SPECTROSCOPIC FACTORS FOR (d,p) STRIPPING REACTIONS WITH DIFFERENT REPULSIVE CORE NUCLEON-NUCLEON POTENTIALS *

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ABSTRACT

Deuteron stripping reactions are considered. The short-range correlations for the nucleons in the nuclei are taken into account. The neutron-proton nuclear potential in the deuteron is taken as a long-range attractive potential surrounded by a short-range repulsive potential. We take different forms for the two parts of the neutron-proton nuclear potential. The differential cross-sections for the two nuclear reactions Si$^{28}$ (d,p)Si$^{29}$ and Ca$^{40}$ (d,p)Ca$^{41}$ have been calculated for each of the nuclear potential forms. Fitting our theoretical calculations, using the DWBA calculations, with the experimental data, a good agreement is obtained. The more reliable spectroscopic factors for these reactions have been extracted.

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I. INTRODUCTION

Deuteron stripping reactions have been proved to be a powerful tool in obtaining information about nuclear structure in nuclei and in nuclear spectroscopy. All the fine effects of the deuteron stripping are included in the form factor integral over the deuteron bound state wave function and the neutron-proton nuclear potential. The theory of these reactions had been considered firstly by Butler on a simple plane wave theory. The finite-range effects were included by many authors and they found that the cross-sections are more reduced than the zero-range theory. This theory has been developed by Huby, Refai and Satchler on a general distorted wave form theory.

In the last few years, the nuclear nucleon-nucleon potential was found to become strongly repulsive at very short distances. For a more reliable representation, the nuclear neutron-proton potential in the deuteron is taken as a short-range repulsive core surrounded by a long-range attractive potential. Taking a gaussian form for each of the two potentials, Osman extracts the exact binding energy for three nucleons on a three-body problem basis. Osman and Zimányi found that the effect of the repulsive core turns out to be a simple multiplicative factor considering the two-nucleon transfer (t,p) reactions. Osman also calculated the differential cross-sections for some direct nuclear reactions. The short-range correlations for (d,p) and (t,p) reactions have been studied in detail by Osman and Zaky, using a gaussian form for both parts of the neutron-proton potential. The spectroscopic factors for (d,p) reactions with this gaussian potential are extracted by Osman and Zaky. The short-range correlations for (d,p) reactions have been reconsidered by Osman and Goumry taking the neutron-proton potential as a Tabakin potential.

In the present work, we introduce a detailed study for deuteron stripping reactions considering the short-range correlations. To include the repulsive core effects in our calculations, we considered the neutron-proton potential as a short-range repulsive core surrounded by a long-range attractive potential. In our present calculations, we take different forms for the two parts of the potential. In each case, for the potential form, we calculated the angular distributions for the two stripping nuclear reactions $^{28}_{\text{Si}}(d,p)^{29}_{\text{Si}}$ and $^{40}_{\text{Ca}}(d,p)^{41}_{\text{Ca}}$. From the best fit between our theoretical calculations for the differential cross-sections and the experimental measurements, we extracted the spectroscopic factors for each reaction in each case.

In Sec. II, the form factors for the nuclear (d,p) stripping reactions including the repulsive core effects are introduced for the different neutron-proton potentials considered. Results of the DWBA calculations for the
differential cross-sections and the extracted spectroscopic factors are presented in Sec. III. Sec. IV is left for discussion.

II. REPULSIVE CORE EFFECTS

Repulsive core effects in (d,p) stripping reactions are studied by considering that the neutron-proton potential is composed of a short-range repulsive core surrounded by a long-range attractive potential. Also, the short-range correlations in the nuclei are introduced by considering that the different wave functions are correlated to become zero within the soft core of the potential.

For the deuteron stripping reactions, the most important features lie in the integral

\[ B_{\ell m} = i^{-\ell} (2\ell + 1)^{-\frac{1}{2}} \int \int \psi_p^* (R + \frac{1}{2} \Delta r) \psi_n^* (R - \frac{1}{2} \Delta r) \mathcal{V}_{np} (r) \]

\[ \phi_d (r) \psi_d (R) \, d\Delta r \, dR, \]

where \( \Delta r = \Delta R - \Delta r \) and \( R = \frac{1}{2} (\Delta r + \Delta R) \).

Expanding both wave functions, \( \psi_p^* (R + \frac{1}{2} \Delta r) \) and \( \psi_n^* (R - \frac{1}{2} \Delta r) \), around the centre-of-mass coordinate \( \mathcal{R} \), we have

\[ B_{\ell m} = i^{-\ell} (2\ell + 1)^{-\frac{1}{2}} \int \int I_{\ell}^{m} \psi_p^* (R) \psi_n^* (R) \phi_d^{(+)} (R) \, dR, \]

(2)
where

\[ \mathbf{I} = \int e^{\frac{i}{\hbar} \mathbf{r} \cdot \left( \mathbf{v}_p - \mathbf{v}_n \right)} V_{n_p}(r) \varphi_d(r) \, dr . \tag{3} \]

In these integrals \( \psi_p^{(-)}(R) \) is the distorted wave function in the final channel representing the interaction between the proton and the residual nucleus with nuclear optical potential \( V_p(R) \); \( \psi_d^{(+)}(R) \) is the distorted wave function in the initial channel representing the interaction between the deuteron and the target nucleus with nuclear optical potential \( V_d(R) \); \( \psi_n(R) \) is the wave function describing the bound state of the captured neutron in the residual nucleus with nuclear potential \( V_{n_t}(R) \).

The integral \( \mathbf{I} \) given by Eq. (3) is the form factor which carries all the finite-range effects and the short-range repulsive core effects.

Expanding the distorted wave functions \( \psi_d^{(+)}(R) \) and \( \psi_p^{(-)}(R) \) in partial waves, taking for \( \psi_n(R) \) a harmonic oscillator wave function and integrating over the angular part \( \Omega_\ell \), we finally get for the differential cross-section of the \((d,p)\) stripping reactions the expression

\[
\frac{d\sigma}{d\Omega} = \frac{m_p^* m_d^*}{(2\pi \hbar^2)^2} \frac{k_p}{k_d} \frac{(2I_{\ell+1})}{(2\ell+1)} \left| \sum_{\ell jm} \mathbf{H}(\ell, j) \right|^2
\]

\[
\times \left\{ \frac{(2\pi \nu)^{\frac{\ell+\frac{3}{2}}{2}}(n-\ell)!}{\left[\Gamma(n+\ell+\frac{1}{2})\right]^3} \right\} \sum_{\lambda \mu} \frac{\lambda - \mu - \ell}{\lambda \mu} (2\mu + 1)
\]

\[
\times \left\{ \frac{(\mu - m)!}{(\mu + m)!} \right\}^2 \mathbf{P}_\ell^{\mu}(\cos \theta) (\ell m \mu - m | \lambda \alpha)(\ell \mu \mu 0 | \lambda \alpha)
\]

\[
\times \int dR R^2 \psi_\lambda^*(R) \psi_\mu^*(R) U_{n\ell}^*(R) F_{RC}^2(R) \right|^2 . \tag{4}
\]
In expression (4), the factor $F^{RC}(r)$ is that factor which carries all the short-range correlations and repulsive core effects in the deuteron stripping reactions. This repulsive core form factor $F^{RC}(r)$, as is clear from Eq. (3), depends on the shapes of both the neutron-proton potential $V_{np}(r)$ and also on the deuteron bound state wave function $\varphi_d(r)$. Firstly, the correlated deuteron wave function $\varphi_d^c(r)$, following Jastrow's method, is given as

$$\varphi_d^c(r) = \mathcal{f}(r) \varphi_d(r), \quad (5)$$

where

$$\mathcal{f}(r) = \begin{cases} 1 - e^{-\frac{r^2 - d^2}{d^2}} & r > d \\ 0 & r \leq d \end{cases} \quad (6)$$

where $d$ is the radius of the repulsive core. For $V_{np}(r)$ and $\varphi_d(r)$, we take several forms, and calculate the corresponding repulsive core form factor $F^{RC}(r)$ in each case.

We have considered the shapes for $V_{np}(r)$ and the corresponding wave function $\varphi_d(r)$, the gaussian, the Yukawa, the Hulthen, the Tabakin and the square well forms.

For the gaussian form:

$$V_{np}^G(r) = -\frac{V_1^G}{\pi^{\frac{3}{2}}} \alpha_G^3 e^{-\alpha_G^2 r^2} + \frac{V_2^G}{\pi^{\frac{3}{2}}} \beta_G^3 e^{-\beta_G^2 r^2} \quad (7)$$

and

$$\varphi_d^G(r) = N_G e^{-\gamma_G^2 r^2}. \quad (8)$$

The repulsive core form factor $F^{RC}(r)$, using a gaussian neutron-proton potential and deuteron wave function, had been calculated earlier by us and is given explicitly by Eq. (5) of our Ref. 10.

For the Yukawa form:

$$V_{np}^Y(r) = -V_1^Y \frac{e^{-\alpha_Y r}}{r} + V_2^Y \frac{e^{-\beta_Y r}}{r} \quad (9)$$
and
\[ \varphi_d^Y(r) = \mathcal{N}_Y \frac{e^{-\gamma_Y r}}{r} . \] (10)

The repulsive core form factor \( F_{Y}^{RC}(R) \) as expressed by Eq. (3) for the Yukawa forms for neutron-proton potential and deuteron wave function, this form factor has been calculated. The explicit form for \( F_{Y}^{RC}(R) \) is given in Appendix A.

For the Hulthén form:
\[ V_{np}^H(r) = -V_1 \frac{e^{-\alpha_H r}}{1 - e^{-\alpha_H r}} + V_2 \frac{\alpha_{\beta_H r}}{1 - e^{-\beta_H r}} \] (11)

and
\[ \varphi_d^H(r) = \mathcal{N}_H \frac{e^{-\gamma_H r}}{r} \left( 1 - e^{-\alpha_H r} \right) , \] (12)

where
\[ \gamma_H = \left( \frac{M \varepsilon_d}{\hbar^2} \right)^{\frac{1}{2}} \] (13)

and
\[ \mathcal{N}_H = \left\{ \frac{2 \gamma_H (\gamma_H + \alpha_H) (2 \gamma_H + \alpha_H)}{\alpha_H^2} \right\}^{\frac{1}{2}} . \] (14)

In this way, and inserting Eqs. (11) and (12) into Eq. (3) making use of Eqs. (5) and (6), the short-range correlation form factor with Hulthén forms for both \( V_{np}(r) \) and \( \varphi_d(r) \) can be calculated. The repulsive core form factor \( F_{np}^{RC}(R) \) in the Hulthén form case is calculated and is given explicitly in Appendix B.
For the Tabakin form: We used the neutron-proton potential suggested by Tabakin, which is a non-local separable potential defined as:

\[ V_{np}(r, r') = \sum_{\alpha M L} \left[ -g_{\alpha L}(r) g_{\alpha L}(r') + h_{\alpha L}(r) h_{\alpha L}(r') \right] \int_{L'}^{L} \phi_{\alpha L}(r) \int_{L'}^{L} \phi_{\alpha L}(r') \]

where \( \alpha \) denotes the quantum numbers JTS. The function \( \phi_{\alpha L}(r) \) is a normalized eigenstate of total angular momentum \( J \) and its z component \( M \); it is a combination of an orbital angular momentum state \( Y_{L}^{M} (\hat{r}) \) and a total spin state \( \chi_{S}^{L} \) and \( \lambda = \frac{n^2}{m} \) (\( m \) is the nucleon mass). The symbols \( g_{\alpha L}(r) \) and \( h_{\alpha L}(r) \) refer to the attractive and repulsive parts of the potential respectively.

The deuteron wave function is taken in the momentum representation as:

\[ \psi_{d}^{T}(k') = \sqrt{\frac{1}{k'^2 + \alpha_{T}^2}} \left( \frac{1}{k'^2 + \beta_{T}^2} \right) \]

where

\[ \sqrt{\frac{1}{T}} = \pi^{-2} \frac{\alpha_{T} \beta_{T}}{(\beta_{T} - \alpha_{T})^{2}} \]

and

\[ \alpha_{T} = \left( \frac{M \epsilon_{d}}{\hbar^2} \right)^{\frac{1}{2}} \]

where \( M \) and \( \epsilon_{d} \) are the reduced nucleon mass and the deuteron binding energy, respectively.

The form factor with short-range correlations for Tabakin neutron-proton potential as given by Eq. (15) has been calculated by Osman and Goumry. These authors calculated \( F_{T}^{RC}(R) \) in two cases. Firstly, by taking for the \( g_{\alpha L}(r) \) and \( h_{\alpha L}(r) \) parts of the potential a Yamaguchi type form where
$F^{RC}_{T}(R)$ is given by Eqs. (3.37) and (3.38) of Ref. 11. In the second case, they calculated the repulsive core form factor $F^{RC}_{T}(R)$ using for $\xi_{AL}(r)$ and $h_{AL}(r)$ a short-range repulsive potential of the Breit et al. 15 form, expressing $F^{RC}_{T}(R)$ explicitly by Eqs. (3.57) and (3.58), of Ref. 11.

For the square well form: We have used for the attractive part of the neutron-proton potential an attractive square well form. For the repulsive part, we take a repulsive hard core. Solving the Schrödinger equations for these different regions of the potentials and matching the wave functions at the boundaries, we get the corresponding bound state deuteron wave function. The square well potential used is expressed as

$$V_{np}^{SW}(r) = \begin{cases} V_2 & 0 < r < d \\ -V_1 & d \leq r \leq \alpha \\ 0 & \alpha < r \leq \infty \end{cases}$$

where $d$ is the hard core radius, $\alpha$ is the range of nuclear forces and $V_1 < V_2$.

III. RESULTS AND CALCULATIONS

In the present calculations, we take into account the short-range correlations between nucleons inside the nuclei. We considered the nucleon-nucleon interaction as a short-range repulsive potential surrounded by a long-range attractive potential, so that the neutron-proton nuclear potential $V_{np}(r)$ and the deuteron bound state wave function $\varphi_d(r)$ are taken in different forms. We considered for them the gaussian, the Yukawa, the Hulthen, the Tabakin and the square well shapes. These forms are given here by Eqs. (7) and (8), (9) and (10), (11) and (12), (15) and (16) and (19), respectively, for both $V_{np}(r)$ and $\varphi_d(r)$, considering the repulsive core effects. The parameters introduced in these forms and present in $F^{RC}_{T}(R)$ are listed in Table I. At the same time, we used in our calculations for all the potential forms considered the values for the repulsive core radius $d = 0.4 \text{ fm}$, the nucleon-nucleon correlation parameter $Z^2 = 6.0 \text{ fm}^{-2}$ and for the bound neutron oscillator size parameter $\nu = 0.48/\lambda^{1/3} \text{ fm}^{-2}$.

For numerical calculations of the DWBA for the $(d,p)$ stripping reactions, a Woods-Saxon form for the optical potentials is suggested as

$$V_{opt.}^{opt.}(R) = -V(1+e^{x})^{-1} - i\left[ W - \frac{dW}{dx}(d_{d})\right](1+e^{x})^{-1} + V_{C}. \quad (20)$$
where
\[ x = \left( R - r \right) A^{1/3} / a \]
\[ x' = \left( R - r' \right) A^{1/3} / a' \]  
and \( V_c \) is the Coulomb potential with radius \( r_c A^{1/3} \).

The parameters of the proton and deuteron optical potentials \( V_p(R) \) and \( V_d(R) \) are listed in Table II.

Two deuteron stripping reactions have been considered. We studied the two nuclear reactions \( Si^{28}(d,p)Si^{29} \) and \( Ca^{40}(d,p)Ca^{41} \). The first reaction is studied at incident deuteron energy of 18.0 MeV, leaving the residual nucleus \( Si^{29} \) in its ground state. The second reaction is considered at incident energy of 7.0 MeV leaving the final nucleus \( Ca^{41} \) in its ground state. The experimental data for these two reactions are taken from Mermaz et al. \(^{16}\) and from Lee et al. \(^{17}\) for the two reactions considered, respectively.

Our theoretical calculations for the differential cross-section for these two reactions have been performed, with short-range correlations taken into account, using the expression \( (4) \). The numerical calculations have been made using the DWBA calculations. We used the computer programme JULIE which is modified by Osman \(^{8}\) to include the repulsive core effects in \( (d,p) \) stripping reactions. We introduce in Table II the optical potential parameters used in our DWBA calculations for the reactions considered. The result of our DWBA calculations are shown in Figs. 1 and 2. In Figs. 1 and 2, the calculated angular distributions have been fitted to experimental data for the two reactions \( Si^{28}(d,p)Si^{29} \) and \( Ca^{40}(d,p)Ca^{41} \), respectively. The heavy points are the experimental data. The curves represent our present calculations. The different curves refer to the form of the neutron-proton potential and the bound state deuteron wave function considered as shown on the figures.

From the fitting of the present calculations of the differential cross-section, taking into account the short-range correlations, with the experimental angular distributions, we extract the spectroscopic factors for these reactions. The extracted spectroscopic factors for these reactions in each case for the neutron-proton potential and the deuteron wave function are introduced in Table III.
IV. DISCUSSION

From the present calculations, it is clear that the DWBA theory enables us to extract more reasonable values for the spectroscopic factors of deuteron stripping reactions. Thus, to improve the simple theory which does not account for all features of the reaction, we must improve the physical basis of the theory by including refinements to both the kinematic part of the calculation and to the optical potential involved.

In the present work, we have considered the effect of the short-range correlations in nuclei as a repulsive soft core in the nucleon-nucleon interaction. So, we take the nucleon-nucleon interaction as a short-range repulsive core surrounded by a long-range attractive potential. From Figs. 1 and 2 and the good agreement between our present calculations and the experimental data, it seems that this representation for the interaction is more realistic. Also, there is no great difference in the calculations, taking different shapes and forms for the neutron-proton potential or the bound state deuteron wave function. At the same time, from Table III, we can conclude that taking short-range correlations into consideration, more correct spectroscopic factors for the deuteron stripping reactions are extracted.

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In this appendix we introduce the repulsive core form factor as

given by the expression (3), for the case of Yukawa forms for both the neutron-proton potential and the deuteron wave function. Let us substitute

\( V_{np}(r) \) and \( \phi_d(r) \) by their expressions given by Eqs. (9) and (10), and introduce these forms into Eq. (3) using the correlated wave function given by Eqs. (5) and (6).

Solving the different integrals and collecting them, we get for the repulsive core form factor \( F_y^{RC}(R) \) in the Yukawa form case, the expression

\[
F_y^{RC}(R) = -8\pi V_1 \sqrt{\frac{\hbar}{2m}} \left\{ \left[ k^2 + K(R)^2 \right]^{-\frac{1}{2}} \tan^{-1}\left\{ \frac{k^2 + K(R)^2}{4L^2} \right\} \right. \\
+ 4\pi V_1 \sqrt{\frac{\hbar}{2m}} \sum_{n} \frac{(-1)^n}{(2n+1)} \left[ 2^{2n+z} \left\{ k^2 + K(R)^2 \right\}^n \right] \\
\left. + \frac{L}{2z} \frac{\Gamma(n+\frac{1}{2})}{\Gamma(n+1)} \left\{ \Gamma(n+\frac{1}{2}) \right\} \right) + \\
+ 8\pi V_2 \sqrt{\frac{\hbar}{2m}} \left\{ k^2 + K(R)^2 \right\}^{-\frac{1}{2}} \tan^{-1}\left\{ \frac{k^2 + K(R)^2}{4M^2} \right\} \\
- 4\pi V_2 \sqrt{\frac{\hbar}{2m}} e^{\frac{M^2}{18z^2} z^2 d + \frac{M^2}{8z^2} \left( \frac{z}{M} \right)^2} e^{-\frac{M^2}{8z^2} z^2 -1} \right.
\]

\[
\sum_{n} \frac{(-1)^n}{(2n+1)} \left[ 2^{2n+z} \left\{ k^2 + K(R)^2 \right\}^n \right] \\
- 4\pi V_2 \sqrt{\frac{\hbar}{2m}} \sum_{n} \frac{(-1)^n}{(2n+1)} \left[ 2^{2n+z} \left\{ k^2 + K(R)^2 \right\}^n \right] 
\]
\[
\left\{ \frac{\Gamma\left(\frac{1}{2}\right)}{\Gamma(n+1)} \Gamma_1\left(n + \frac{1}{2}; \frac{1}{2}; \frac{M^2}{4z^2}\right) +
\right.
\]
\[
+ \frac{m}{2z^2} \frac{\Gamma\left(-\frac{1}{2}\right)}{\Gamma(n + \frac{1}{2})} \Gamma_1\left(n + 1; \frac{3}{2}; \frac{M^2}{4z^2}\right) \right\},
\]
(A.1)

where
\[
k^2 = \frac{4m}{\hbar^2} \left( E_d + E_{ex} \right)
\]
(A.2)

and
\[
K^2(R) = \frac{4m}{\hbar^2} \left[ V_p(R) + V_n(R) - V_d(R) \right],
\]
(A.3)

\(E_d\) is the deuteron binding energy and \(E_{ex}\) is the excitation energy of the residual nucleus. Also, we have
\[
L = \alpha_Y + \gamma_Y,
\]
\[
M = \beta_Y + \gamma_Y.
\]
(A.4)

The finite-range effects without repulsive forces can be obtained from \(F^{RC}_Y(R)\), given by the expression (A.1), by letting \(V_2 = 0, d = 0\) and \(Z \rightarrow \infty\).

If we denote the finite-range form factor without repulsive forces as \(F^{FR}_Y(R)\), we obtain
\[
F^{FR}_Y(R) = -8\pi V_1 N_Y \left\{ K^2(R) \right\}^{-\frac{1}{2}} \tan^{-1} \left\{ \frac{K^2(R)^{\frac{1}{2}}}{4L^2} \right\}.
\]
(A.5)
Also, the zero-range form factor $F^Z_R(R)$ can be obtained from (A.5) by further letting $L \to 0$, which gives

$$F^Z_R(R) = -\frac{4\pi^2}{k} V_1.$$  \hfill (A.6)
Here, we have to calculate the short-range correlation form factor in the case where a Hulthén form is taken for both the neutron-proton potential and for the deuteron bound state wave function. The Hulthén shapes for both $V_{np}(r)$ and $\varphi_d(r)$ are given by Eqs. (11) and (12) respectively. Introducing Eqs. (11) and (12) with Eqs. (5) and (6) into Eq. (3), we get the integral which carries all the short-range correlations with the finite-range effects. Integrating over $r$ the different components of the integral, and collecting them, we obtain the following expression for the repulsive core form factor with Hulthén forms:

$$F_R^C(R) = -16\pi N_H V_1 \left[ 4 \left( \chi_H + \chi_H \right)^2 + \left\{ k^2 + K^2(R) \right\}^2 \right]^{-1}$$

$$-2i\pi^{3/2} N_H V_1 \left\{ k^2 + K^2(R) \right\}^{-1/2} e^{z^2d^2}$$

$$\times \left\{ e^{\frac{2(\chi_H + \chi_H) - i\left\{ k^2 + K^2(R) \right\}^{1/2}}{4z}} \right\} \times \left[ 1 - \Phi \left( \frac{2(\chi_H + \chi_H) - i\left\{ k^2 + K^2(R) \right\}^{1/2}}{4z} \right) \right]$$

$$-e^{\frac{2(\chi_H + \chi_H) + i\left\{ k^2 + K^2(R) \right\}^{1/2}}{4z}} \times \left[ 1 - \Phi \left( \frac{2(\chi_H + \chi_H) + i\left\{ k^2 + K^2(R) \right\}^{1/2}}{4z} \right) \right]$$

-14-
\[-4 i \pi N_H V_2 \beta_H^{-1} \left\{ k^2 + K^2(R) \right\}^{3/2}.\]

\[
\left\{ \begin{array}{l}
\Psi \left( \frac{2(\gamma_H + \beta_H) + i \xi k^2 + K^2(R) \beta_H^{-1/2}}{2 \beta_H} \right) \\
- \Psi \left( \frac{2(\gamma_H + \beta_H) - i \xi k^2 + K^2(R) \beta_H^{-1/2}}{2 \beta_H} \right)
\end{array} \right. \\
- \left[ \Psi \left( \frac{2(\gamma_H + \alpha_H + \beta_H) + i \xi k^2 + K^2(R) \beta_H^{-1/2}}{2 \beta_H} \right) \\
- \Psi \left( \frac{2(\gamma_H + \alpha_H + \beta_H) - i \xi k^2 + K^2(R) \beta_H^{-1/2}}{2 \beta_H} \right) \right] \right\}
\]

\[-\pi^n N_H V_2 e^{\frac{z^2}{2}} \sum_{nm} (-1)^n \binom{2m+1}{2n+1}^{-1} 4m-4n+1.\]

\[
\frac{z^{2m-2n-1}}{\beta_H^{2m-1}} \left\{ k^2 + K^2(R) \right\}^n.
\]
where $k^2$ and $k^2(R)$ have the same expressions (A.2) and (A.3) introduced in Appendix A.

The finite-range effects without repulsive forces can be obtained from (B.1) by letting $V_2 = 0$, $d = 0$ and $Z \to \infty$. The finite-range form factor $F^{FR}(R)$ without repulsive forces for Hulthén forms is given by

$$
F^{FR}_H (R) = -16 \pi \chi H \chi_1 \left[ 4 (\chi_H + \chi_1) + \left\{ k^2 + k^2(R) \right\} \right]^{-1}.
$$

(B.2)
REFERENCES


### TABLE I

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<th>β</th>
<th>γ</th>
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<td>(fm$^{-1}$)</td>
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<td>115.9</td>
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</table>

a) See Ref. 6.
b) See Ref. 11.
## TABLE II

### Optical model parameters

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<tr>
<th>Channel</th>
<th>Number of set</th>
<th>V (MeV)</th>
<th>r_v (fm)</th>
<th>a_r (fm)</th>
<th>W (MeV)</th>
<th>W_d (MeV)</th>
<th>r_i (fm)</th>
<th>a_i (fm)</th>
<th>r_c (fm)</th>
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</thead>
<tbody>
<tr>
<td>Proton</td>
<td>I</td>
<td>44.0</td>
<td>1.25</td>
<td>0.65</td>
<td>0</td>
<td>38.5</td>
<td>1.25</td>
<td>0.47</td>
<td>1.25</td>
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<td></td>
<td>II</td>
<td>50.3</td>
<td>1.20</td>
<td>0.65</td>
<td>0</td>
<td>44.0</td>
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<td>0.47</td>
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<td>Deuteron</td>
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<td>0.943</td>
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### Extracted spectroscopic factors from the DWBA calculations

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<th>Reaction</th>
<th>n-p potential form</th>
<th>Incident energy (MeV)</th>
<th>$\lambda$</th>
<th>$J^\pi$</th>
<th>Spectroscopic factors Present work</th>
<th>Spectroscopic factors Previous work</th>
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</thead>
<tbody>
<tr>
<td>$^{28}\text{Si}(d,p)^{29}\text{Si}$</td>
<td>Gaussian</td>
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<tr>
<td>$^{40}\text{Ca}(d,p)^{41}\text{Ca}$</td>
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</table>
FIGURE CAPTIONS

Fig. 1  The angular distributions of the nuclear stripping reaction $^{28}\text{Si}(d,p)^{30}\text{Si}$ of incident deuteron energy 18.0 MeV leaving the residual nucleus $^{30}\text{Si}$ in its ground state. The solid curves are our present calculations. The optical model parameters used in our calculations are set I for the proton and set I for the deuteron, as listed in Table II. The experimental data are taken from Ref. 16.

Fig. 2  The angular distributions of the nuclear stripping reaction $^{40}\text{Ca}(d,p)^{42}\text{Ca}$ of incident deuteron energy 7.0 MeV leaving the residual nucleus $^{42}\text{Ca}$ in its ground state. The solid curves are our present calculations. The optical model parameters used in our calculations are set II for the proton and set II for the deuteron, as listed in Table II. The experimental data are taken from Ref. 17.
Si$^{28}$ (d,p)Si$^{29}$

$E_d = 18.0$ MeV

$\ell = 0; J^\pi = \frac{1^+}{2}$
$^{40}\text{Ca}^{(d,p)}{^{41}\text{Ca}}$

$E_d = 7.0$ MeV

$l = 3; J^\pi = \frac{T^-}{2}$

---

**Fig. 9**