion-number has a component which transforms as an octet with respect to the column SU(3)'s.

No mass terms are permissible unless we discard the column SU(3)'s and restrict the row SU(3)'s to O(3)'s. This would permit the "off-diagonal" mass terms, consistent with F conservation

$$M_1\, \Phi^1_{+i}\, \Phi^3_{+i} + M_2\, \Phi^2_{+i}\, \Phi^3_{+i} + \text{h.c.} \qquad (3.6)$$

and will produce masses for at least some of the states.

The possibility that mass can be generated spontaneously through the radiative mechanism of Coleman and Weinberg remains to be investigated. To realize this, one may consider the economical model (3.5) above, together with a (parity-conserving) gauging of the column $SU(3)_+ \times SU(3)_-$. The resulting Lagrangian will contain the coupling parameter $f$ in addition to the gauge parameters $g$. Barring an unexpected catastrophe, following Coleman and Weinberg, the existence of the two parameters ($f$ and $g$) should make it possible (even after dimensional transmutation) to compute all the masses in the model, even if no intrinsic mass term is introduced.



# IV. A JUDICIOUS MODEL

A quite different scheme for obtaining a conserved fermion-number is obtained by the judicious combination of the fermion parts of a gauge potential and a real matter supermultiplet. Consider the case of local SU(3),

$$\Phi_\pm \to e^{i\Lambda_\pm}\, \Phi_\pm\, e^{-i\Lambda_\pm},$$

where $\Phi_\pm$ and $\Lambda_\pm$ are traceless $3 \times 3$ matrix superfields subject to the reality conditions

$$\Phi_- = \Phi_+^\dagger \quad \text{and} \quad \Lambda_- = \Lambda_+^\dagger .$$

The spinor part of $\Phi_\pm$ is a Majorana octet, $\psi(x)$. Likewise, the spinor part of the gauge potential $\Psi$ is a Majorana octet $\lambda$. It is only a question of straightforward, though tedious, computation to show that the spinor contributions to the gauge invariant Lagrangian can be expressed in the form

$$\bar{\chi} i \not{\partial} \chi - ig\, \chi \times (A + \gamma_5 B) \cdot \chi ,$$

where $\chi$ denotes the complex octet

$$\chi = \frac{1}{\sqrt{2}} (\lambda + i\psi)$$

and where $A$ and $B$ denote the (real) scalar and pseudoscalar parts of the matter octet. Invariance under phase transformations on $\chi$ is evident.

In this somewhat accidental fashion, two real supermultiplets are made to co-operate in such a way as to yield a conserved fermion-number. Again there is no mass term (and no self-interaction) for the matter supermultiplet. *)

The Lagrangian for the model is given by

$$\mathcal{L} = -\frac{1}{4} V_{\mu\nu}^{\,2} + \frac{1}{2}(\nabla_\mu A)^2 + \frac{1}{2}(\nabla_\mu B)^2 - \frac{g^2}{2}(A \times B)^2$$
$$+ \bar{\chi} i \nabla\!\!\!/ \chi - ig\,\bar{\chi} \times (A + \gamma_5 B)\cdot\chi \quad ,$$

where

$$V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu + g\, V_\mu \times V_\nu$$

$$\nabla_\mu A = \partial_\mu A + g\, V_\mu \times A$$

$$\nabla_\mu \chi = \partial_\mu \chi + g\, V_\mu \times \chi$$

This model is extremely tight; it contains just one coupling parameter, and no parameter with dimensions of length.

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*) In the one-loop approximation there appears to be a difficulty associated with this model; it has no stable Poincaré invariant ground state. This peculiarity was discovered in the course of an attempt (carried out together with Dr. M.J. Duff) to generate particle masses through radiative corrections. The one-loop contribution to the effective potential $V(A,B)$ is finite and complex (for real A and B). It is possible that a more elaborate computation of the potential, following the methods recently discussed by Dolan and Jackiw [5], may cure the sickness of the one-loop approximation in this otherwise truly remarkable [3] model.



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