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FEYNMAN RULES FOR SUPERFIELDS

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ABSTRACT

The notion of functional differentiation with respect to superfields is defined and used to set up formal rules for computing Green's functions and scattering amplitudes in supersymmetric models.

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I. INTRODUCTION

A number of renormalizable lagrangian models have been given recently which exhibit an underlying symmetry between fermions and bosons [1]. Although this so-called supersymmetry has yet to manifest itself in the real world, it is characterized by some unusual features which make it worthy of study [2].

Fundamentally, supersymmetry represents an extension of the Poincaré group [3] and could, indeed, be looked upon as a reflection of the structure of space-time. In this quasi-geometrical view, the extended Poincaré group can be made to act upon an 8-dimensional "extended spacetime" whose points are labelled by the pair (x_μ, θ_α), where x_μ denotes the usual minkowskian coordinate and θ_α is an <u>anticommuting</u> c-number Majorana spinor. The action of the new transformations on this space is given by

$$x_\mu \to x_\mu + \frac{i}{2}\bar{\varepsilon}\gamma_\mu\theta \ ,$$

$$\theta_\alpha \to \theta_\alpha + \varepsilon_\alpha \ , \qquad (1.1)$$

where the parameter ε_α is an anticommuting Majorana spinor. Our purpose here is to discuss the formulation of field theories on the extended space-time *).

A typical field, the scalar $\Phi(x, \theta)$, may be expanded in powers of θ. Owing to the anticommutation property, $\{\theta_\alpha, \theta_\beta\} = 0$, such expansions must terminate. In fact only sixteen independent monomials can be made from the θ's. A convenient form [4] of the expansion is given by

$$\begin{aligned}\Phi(x,\theta) \ = \ & A(x) + \bar{\theta}\psi(x) \\ & + \frac{1}{4}\bar{\theta}\theta\ F(x) + \frac{1}{4}\bar{\theta}\gamma_5\theta\ G(x) + \frac{1}{4}\bar{\theta}i\gamma_\nu\gamma_5\theta\ A_\nu(x) \\ & + \frac{1}{4}\bar{\theta}\theta\ \bar{\theta}\chi(x) + \frac{1}{32}(\bar{\theta}\theta)^2\ D(x) \ ,\end{aligned} \qquad (1.2)$$

where the coefficients are ordinary fields : A, F, D are scalars, G is a pseudoscalar, A_ν is an axial vector, ψ and χ are Dirac spinors. One sees that half of the components are bosons and the other half fermions. The new dimensions of the extended space-time thus give rise to new degrees of freedom that are spin-like and finite in number.

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*) We shall be concerned with linear realizations only. One can avoid the extension of space-time by working with non-linear realizations wherein the supersymmetry is broken spontaneously. This approach was taken by Volkov and Akulov, Ref.3, and extended by Zumino, Ref.2.

It is possible to define the operation of differentiation with respect to the anticommuting coordinate θ and, in particular, the covariant derivative

$$D_\alpha \Phi = \frac{\partial\Phi}{\partial\bar{\theta}^\alpha} - \frac{i}{2} (\gamma_\mu \theta)_\alpha \frac{\partial\Phi}{\partial x_\mu} \quad . \tag{1.3}$$

This operator transforms as a spinor under the action of the Lorentz group and is invariant with respect to the "supertranslations" (1.1) whose action on Φ is given by

$$\delta\Phi = -\bar{\varepsilon}^\alpha \left(\frac{\partial\Phi}{\partial\bar{\theta}^\alpha} + \frac{i}{2} (\gamma_\mu \theta)_\alpha \frac{\partial\Phi}{\partial x_\mu}\right) \quad . \tag{1.4}$$

A number of properties of the operator D_α is derived in Ref. 4. The most important of these is the anticommutation rule

$$\{D_\alpha, D_\beta\} = -(\gamma_\mu C)_{\alpha\beta}\, i \frac{\partial}{\partial x_\mu} \quad , \tag{1.5}$$

where C denotes the charge conjugation matrix.

The operator D_α is used to define the so-called chiral superfields $\Phi_\pm(x, \theta)$ by means of the covariant differential constraints

$$(1 \mp i\gamma_5) D\, \Phi_\pm(x, \theta) = 0 \quad . \tag{1.6}$$

These equations imply a number of relations among the coefficients in the expansion (1.2), viz.,

$$G_\pm = \pm iF_\pm \,, \qquad A_{\nu\pm} = \pm i\partial_\nu A_\pm \,, \qquad D_\pm = -\partial^2 A_\pm \,, \qquad \chi_\pm = -i \not\partial \psi_\pm \,,$$

where $(1 \mp i\gamma_5)\psi_\pm = 0$. It follows that the chiral superfields can be represented by the expansions:

$$\Phi_{\pm}(x,\theta) = \exp\left(\mp\tfrac{1}{4}\bar{\theta}\gamma_\mu\gamma_5\theta \frac{\partial}{\partial x_\mu} \right)\left[A_{\pm}(x) + \bar{\theta}\psi_{\pm}(x) + \tfrac{1}{4}\bar{\theta}(1 \pm i\gamma_5)\theta\, F_{\pm}(x)\right] \quad . \qquad (1.7)$$

The set of chiral superfields is closed under multiplication,

$$\Phi_+(x, \theta)\ \Phi'_+(x, \theta) = \Phi''_+(x, \theta) \ .$$

This property, which is a direct consequence of the linear defining condition (1.6), is very useful in the construction of renormalizable Lagrangians, and the following sections will deal exclusively with chiral superfields.

It can be objected that the proposed transformation law (1.1) is not consistent in that to the coordinates x_μ , which are usually taken to be real numbers, are added the quantities $(i/2)\bar{\varepsilon}\gamma_\mu\theta$ which are not numbers. (Notice, for example, that $(\bar{\varepsilon}\gamma_\mu\theta)^5 = 0$.) Clearly some generalized interpretation of the space-time coordinate x_μ is needed here. However, this is not a matter of any practical importance since one could easily suppress all reference to anticommuting c-numbers. Physical consequences of the supersymmetry could be extracted by requiring the scattering operator to commute with the "supercharge", S_α, whose action on the field components is defined by substituting

$$\delta\Phi = [\,\Phi,\ i\bar{\varepsilon}^\alpha S_\alpha]$$

on the left-hand side of (1.4) and comparing coefficients. (For consistency, one must suppose that the spinorial quantities ε and θ anticommute among themselves as well as with the fermionic components ψ and χ.) The commutation rules satisfied by P_μ and $J_{\mu\nu}$, the generators of infinitesimal Poincaré transformations, must be supplemented by new rules involving S_α . These are:

$$[S_\alpha, P_\mu] = 0 \ ,$$

$$[S_\alpha, J_{\mu\nu}] = \tfrac{1}{2}(\sigma_{\mu\nu}S)_\alpha \ ,$$

$$\{S_\alpha, S_\beta\} = -(\gamma_\mu C)_{\alpha\beta}\, P_\mu \ . \qquad (1.8)$$

The problem of constructing unitary representations of this system and deriving selection rules has been considered elsewhere [5]. In this paper we are attempting to show that adherence to superfields and anticommuting c-numbers does not appear to lead to any difficulties.

Up to now the concept of the superfield has played a subsidiary though useful role in guiding the construction of lagrangian models. In effect these fields have been used to make action functionals which are manifestly supersymmetric. To proceed with any dynamical calculation, however, one was forced to relinquish the manifest symmetry and express the Lagrangian in terms of component Fermi and Bose fields of the usual kind. With the Lagrangian so expressed, one could follow the standard programme for computing Green's functions and scattering amplitudes. This approach seems to us a retrograde one. It would be more satisfying if one could set up the Feynman rules and make calculations in the manifestly covariant notation. The purpose of this paper is to show that such a course can indeed be followed. We do not claim any computational advantage for the methods presented here. Rather, we should like this formulation to be seen as a preliminary exploration of a methodology wherein supersymmetry is realized in a succint and obvious manner.

II. THE FUNCTIONAL DERIVATIVE

For simplicity, we shall begin by considering functionals of a neutral scalar supermultiplet $\Phi_\pm(x,\theta)$. The chiral components of $\Phi_\pm$ are defined by the expansions

$$\Phi_\pm(x,\theta) = \exp\left[\mp \frac{1}{4}\,\bar\theta \not\partial \gamma_5 \theta\right] \left(A_\pm(x) + \bar\theta\,\frac{1 \pm i\gamma_5}{2}\,\psi(x) + \frac{1}{4}\,\bar\theta(1 \pm i\gamma_5)\theta\, F_\pm(x)\right) , \tag{2.1}$$

where $\psi(x)$ is a Majorana spinor and $A_\pm$, $F_\pm$ are complex boson fields subject to the reality conditions $A_- = A_+^*$, $F_- = F_+^*$. Under space reflections the fields A_+ and F_+ are carried into A_- and F_-, respectively.

Let Γ be a functional of Φ_+ and Φ_-. Corresponding to the infinitesimal variations $\delta\Phi_\pm$ the variation in Γ may be written *)

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*) This integral defines an invariant bilinear form. The forms

$$\int dx \left(-\frac{1}{2}\,\bar D D\right) \left(\Phi_\pm^{(1)}\,\Phi_\pm^{(2)}\right) = \int dx \left(A_\pm^{(1)}\,F_\pm^{(2)} + F_\pm^{(1)}\,A_\pm^{(2)} - \bar\psi_\pm^{(1)c}\,\psi_\pm^{(2)}\right) ,$$

are invariant under supertransformations and proper Lorentz transformations.

$$\delta\Gamma = \int dx \left(-\frac{1}{2}\bar{D}D\right) \left(\delta\Phi_+(x,\theta) \frac{\delta\Gamma}{\delta\Phi_+(x,\theta)} + \delta\Phi_-(x,\theta) \frac{\delta\Gamma}{\delta\Phi_-(x,\theta)}\right) . \tag{2.2}$$

This formula serves to define the derivatives of Γ with respect to Φ_+ and Φ_-. These derivatives, $\delta\Gamma/\delta\Phi_\pm$, are themselves chiral superfields and their components may be expressed in terms of the derivatives of Γ with respect to the component fields $A_\pm(x)$, etc. It is natural to define these latter derivatives by the linear form,

$$\delta\Gamma = \int dx \left(\delta A_+(x) \frac{\delta\Gamma}{\delta A_+(x)} + \delta A_-(x) \frac{\delta\Gamma}{\delta A_-(x)} + \delta\bar{\psi}(x) \frac{\delta\Gamma}{\delta\bar{\psi}(x)} + \delta F_+(x) \frac{\delta\Gamma}{\delta F_+(x)} + \delta F_-(x) \frac{\delta\Gamma}{\delta F_-(x)}\right) . \tag{2.3}$$

Comparing the expressions (2.2) and (2.3) one finds

$$\frac{\delta\Gamma}{\delta\Phi_\pm(x)} = \exp\left[\mp \frac{1}{4}\bar{\theta}\not{\partial}\gamma_5\theta\right] \left(\frac{\delta\Gamma}{\delta F_\pm(x)} - \bar{\theta}\frac{1 \pm i\gamma_5}{2} \frac{\delta\Gamma}{\delta\bar{\psi}(x)} + \frac{1}{4}\bar{\theta}(1 \pm i\gamma_5)\theta \frac{\delta\Gamma}{\delta A_\pm(x)}\right) . \tag{2.4}$$

This formula serves quite generally for translating ordinary functional derivatives into the superfield form. A particular application of it - with Γ set equal to $\Phi_\pm(x',\theta')$ - serves to define what might be called the "super delta-function". Thus, one finds

$$\frac{\delta\Phi_\pm(x',\theta')}{\delta\Phi_\pm(x,\theta)} = \exp\left[\mp \frac{1}{4}\bar{\theta}\not{\partial}\gamma_5\theta \mp \frac{1}{4}\bar{\theta}'\not{\partial}'\gamma_5\theta'\right] \frac{1}{4}(\bar{\theta}-\bar{\theta}')(1\pm i\gamma_5)(\theta-\theta')\,\delta(x-x')$$

$$\equiv \delta_\pm(x,\theta;x'\theta') \tag{2.5}$$

and, on the other hand,

$$\frac{\delta\Phi_\mp(x',\theta')}{\delta\Phi_\pm(x,\theta)} = 0 . \tag{2.6}$$

The functions $\delta_\pm(x,\theta;\, x'\theta')$ enjoy the following properties:

(a) symmetry under the interchange $(x,\theta) \leftrightarrow (x',\theta')$;

(b) Poincaré invariance;

(c) supersymmetry, viz.,

$$\delta_\pm(x + \frac{i}{2}\,\varepsilon\gamma\theta\ ,\ \theta + \varepsilon\ ;\ x' + \frac{i}{2}\,\varepsilon\gamma\theta'\ ,\ \theta' + \varepsilon) = \delta_\pm(x,\theta;x',\theta') \quad ; \tag{2.7}$$

(d) chirality

$$(1 \mp i\gamma_5)D\ \delta_\pm(x,\theta;\, x',\theta') = 0\ ; \tag{2.8}$$

(e) the integral identities

$$\int dx(-\tfrac{1}{2}\bar{D}D)\Big(\Psi_\pm(x,\theta)\ \delta_\pm(x,\theta;x',\theta')\Big) = \Psi_\pm(x',\theta') \tag{2.9}$$

for arbitrary chiral superfields $\Psi_\pm$ (this means that the functions $\delta_\pm$ serve as identity distributions).

It would appear that, although we do not have a measure of the usual sort on the space of x and θ, it is possible to treat the operation

$$\int dx(-\tfrac{1}{2}\bar{D}D)$$

as a kind of generalized integration. Pursuing the analogy, we may define the functional Taylor expansion,

$$\Gamma(\Phi_+,\Phi_-) = \sum_{n,m} \frac{1}{n!m!} \prod_1^{n+m} \int dx_j\Big(-\frac{1}{2}\,\bar{D}_j\ D_j\Big)\Big[K_{nm}(x_1\theta_1,\ldots,x_{n+m}\ \theta_{n+m})\ \cdot$$
$$\cdot\ \Phi_+(x_1\theta_1)\cdots\ \Phi_+(x_n\theta_n)\ \Phi_-(x_{n+1}\ \theta_{n+1})\cdots\ \Phi_-(x_{n+m}\ \theta_{n+m})\Big]\ , \tag{2.10}$$

where the coefficient distributions K_{nm} satisfy the chiral conditions

$$(1 - i\gamma_5)D_j\, K_{nm}(1,\ldots,\; n+m) = 0 \qquad \text{for } j = 1,\ldots,n \;,$$

$$(1 + i\gamma_5)D_j\, K_{nm}(1,\ldots,\; n+m) = 0 \qquad \text{for } j = n+1,\ldots,n+m \,.$$

A typical example of a functional defined over the superfields $\Phi_\pm$ is the classical (local) action

$$S(\Phi_+,\Phi_-) = \int dx \left[\frac{1}{8}\,(\bar{D}D)^2(\Phi_-\Phi_+) - \frac{1}{2}\,\bar{D}D\big(V(\Phi_+) + V(\Phi_-)\big)\right] \;, \qquad (2.11)$$

where V is a polynomial *). To evaluate the derivatives of S it is necessary to make use of the operator identities [4]

$$\begin{aligned} \frac{1}{2}(\bar{D}D)^2 &= \bar{D}\,\frac{1 - i\gamma_5}{2}\,D\,\bar{D}\,\frac{1 + i\gamma_5}{2}\,D + \bar{D}\not{\partial}\gamma_5 D \\ &= \bar{D}\,\frac{1 + i\gamma_5}{2}\,D\,\bar{D}\,\frac{1 - i\gamma_5}{2}\,D - \bar{D}\not{\partial}\gamma_5 D \;. \end{aligned} \qquad (2.12)$$

Make an infinitesimal variation in (2.11),

$$\begin{aligned} \delta S &= \int dx \left[\frac{1}{8}\,(\bar{D}D)^2\left(\delta\Phi_-\Phi_+ + \delta\Phi_+\Phi_-\right) - \frac{1}{2}\,\bar{D}D\left(\delta\Phi_+\, V'(\Phi_+) + \delta\Phi_-\, V'(\Phi_-)\right)\right] \\ &= \int dx \left[\frac{1}{4}\,\bar{D}\,\frac{1 - i\gamma_5}{2}\,D\,\bar{D}\,\frac{1 + i\gamma_5}{2}\,D\,(\delta\Phi_-\Phi_+)\right. \\ &\quad + \frac{1}{4}\,\bar{D}\,\frac{1 + i\gamma_5}{2}\,D\,\bar{D}\,\frac{1 - i\gamma_5}{2}\,D\,(\delta\Phi_+\Phi_-) \\ &\quad + \frac{1}{4}\,\bar{D}\not{\partial}\gamma_5 D\left(\delta\Phi_-\Phi_+ - \delta\Phi_+\Phi_-\right) \\ &\quad \left. - \frac{1}{2}\,\bar{D}D\left(\delta\Phi_-\, V'(\Phi_-) + \delta\Phi_+\, V'(\Phi_+)\right)\right] \;. \end{aligned}$$

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*) One may ask whether the action functional (2.11) can be expressed in the multilocal form (2.10). The answer is, indeed, yes. The coefficient K_{11}, in particular, is given by

$$K_{11}(1,2) = -\frac{1}{4}\,(\bar{D}D)_1\,\delta_-(1,2) - \frac{1}{4}\,(\bar{D}D)_2\,\delta_+(1,2) \;.$$

Making use of the chiral properties of $\Phi_\pm$ and $\delta\Phi_\pm$ one can reduce this immediately to the form

$$\delta S = \int dx \left(-\frac{1}{2}\bar{D}D\right)\left[\delta\Phi_+\left(-\frac{1}{2}\bar{D}D\Phi_- + V'(\Phi_+)\right) - \delta\Phi_-\left(-\frac{1}{2}\bar{D}D\Phi_+ + V'(\Phi_-)\right)\right] + \oint d\Sigma_\mu \frac{1}{4}\bar{D}\gamma_\mu\gamma_5 D\left(\Phi_+\delta\Phi_- - \Phi_-\delta\Phi_+\right) . \tag{2.13}$$

Discarding the surface term *) and comparing the remainder with (2.2), one obtains the functional derivatives

$$\frac{\delta S}{\delta\Phi_\pm} = -\frac{1}{2}\bar{D}D\Phi_\mp + V'(\Phi_\pm) . \tag{2.14}$$

Setting these equal to zero gives the classical equations of motion in superfield form. In Sec. III the problem of quantizing this system will be considered.

The functional derivatives with respect to chiral superfields are now seen to be satisfactorily defined by (2.1). Derivatives with respect to non-chiral superfields, however, must be independently defined. Such non-chiral fields are needed to represent the gauge dependent potentials in models which carry a local symmetry [4]. A suitable definition for the derivative with respect to the non-chiral field $\Psi(x,\theta)$ is given by

$$\delta\Gamma = \int dx \frac{1}{2}(\bar{D}D)^2 \left(\delta\Psi(x,\theta)\frac{\delta\Gamma}{\delta\Psi(x,\theta)}\right) . \tag{2.15}$$

With the component structure of Ψ defined by the expansion (1.2) and derivatives with respect to these components defined by:

*) The surface term is of course important for the derivation of conserved currents but we shall not consider them here.

$$\delta\Gamma = \int dx \left\{ \delta A \frac{\delta\Gamma}{\delta A} + \delta\bar{\psi} \frac{\delta\Gamma}{\delta\bar{\psi}} + \delta F \frac{\delta\Gamma}{\delta F} + \delta G \frac{\delta\Gamma}{\delta G} \right.$$

$$\left. + \delta A_\nu \frac{\delta\Gamma}{\delta A_\nu} + \delta\bar{\chi} \frac{\delta\Gamma}{\delta\bar{\chi}} + \delta D \frac{\delta\Gamma}{\delta D} \right\} , \tag{2.16}$$

one finds the following component structure for the derivative with respect to $\Psi(x,\theta)$:

$$\frac{\delta\Gamma}{\delta\bar{\psi}(x,\theta)} = \frac{\delta\Gamma}{\delta D(x)} - \frac{1}{2}\bar{\theta} \frac{\delta\Gamma}{\delta\bar{\chi}(x)}$$

$$+ \frac{1}{8}\bar{\theta}\theta \frac{\delta\Gamma}{\delta F(x)} + \frac{1}{8}\bar{\theta}\gamma_5\theta \frac{\delta\Gamma}{\delta G(x)} + \frac{1}{8}\bar{\theta}i\gamma_\nu\gamma_5\theta \frac{\delta\Gamma}{\delta A_\nu(x)}$$

$$- \frac{1}{8}\bar{\theta}\theta\,\bar{\theta} \frac{\delta\Gamma}{\delta\bar{\psi}(x)} + \frac{1}{32}(\bar{\theta}\theta)^2 \frac{\delta\Gamma}{\delta A(x)} . \tag{2.17}$$

The corresponding super delta-function is given by

$$\frac{\delta\Psi(x,\theta)}{\delta\Psi(x',\theta')} = \delta(x,\theta;x',\theta')$$

$$= \frac{1}{32}\left[(\bar{\theta} - \bar{\theta}')(\theta - \theta')\right]^2 \delta(x - x') . \tag{2.18}$$

This invariant function acts as the identity distribution,

$$\int dx \frac{1}{2}(\bar{D}D)^2 \left[\Psi(x,\theta)\ \delta(x,\theta;x',\theta')\right] = \Psi(x',\theta') . \tag{2.19}$$

III. COVARIANT QUANTIZATION

The formal derivation of Feynman rules proceeds very much as in any ordinary quantized field theory. To illustrate this we consider the case of a neutral scalar supermultiplet whose dynamics at the classical level is governed by the action functional (2.11). A straightforward method to obtain the Feynman rules is to perturb the system by means of an external current distribution, $J_{\pm}(x,\theta)$,

$$S \to S + \int dx(-\frac{1}{2}\bar{D}D)\left[J_+\Phi_+ + J_-\Phi_-\right] \quad ,$$

and represent the corresponding vacuum transition amplitude by a path integral,

$$\exp\frac{i}{\hbar}Z(J_+,J_-) = \int (d\Phi_+\ d\Phi_-)\ \exp\frac{i}{\hbar}\left[S(\Phi_+,\Phi_-) + \int dx\ (-\frac{1}{2}\bar{D}D)(J_+\Phi_+ + J_-\Phi_-)\right] \quad . \tag{3.1}$$

One proceeds by separating the classical action into a free (bilinear) and an interaction piece,

$$V(\Phi_{\pm}) = \frac{M}{2}\Phi_{\pm}^2 + V_{int}(\Phi_{\pm}) \quad , \tag{3.2}$$

where V_{int} contains cubic and higher order terms *). In (3.1) make the formal replacement

$$V_{int}(\Phi_{\pm}) \to V_{int}\left(\frac{\hbar}{i}\frac{\delta}{\delta J_{\pm}}\right)$$

*) In general the expansion of S must be taken about a stationary point. For simplicity, we are here assuming that $\Phi_{\pm} = 0$ is such a point.

so as to represent the vacuum amplitude in the form

$$\exp \frac{i}{\hbar} Z(J_+,J_-) = \left[\exp \frac{i}{\hbar} \int dx\left(-\frac{1}{2}\bar{D}D\right)\left(V_{int}\left(\frac{\hbar}{i}\frac{\delta}{\delta J_+}\right) + V_{int}\left(\frac{\hbar}{i}\frac{\delta}{\delta J_-}\right)\right)\right] \exp \frac{i}{\hbar} Z_0(J_+,J_-), \tag{3.3}$$

where Z_0 is represented by a gaussian path-integral,

$$\exp \frac{i}{\hbar} Z_0(J_+,J_-) = \int (d\Phi_+ \, d\Phi_-) \exp \frac{i}{\hbar} \int dx \left[\frac{1}{8}(\bar{D}D)^2 (\Phi_+\Phi_-) - \frac{1}{2}\bar{D}D\left(\frac{M}{2}(\Phi_+^2 + \Phi_-^2) + (J_+\Phi_+ + J_-\Phi_-)\right)\right] . \tag{3.4}$$

Being gaussian, this integral can be evaluated by the usual trick of translating the integration variables. An equivalent method is to obtain functional differential equations for Z_0 from the identities

$$0 = \int (d\Phi_+ \, d\Phi_-) \frac{\delta}{\delta\Phi_\pm(x',\theta')} \exp \frac{i}{\hbar} \int dx \left[\frac{1}{8}(\bar{D}D)^2 (\Phi_+\Phi_-) - \frac{1}{2}\bar{D}D\left(\frac{M}{2}(\Phi_+^2 + \Phi_-^2) + (J_+\Phi_+ + J_-\Phi_-)\right)\right]$$

$$= \int (d\Phi_+ \, d\Phi_-) \frac{i}{\hbar}\left[-\frac{1}{2}\bar{D}D\Phi_\mp + M\Phi_\pm + J_\pm\right] \exp \frac{i}{\hbar} \int dx \left[\ '' \ \right]$$

$$= \frac{i}{\hbar}\left[-\frac{1}{2}\bar{D}D \frac{\hbar}{i}\frac{\delta}{\delta J_\mp} + M\frac{\hbar}{i}\frac{\delta}{\delta J_\pm} + J_\pm\right] \exp \frac{i}{\hbar} Z_0$$

$$= \frac{i}{\hbar}\left(\exp \frac{i}{\hbar} Z_0\right)\left[-\frac{1}{2}\bar{D}D \frac{\delta Z_0}{\delta J_\mp} + M\frac{\delta Z_0}{\delta J_\pm} + J_\pm\right] .$$

The linear inhomogeneous equations

$$-\frac{1}{2}\bar{D}D\frac{\delta Z_0}{\delta J_\mp} + M\frac{\delta Z_0}{\delta J_\pm} = -J_\pm \tag{3.5}$$

are easily solved (with the help of the identity, $(\bar{D}D)^2 = -4\partial^2$, on chiral fields [4]) to give

$$\frac{\delta Z_0}{\delta J_\pm} = \frac{1}{\partial^2 + M^2}\left(-\frac{1}{2}\bar{D}D\, J_+ - M\, J_\pm\right) \quad . \tag{3.6}$$

These equations in turn can be integrated to give

$$Z_0(J_+,J_-) = \int dx \left[\frac{1}{8}(\bar{D}D)^2\left(J_+ \frac{1}{\partial^2 + M^2} J_-\right) - \frac{1}{2}\bar{D}D\left(J_+ \frac{-M/2}{\partial^2 + M^2} J_+ + J_- \frac{-M/2}{\partial^2 + M^2} J_-\right)\right] \quad . \tag{3.7}$$

The perturbation development of the connected vacuum amplitude $(i/\hbar)Z$ is obtained from (3.3) by expanding in powers of V_{int} and substituting the expression (3.7) for Z_0 . The bare propagators are of course the second-order functional derivatives of Z_0 . Their explicit forms are given by

$$\frac{i}{\hbar}\left\langle T\Phi_\pm(x,\theta)\,\Phi_\pm(x',\theta')\right\rangle_0 = \frac{-M}{\partial^2 + M^2}\,\delta_\pm(x,\theta\,;\,x',\theta') \quad ,$$

$$\frac{i}{\hbar}\left\langle T\Phi_\pm(x,\theta)\,\Phi_\mp(x',\theta')\right\rangle_0 = \frac{1}{\partial^2 + M^2}\left(-\frac{1}{2}\bar{D}D\right)\delta_\mp(x,\theta\,;\,x'\theta') \quad , \tag{3.8}$$

where the delta-functions $\delta_\pm$ are defined in (2.5). In terms of the causal function

$$\Delta_F(x - x';M) = \int \frac{d_4k}{(2\pi)^4} \frac{i\hbar}{k^2 - M^2 + i\varepsilon} \exp[-ik(x - x')] \quad ,$$

the propagators (3.8) can be written,

$$\left\langle T\Phi_\pm(x,\theta)\, \Phi_\pm(x',\theta')\right\rangle_0 =$$

$$= - M\left[\exp\left(\mp \frac{1}{4}\right) \left(\bar\theta\partial\!\!\!/\gamma_5\theta + \bar\theta'\partial\!\!\!/'\gamma_5\theta'\right)\right] \frac{1}{4}\,(\bar\theta-\bar\theta')\ (1\pm i\gamma_5)\ (\theta-\theta')\ \Delta_F$$

$$\left\langle T\Phi_\pm(x,\theta)\ \Phi_\mp(x',\theta')\right\rangle_0 =$$

$$= \left[\exp\left(\mp \frac{1}{4}\right) \left(\bar\theta\partial\!\!\!/\gamma_5\theta - \bar\theta'\partial\!\!\!/'\gamma_5\theta'\right)\right] \left[\exp \frac{i}{2}\,\bar\theta(1\pm i\gamma_5)\partial\!\!\!/\theta'\right] \Delta_F \quad . \tag{3.9}$$

These expressions are to be associated with the internal lines of any graph. Vertices, on the other hand, are associated with the expression

$$\frac{i}{\hbar}\, g_n \int dx(-\tfrac{1}{2}\bar{D}D) \, , \tag{3.10}$$

where g_n is a coupling constant from the expansion *)

$$V_{int}(\Phi_\pm) = \frac{g_3}{3!}\,\Phi_\pm^3 + \frac{g_4}{4!}\,\Phi_\pm^4 + \cdots \tag{3.11}$$

The integration over x at each vertex - which effects the conservation of 4-momentum - gives rise to some simplifications. Many of the exponentiated factors in (3.9) can be seen to cancel so that the internal lines are associated with the "effective" propagators

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*) If the model is to be renormalizable, then only g_3 can differ from zero.

$$\left\langle T\Phi_\pm(x,\theta)\ \Phi_\pm(x',\theta')\right\rangle_{eff} = -\frac{M}{4}(\bar\theta-\bar\theta')\ (1\pm i\gamma_5)\ (\theta-\theta')\ \Delta_F$$

$$\left\langle T\Phi_\pm(x,\theta)\ \Phi_\mp(x',\theta')\right\rangle_{eff} = \left[\exp \frac{i}{2}\bar\theta(1\pm i\gamma_5)\not\partial\theta'\right]\Delta_F \quad . \tag{3.12}$$

External lines should be associated with the wave functions

$$\varphi_\pm^{ext}(x,\theta) = A_\pm^{ext}(x) + \bar\theta\,\frac{1\pm i\gamma_5}{2}\,\psi^{ext}(x) + \frac{1}{4}\,\bar\theta(1\pm i\gamma_5)\theta\ F_\pm^{ext}(x) \quad . \tag{3.13}$$

The momentum space Feynman rules may be summarized as follows:

(1) Draw diagrams in the usual way with vertices corresponding to the various types of monomial $(g_n/n!)\,\Phi_+^n$ and $(g_n/n!)\,\Phi_-^n$ in the interaction Lagrangian.

(2) With each n-leg vertex associate the factor

$$(2\pi)^4\ \delta(p_1+\cdots+p_n)\ \frac{i}{\hbar}\ g_n \quad .$$

(3) With each line joining a pair of vertices of the same chirality (±) associate the effective propagator

$$\frac{\hbar}{i}\left(-\frac{M}{2}\right)\ (\bar\theta_1-\bar\theta_2)\ \frac{1\pm i\gamma_5}{2}\ (\theta_1-\theta_2)\ \frac{1}{-p^2+M^2-i\varepsilon} \quad .$$

(4) With each line joining a pair of vertices with opposite chirality associate the effective propagator

$$\frac{\hbar}{i}\exp\left(\theta_1\ \frac{1+i\gamma_5}{2}\ \not p\ \theta_2\right)\ \frac{1}{-p^2+M^2-i\varepsilon} \quad ,$$

where the 4-momentum is directed from the +type vertex (1) to the -type vertex (2).

(5) With each external line associate the wave function

$$\varphi_{\pm}^{ext} = A_{\pm}^{ext}(p) + \bar{\theta}\,\frac{1 \pm i\gamma_5}{2}\,\psi^{ext}(p) + \frac{1}{2}\,\bar{\theta}\,\frac{1 \pm i\gamma_5}{2}\,\theta\, F_{\pm}^{ext}(p) \quad .$$

(6) At each vertex apply the differential operator

$$-\tfrac{1}{2}\bar{D}D = -\tfrac{1}{2}(C^{-1})^{\beta\alpha}\,\frac{\partial}{\partial\bar{\theta}^{\alpha}}\,\frac{\partial}{\partial\bar{\theta}^{\beta}} \quad ,$$

i.e. extract the coefficient of the term

$$\prod_{\text{vertices}} \frac{1}{4}\,\bar{\theta}(1 \pm i\gamma_5)\theta \ .$$

(7) Integrate over the momenta,

$$\prod_{\text{lines}} \int \frac{dp}{(2\pi)^4} \quad .$$

To pick out a given kind of process, one would of course specialize the external wave functions. For example, an external on-shell 0^+ particle would be represented by the wave functions

$$\varphi_{\pm}^{ext}(x,\theta) = \frac{1}{(2\pi)^3}\,\exp[-ikx]\,\left[1 - \frac{M}{4}\,\bar{\theta}(1 \pm i\gamma_5)\right] \quad .$$

The efficacy of this kind of approach has been demonstrated in the work of Capper and Delbourgo [6] who show that the divergence softening characteristic of supersymmetric models can be understood in terms of the properties of superfield graphs. In particular, Capper and Leibbrandt [7] have developed a formula for the superficial degree of divergence of such graphs.

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