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ON FERMION-NUMBER AND ITS CONSERVATION *

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ABSTRACT

We raise the question if fermion-number ($F = B + L$) is absolutely conserved. Neutrinoless double β-decay experiments give upper limits on $|\Delta L| = 2$ transitions. Experiments on the stability of the deuteron against decay into pions would give analogous limits on $|\Delta B| = 6$ transitions, while proton stability experiments set limits on $|\Delta F| = 0$, $\Delta B = -\Delta L$ as well as on transitions involving $|\Delta F| \neq 0$. Here $B = +1$ for (integer or zero charge) quarks, $B = -1$ for antiquarks while L is the lepton-number. Remarking that the maximal symmetry group for the kinetic energy terms of a set of n four-component Dirac fields is $SU(2n)$ rather than $U(1) \times SU_L(n) \times$ $\times\, SU_R(n)$ (with the extra gauge currents carrying fermion-number ± 2), we briefly investigate the possibility of constructing spontaneously broken gauge theories where fermion-number appears as a non-abelian generator.

I. INTRODUCTION

One question which neutrinoless double β-decay experiments are designed to answer is this - to what extent is neutrino-number N_ν conserved? Equivalently, is the neutrino described by a Dirac spinor or by a time-varying mixture of Majorana spinors?

An analogous question could equally be asked for neutral baryons; for example, the neutrons or Λ's . To be concrete, if one believes in an integer charge quark model, one could pose the question - to what extent is a physical neutral quark a mixture of a bare quark plus bare antiquark; to what extent is the quark-number N_q conserved modulo two units (one needs two units in order to conserve angular momentum)? Equivalently, does a neutral diquark system possess components with the quantum numbers of the vacuum or, equally, does the positively-charged diquark possess components with the quantum number of π^+ , ρ^+ and A^+ , etc? Thus, to the extent that a deuteron may be considered as a three (integer-charge) diquark composite, the deuteron's stability would (for example) provide a measure of the type of admixture mentioned above.

The questions above - though logically posable independently - have, in our work, been prompted by[1)] a desire to extend the $SU(4) \times SU(4')$ model of unified particle interactions, where integer charge quarks and leptons are combined in the basic fermion multiplet

$$F = \begin{vmatrix} p_a^0 & p_b^+ & p_c^+ & \nu_e \\ n_a^- & n_b^0 & n_c^0 & e^- \\ \lambda_a^- & \lambda_b^0 & \lambda_c^0 & \mu^- \\ \chi_a^0 & \chi_b^+ & \chi_c^+ & \nu_\mu \end{vmatrix}$$

In this model, it is possible to define a fermion-number F, which is the sum of baryon-number (or, more precisely, quark-number which we designate B) plus electron-number L_{e^-} (which is a sum of N_{e^-} and N_{ν_e}) plus muon-number L_{μ^-} (= $N_{\mu^-} + N_{\nu_\mu}$), i.e.

$$F = B + L_{e^-} + L_{\mu^-}$$

It was postulated that this number (represented by an abelian generator U(1) which lies outside $SU(4) \times SU(4')$) is conserved absolutely even though [2] B, L_{e^-} and L_{μ^-} may not be conserved individually ($\Delta F = 0$, $\Delta B = -\Delta L$). In this paper we wish to relax this assumption of F-number conservation, permitting the three independent $|\Delta F| = 2$ possibilities ($|\Delta L| = 2$, $|\Delta B| = 0$), ($|\Delta L| = 0$, $|\Delta B| = 2$) and ($\Delta L = \Delta B = +1$ or -1) . To construct an elegant theory we shall need to embed the structure $U(1) \times SU(4) \times SU(4')$ into a higher non-abelian group structure. A gauging of this new structure would give rise to currents carrying $F = \pm 2 \left\{ \begin{cases} B = \pm 2 \\ L = 0 \end{cases} \text{or} \begin{cases} L = \pm 2 \\ B = 0 \end{cases} \text{or} \begin{cases} L = \pm 1 \\ B = \pm 1 \end{cases} \right\}$ quantum numbers in addition to the currents (previously introduced) corresponding to the symmetry $U(1) \times SU(4) \times SU(4')$ and which carry zero-fermion-number only ($F = 0$; $B = 0$, $L = 0$ or $B = \pm 1$, $L = \mp 1$). A spontaneous breaking of this higher symmetry through a mixing between $|F| = 2$ and $F = 0$ currents (briefly motivated in this paper) could lead to F-violating interactions in a hierarchical fashion, with effective coupling strengths which are weaker than the effective strengths of the interactions which conserve F .

The physical reason why one contemplates eventual break-down of fermion-number conservation is this. Contrast the case of F conservation with the case of the only other (presumably) absolutely conserved quantity - the electric charge Q.

Of these two numbers, Q and F, one, the electric charge, is the source of a massless vector field [3] ($m_\gamma < 4 \times 10^{-48}$ gm). If fermion-number too is the source of a massless vector field, then, from the well-known argument of Lee and Yang [4], its coupling strength must be weaker than the gravitational coupling by a factor lying between 10^{-5} and 10^{-8} . Assuming, therefore, that such a massless vector field does not exist, even while fermion-number is conserved absolutely, we are met with Wheeler's famous dilemma. That is, when a quantity of matter passes through a black-hole horizon, its fermion-number, though conserved, becomes impossible to measure. This does not happen with the other absolutely conserved quantities, charge, mass and angular momentum, which are associated with long-range fields. In Wheeler's courteous phrase, the fermion-number is "transcended". In order to avoid this dilemma of transcendence, it appears more natural to us to suppose that fermion-number may eventually be violated and that all neutral particles end up as Majorana fermions. (This is in line with and an extension of our attitude towards the possible violation of the baryon and lepton numbers [2].)

In the following we shall assume that fermion-number is associated with a broken symmetry. Sec. II is concerned with the setting of upper limits on the strength of the supposed symmetry-breaking which are implied by experiment. A formulation of gauge theories in which spontaneous breaking could be expected is given in rather general terms in Sec. III. In Sec. IV, the formalism is applied for purposes of illustration to a simple (though unrealistic) example, with a brief discussion of the $U(1) \times SU(4) \times SU(4')$ case.

II. EXPERIMENTAL UPPER LIMITS

From double β-decay [5] and stability of matter [6] experiments, one can set limits on the observed degree of fermion-number violation. We consider these in turn.

(A) Double β-decay $|\Delta L| = 2$, $|\Delta B| = 0$, $|\Delta F| = 2$

To describe neutrinoless double β-decay experiments, one may write the lepton current in the weak Hamiltonian in the form

$$L_\mu = \bar{e}\,\gamma_\mu \left[(1 - i\gamma_5) + \eta(1 + i\gamma_5)\right] (\nu + \xi_\nu\, \nu^c) \quad ,$$

where ν^c denotes the charge conjugate of ν and ξ_ν is a parameter which measures the fermion-number violation. The present experimental estimates [7] (from Te^{130}) give,

$$|\eta\xi_\nu| \leq 3 \times 10^{-4} .$$

Assuming a lower limit on the $V + A$ admixture measured by the parameter η (obtained from measurements of the longitudinal polarization of electrons emitted in β-decay), i.e.

$$|\eta| \approx \frac{1}{10} \quad ,$$

one obtains the rather mild limit on N_ν violation ($|\Delta N_\nu| = 2$) :

$$|\xi_\nu| < 3 \times 10^{-3} .$$

(B) Quark-antiquark mixing, $|\Delta B| = 2$, $|\Delta L| = 0$, $|\Delta F| = 2$

Let ξ_q denote the bare quark-antiquark admixture in the composition of the "physical quark":

$$q_{phy} = q + \xi_q q^c \quad ,$$

and let β_D be the amplitude for the deuteron to exist as a three-diquark composite. Then the amplitude for the deuteron to decay (for example) into pions is approximately proportional to $\beta_D \xi_q^3$. Since there is no direct experiment to place limits on deuteron's stability, we wish to use the experiments of Reines and co-workers [5] on proton's half life to place limits on ξ_q.

(C) Quark-lepton ($\Delta F = 0$, $\Delta B = -\Delta L \neq 0$) and quark-antilepton transitions ($\Delta F = 2$, $\Delta B = \Delta L = \pm 1$)

Let ζ_ℓ and $\xi_{q\ell}$ in $q_{phy} = q + \xi_q q^c + \zeta_\ell \ell + \xi_{q\ell}\bar{\ell}$ give the quark-lepton and quark-antilepton admixture-parameters [8]. Since the proton is a three-quark composite, the amplitudes for

$$\begin{aligned} \text{proton} &\rightarrow 3 \text{ lepton} + \text{pions} \,, & |\Delta F| &= 0 \\ &\rightarrow \text{diquark} + \text{quark} \rightarrow \text{pion} + \text{lepton} \,, & |\Delta F| &= 2 \\ &\rightarrow \qquad \text{pion} + \text{antilepton} \,, & |\Delta F| &= 4 \end{aligned}$$

are proportional to ζ_ℓ^3 , $\zeta_\ell \xi_q$, $\xi_q \xi_{q\ell}$. From empirical proton-half-life of $\gtrsim 10^{30}$ years, all one might infer [8] (so far as orders of magnitudes are concerned) is that:

$$\zeta_\ell \approx \xi_{\ell q} \approx \xi_q^{\frac{1}{2}} \lesssim 10^{-8} - 10^{-9} \quad .$$

These upper limits 10^{-3} for ξ_ν , $10^{-8} - 10^{-9}$ for $\xi_{q\ell}$ and ζ_ℓ and $10^{-16} - 10^{-18}$ for ξ_q, do not give any uniform experimental picture of the strength of fermion-number violation, if any. One may possibly conjecture that there is a hierarchy of symmetry-breaking parameters and that fermion-number violation is associated with the smallest of these effective couplings (specifically, with a coupling which is smaller than the F-conserving baryon-lepton transition parameter $\zeta_\ell \approx 10^{-8} - 10^{-9}$). In this connection one may even conjecture that for the quark-antilepton transition a two-step relation like $q \rightarrow \ell \rightarrow \bar{\ell}$ may hold, signifying $\xi_{q\ell} \approx \zeta_\ell \xi_\nu \lesssim 10^{-9} \times 10^{-3} \lesssim 10^{-12}$. With a view to formalizing such a hierarchy, we consider gauge models of fermion-number violation in the next section.

III. FERMION NUMBER AND LOCAL SYMMETRY

Our purpose is to set up a gauge theory in which the fermion-number current is coupled to a massive vector field. There are a number of ways in which this can be achieved.

Suppose firstly that the conservation of fermion-number results from invariance of the Lagrangian against simple phase transformation of the Dirac fields. Coupling to a vector field is obtained in the usual way by inserting covariant derivatives on the Dirac fields. Since, in this abelian context the vector field does not itself carry fermion-number, one is free to add a mass term for it. (Such a mass term would break the local symmetry but would not distrub the renormalizability of the system nor the conservation of fermion-number.) Alternatively, one could introduce a set of scalar fields which carry fermion-number and whose self-interactions are arranged so as to favour the emergence of a symmetry-breaking ground state. In this way a vector mass would result from the spontaneous breakdown mechanism and, again, the renormalizability would be preserved.

The first approach cannot be adopted if the ultimate purpose is to deal with broken fermion-number symmetry. Renormalizability would be lost with the local symmetry. The second approach is to be preferred so long as the scalar system has non-trivial interaction with the fermions: such direct interactions are needed if the symmetry-breaking effects are to involve the fermions.

A more interesting scheme may result from generalizing the fermion-number symmetry and making it part of a non-abelian local symmetry, which arises if we gauge the maximal local symmetry obtainable in the space of a set of 4-component Dirac spinors. Such a symmetry contains fermion-number among its generators and spontaneous breaking of this local symmetry not only would provide mass to the gauge boson coupled to the fermion-number,

but could also induce a non-conservation of the fermion-number. There are two likely advantages of gauging the fermion-number symmetry as part of the maximal non-abelian symmetry rather than as an abelian symmetry:

1) With a non-abelian group one may hope to realize asymptotic freedom for the complete theory if spontaneous symmetry breaking is dynamical or the model is supersymmetric.
2) All elementary interactions would be described by a single unifying coupling constant.

A generalized symmetry (containing fermion-number in its global form) was in fact considered by several authors [9] many years ago. A brief resumé will serve to fix the notation and motivate the extension of gauge ideas to it.

Let $\Psi_L = \{\Psi_{L1}, \Psi_{L2}, \ldots, \Psi_{LN}\}$ denote a column of N independent left-handed spinors

$$(1 + i\gamma_5)\,\psi_{Lp} = 0 \quad , \qquad p = 1,2,\ldots,N \quad .$$

The adjoint $\bar{\Psi}_L = \{\psi^\dagger_{L1}\,\gamma_0, \psi^\dagger_{L2}\,\gamma_0, \ldots\}$ is an N-component row. The maximal invariance group of the kinetic bilinear, $\bar{\Psi}_L i\not\partial \Psi_L$, is clearly $U(N) \approx U(1) \times SU(N)$. This group differs from the usual sort of internal symmetry in that we shall include both particles and antiparticles in the single column Ψ_L. There is no restriction here for N to be even; if, however, $N = 2n$, then the free kinetic energy term can equivalently be written in terms of n four-component Dirac fields. The $U(1)$ abelian group specified above corresponds to a γ_5 transformation. This particular $U(1)$ will play no role in our future consideration.

[Right-handed components can be defined by complex conjugation,

$$\Psi_R = \beta\, C\, \bar{\Psi}_L^T \quad ,$$

where C denotes the usual charge conjugation matrix and β is any conveniently chosen $N \times N$ matrix which operates on the internal indices. In the usual presentation Ψ_R and $C\bar{\Psi}_L^T$ are independent in that, applied to the vacuum, the first creates a right-handed antiparticle while the second creates a right-handed particle. In the scheme considered here, both right-handed particles <u>and</u> antiparticles are created by $C\bar{\Psi}_L^T$. The fields Ψ_R are, indeed, redundant.]

Among the subgroups of SU(N) we shall suppose there is a U(1) which can be associated with fermion-number. The remaining $N^2 - 2$ generators will then carry a well defined fermion-number and the irreducible representations will be classifiable with respect to their fermion-number content.

It is necessary first to discuss the parity assignments. Under space reflections we expect to have

$$\Psi_L \overset{P}{\rightarrow} \omega\ \gamma_0\ C\ \bar{\Psi}_L^T$$

$$\bar{\Psi}_L \overset{P}{\rightarrow} -\Psi_L^T\ C^{-1}\ \gamma_0\ \omega^{-1} \quad ,$$

where ω is a unitary $N \times N$ matrix. This transformation preserves the kinetic bilinear. A necessary condition is that the square of the space reflection operator $\mathbb{P}$ must equal ± 1. This implies

$$\mathbb{P}^2 = -\omega\,\omega^{-1T} = -\omega\,\omega^* = \pm 1 \ , \quad \text{i.e.} \quad \omega = \mp\,\omega^T \ .$$

(Thus, for the case $N = 1$ and Ψ_L is necessarily the left-handed part of a Majorana spinor, one must take $\mathbb{P}^2 = -1$.)

The parities of the SU(N) generators are easily discovered by examining the corresponding current desities,

$$J_0(\lambda) = \bar{\Psi}_L\,\gamma_0\,\lambda\,\Psi_L \quad ,$$

where λ is a hermitian $N \times N$ matrix. Under space reflections one finds

$$J_0(\lambda) \rightarrow J_0(-\omega\,\lambda^T\,\omega^{-1}) \quad .$$

Since, by definition, both particles and their antiparticles belong to the same irreducible multiplets, it follows that the operation of antiparticle conjugation is one of the group transformations. (As an example consider SU(6), where a suitable choice would be

$$e^{-i\frac{\pi}{2}\tau_2} = \omega^{-1} \quad ,$$

which gives the usual C-properties to the vector and axial currents.) Fermion-number itself must be identified with one of the U(1) generators λ_F among the SU(N) which is reflected by ω, $\omega^{-1}\lambda_F\,\omega = -\lambda_F$. For the case $N = 3$, for example, we could choose λ_3 but not λ_8. (Notice that this choice gives a particularly simple form to the CP transformation, $J_0(\lambda) \rightarrow J_0(-\lambda^T)$.)

Since space reflections carry representations of SU(N) into their conjugates, there is generally a parity doubling. Only the real representations can be assigned an intrinsic parity.

The Yukawa couplings of the quarks Ψ_L and $\bar{\Psi}_L$ are quite restricted. Since $\bar{\Psi}_L\Psi_L = 0$, the only possible non-derivative coupling of zero-spin fields is through the term

$$f\ \bar{\Psi}_L\ \Phi\ C\ \bar{\Psi}^T + \text{h.c.}$$

The complex scalars Φ must belong to the <u>symmetric tensor</u> (N(N+1)/2 - dimensional) representation of SU(N).

Gauge couplings are introduced in the usual way via the covariant derivative. For example,

$$\nabla_\mu \Psi_L = \partial_\mu \Psi_L - i\,g\,W_\mu\,\Psi_L$$

$$\nabla_\mu \Phi = \partial_\mu \Phi - i\,g\,(W_\mu\,\Phi + \Phi\,W_\mu^T)$$

$$W_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu - i\,g\,[W_\mu , W_\nu] \quad ,$$

where W_μ is a traceless hermitian matrix belonging to the adjoint representation of SU(N).

The group, as it stands, would lead to Adler-Bell-Jackiw anomalies, the resolution of which would have to depend upon either the introduction of a new set of fermions F' (F and F' being coupled with opposite chiralities to the same set of gauge bosons) or gauging a suitable anomaly-free subgroup of SU(N), which nevertheless preserves the qualitative features of interest in this note. We have not pursued this question at present and ignore it in the discussion to follow.

In general, it will be not only possible but necessary to introduce scalar fields belonging to more than one representation of SU(N). The first problem is to cause a spontaneous breaking of SU(N) to its $\Delta F = 0$ subgroup such that all the $|F| = 2$ gauge mesons become superheavy. Further breakings are then invoked to obtain the spectra of mesons and fermions of a model such as that of Ref. 1.

For illustration consider the case $N = 6$. The 36 independent generators are conveniently labelled by the direct products $\lambda_j \tau_a$ where the λ_j ($j = 0,1, \ldots, 8$) are Gell-Mann's matrices and the τ_a ($a = 0,1,2,3$) are Pauli's. Let the fermion-number be associated with $\lambda_0 \tau_3$. In order that this generator have even parity, we choose $\omega = i\tau_2$. The generators which carry no fermion-number are those which commute with $\lambda_0 \tau_3$; there are eighteen of these, $\lambda_j \tau_0$ and $\lambda_j \tau_3$ ($j = 0, \ldots, 8$). Discarding the pair with $j = 0$, we can separate the remaining 16 into two eightfolds,

$$\lambda_j^{(a)} , \; \lambda_j^{(s)} \; \tau_3 \qquad \text{(even parity)}$$

and

$$\lambda_j^{(a)} \tau_3 , \; \lambda_j^{(s)} \qquad \text{(odd parity)}$$

where the symmetric and antisymmetric members are denoted $\lambda^{(s)}$ and $\lambda^{(a)}$, respectively. It is more instructive to take sums and differences of these generators, viz.

$$V + A \sim \lambda_j \frac{1 + \tau_3}{2} \; , \qquad V - A \sim (-\lambda_j^T) \frac{1 + \tau_3}{2}$$

$$= (8,1)_{F = 0} \qquad\qquad = (1,8)_{F = 0} \; .$$

This arrangement makes it clear that the $\Delta F = 0$ currents generate the algebra of chiral $SU(3) \times SU(3)$. Moreover, it appears that the basic six-fold Ψ_L decomposes relative to this algebra according to

$$\Psi_L = (3,1)_{F = 1} + (1,\bar{3})_{F = -1} \; .$$

The remaining eighteen generators of the SU(6) algebra which carry $\Delta F = \pm 2$ are easily seen to belong to the representations (3,3) and $(\bar{3},\bar{3})$, i.e.

$$J_0(\lambda_j \tau_+) = (3,3)_{F = 2} \qquad (3V + 6A) \; ,$$

$$J_0(\lambda_j \tau_-) = (\bar{3},\bar{3})_{F = -2} \qquad \text{"} \; .$$

[In this example Φ would have 21 complex components with the $SU(3) \times SU(3) \times U(1)_F$ content,

$$21 = (6,1)_{F = 2} + (1,\bar{6})_{F = -2} + (3,\bar{3})_{F = 0} \; .]$$

To cause the principal symmetry breakdown $SU(6) \to SU(3) \times SU(3) \times U(1)_F$, introduce a 35-fold of scalar fields Σ with the self-interaction term

$$V_0(\Sigma) = - \frac{\mu^2}{2} \operatorname{Tr}(\Sigma^2) + \frac{\lambda_1}{4} (\operatorname{Tr}(\Sigma^2))^2 + \frac{\lambda_2}{4} \operatorname{Tr}(\Sigma^4) \; ,$$

with positive λ_1 and λ_2. To get a fermion mass term it is necessary to involve the 21-folds Φ and $\bar{\Phi}$ in the self-interactions. Thus the potential must in general contain fields belonging to more than one representation and the complexion of the resulting symmetry breakdown can be found only by a detailed investigation of specific cases.

IV. PARITY AND F-CONTENT OF REPRESENTATIONS OF SU(2n)

Dimensionality	J^P	F	Number of components
$4n^2 - 1$	0^+ or 1^-	2	$\frac{n(n-1)}{2}$
		0	n^2
		-2	$\frac{n(n-1)}{2}$
	0^- or 1^+	2	$\frac{n(n+1)}{2}$
		0	$n^2 - 1$
		-2	$\frac{n(n+1)}{2}$
$n(2n+1) + \overline{n(2n+1)}$	0^-	2	$\frac{n(n+1)}{2}$
		0	n^2
		-2	$\frac{n(n+1)}{2}$
	0^+	2	$\frac{n(n+1)}{2}$
		0	n^2
		-2	$\frac{n(n+1)}{2}$
$n(2n-1) + \overline{n(2n-1)}$	0^-	2	$\frac{n(n-1)}{2}$
		0	n^2
		-2	$\frac{n(n-1)}{2}$
	0^+	2	$\frac{n(n-1)}{2}$
		0	n^2
		-2	$\frac{n(n-1)}{2}$

V. CONCLUSIONS

In view of the extraordinary difficulty of neutrinoless double β-decay experiments as well as experiments on proton and deuteron stability, the hypothesis of ultimate fermion-number violation $|\Delta F| = 2$, with $(|\Delta L| = 2, \Delta B = 0)$ or $(\Delta L = +1, \Delta B = +1)$ or $(|\Delta B| = 2, \Delta L = 0)$ and with an effective strength even weaker than the F-conserving (but B- and L-violating $\Delta F = 0$, $\Delta B = -\Delta L$) interaction, has at present no experimental basis. The only justification we can give for making such a hypothesis is the theoretical one — given n 4-component Dirac fields, the maximal symmetry group for the free kinetic energy terms is not $U(1) \times SU(n) \times SU(n)$ but $SU(2n)$, as can be seen simply if one writes out the Lagrangian in terms of the 2n fields ψ_L and $(\psi^c)_L$. For the quark-lepton unified model studied in Ref.1, the maximal group with a total of sixteen 4-component Dirac fields, the symmetry group, is thus SU(32) (or SU(34) when the ζ particles of Ref.1(B) (introduced in Sec.V.2) are also taken into account). Thus SU(32) has $U(1) \times SU_L(16) \times SU_R(16)$ as a subgroup. In Ref.1 one gauged a sub-subgroup of this - i.e. $SU_L^{I+II}(2) \times SU_R^{I+II}(2) \times SU(4')$. Following Fritsch and Minkowski, [10] one may, however, set up a hierarchy of interactions starting with the full SU(32), <u>with but one basic coupling parameter</u>, and endow the gauge mesons with a succession of masses arranged through an appropriate spontaneous symmetry-breaking mechanism, generating thereby a hierarchy of effective strengths, each successively weaker than the one before [11].

One final remark; the idea of a conserved fermion-number has no natural place in the simpler versions of supersymmetric renormalizable Lagrangian theories where Majorana (rather than Dirac) fields play a fundamental role. It is difficult to construct supersymmetric theories where some bosons do not carry fermion-number two [12]. This will be discussed in detail in a separate note.

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11) One may also consider confining oneself to SU(30) (i.e. with two 2-component neutrinos $\nu_L = \nu_e$ and $(\nu^c)_L = \nu_\mu$ rather than the two 4-component neutrinos as was the case for SU(32)). The resulting theory will differ from the model studied in Ref.1(B) - particularly in respect of the exotic mesons which are relevant to $e^+ + e^- \to$ hadrons.

12) See Abdus Salam and J. Strathdee, ICTP, Trieste, preprint IC/74/42 (to be published in Phys. Rev.) for a brief discussion.