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SU(6) AND SUPERSYMMETRY

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ABSTRACT

Some qualitative aspects of an isospin containing supersymmetry are considered.

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I. INTRODUCTION

In an earlier paper [1] it was indicated that supersymmetric theories can support spin-containing symmetries like $SU(4)$ or $SU(6)$ in a manner which is consistent with special relativity and unitarity. In this paper we wish to outline the characteristic features of such symmetries. It appears that these spin-containing symmetries, based as they are on Clifford algebras, exhibit a number of unusual features and it is not quite clear if they can be identified with Wigner's $SU(4)$ or Gürsey-Radicati-Sakita's $SU(6)$.

II. LINEAR REALIZATIONS OF SUPERSYMMETRY AND SPIN-CONTAINING SYMMETRIES

Let us assume that the Majorana charges which generate supersymmetry $S_{\alpha i}$ belong to some representation of the direct product of the Lorentz group with an internal symmetry group such as $U(1)$, $SU(2)$, $SU(3)$ etc. (The index α refers to the Lorentz group; the index i is an internal symmetry label.) Postulate the basic commutation rules,

$$\left\{S_{\alpha i},\, S_{\beta j}\right\} = \delta_{ij}(\gamma_r C)_{\alpha\beta}\, P_\mu \;,$$

$$[S_{\alpha i},\, P_\mu] = 0,\; [P_\mu, P_\nu] = 0 \;, \qquad (1)$$

where $C_{\alpha\beta}$ is the charge-conjugation matrix and P_μ is the generator of space-time translations. The one and only finite-dimensional representations of the system (1) - with P_μ restricted to be a fixed time-like vector - has the dimension 2^{2n} where n denotes the dimension of the internal symmetry representation to which S belongs.

If the representation to which S is assigned is a complex one (of n dimensions) we must treat $S_{\alpha i}$ and $\bar{S}^{\alpha i}$ as independent operators, replacing (1) by

$$\left\{S_{\alpha i},\, S_{\beta j}\right\} = 0 \;, \qquad \left\{\bar{S}^{\alpha i},\, \bar{S}^{\beta j}\right\} = 0 \;,$$

$$\left\{S_{\alpha i},\, \bar{S}^{\beta j}\right\} = \delta_i^j (\gamma_\mu)_\alpha^{\ \beta}\, P_\mu \;. \qquad (2)$$

In this case the fundamental representation has dimension 2^{4n}.

To illustrate the emergence of the spin-containing symmetry $SU(6)$, consider the internal symmetry $O(3)_I$ and require the generators S_{ai} to transform as a 3-vector of this group. In the rest frame, $P_\mu = (M,\vec{0})$, the anti-commutators (1) take the form

$$\{S_{ai}, S_{bj}\} = 0 \quad , \qquad \{S^+_{ai}, S^+_{bj}\} = 0 \; ,$$

$$\{S_{ai}, S^+_{bj}\} = M\,\delta_{ab}\delta_{ij} \; , \tag{3}$$

where $a = 1,2$. An orthogonal system of vectors is easily contructed by repeated application of the six operators S_{ai} to a "ground state" which is annihilated by the S^+_{ai}. For example, the fundamental representation is spanned by the states

$$|0\rangle, \quad S_{ai}|0\rangle, \quad S_{ai}\,S_{bj}|0\rangle, \;\ldots\;, \quad S_{a_1 i_1} \cdots S_{a_6 i_6}|0\rangle \; , \tag{4}$$

among which there are 64 independent ones. These states can be looked upon as antisymmetric SU(6) tensors and we have the decomposition

$$\underset{\sim}{64} = \underset{\sim}{1} + \underset{\sim}{6} + \underset{\sim}{15} + \underset{\sim}{20} + \underset{\sim}{\overline{15}} + \underset{\sim}{\overline{6}} + \underset{\sim}{1} \tag{4'}$$

relative to the infinitesimal algebra of SU(6) which is generated by the bilinear operators

$$\frac{1}{2M}\,[S_{ai}, S^+_{bj}] \; . \tag{5}$$

Insofar as the Hamiltonian P_0 commutes with these generators, (5), SU(6) is a rest-subsymmetry of the theory. But the emergence of this rest-symmetry is quite peculiar, in at least two respects:

i) The fundamental supersymmetric representation $\underset{\sim}{64}$ contains not only the (Majorana) 6-fold quark but also the higher totally <u>anti-symmetric</u> tensors of SU(6) (diquarks, triquarks etc.) like $\underset{\sim}{15}$, $\underset{\sim}{20}$, ...

ii) The general (rest-frame) irreducible representation of the supersymmetry can be classified with respect to $SU(6) \times SU(2)_{j} \times O(3)_{\mathcal{J}}$, (the fundamental $\underset{\sim}{64}$ corresponding to $j = \mathcal{J} = 0$). The peculiarity of these representations is that the <u>same</u> set of SU(6) representations ($\underset{\sim}{1}$, $\underset{\sim}{6}$, $\underset{\sim}{15}$, $\underset{\sim}{20}$, ...) is involved in every supersymmetric representation, so that one may speak of these as (j, $\mathcal{J}$) excited antisymmetric SU(6) states (somewhat analogous to the familiar ℓ-excited representations of $SU(6) \times O_\ell(3)$).

This persistence of the antisymmetric SU(6) states, though not unexpected from the Clifford nature of the underlying supersymmetry, is truly uncanny. Consider the decomposition into irreducible representations of the direct product of two irreducible representations. Remarking that, applied to product states, the operators S, like the Poincaré generators are <u>additive</u> and conserved in the sense that

$$S^{(1)} + S^{(2)}\Big|_{\text{initial}} = S^{(3)} + S^{(4)}\Big|_{\text{final}} \quad , \qquad (6)$$

one can find a total of 64 "ground" states in the centre of mass which are annihilated by the six lowering operators $(S_{ai}^{(1)} + S_{ai}^{(2)})^{+}$. On each of these "ground" states, an irreducible 64-dimensional representation of the centre-of-mass Clifford algebra can be erected by applying the raising operators $(S_{ai}^{(1)} + S_{ai}^{(2)})$.

It may be instructive to list these 64 "ground states", labelled as multiplets of $O(3)_{\mathcal{J}}$ and (ignoring relative momentum for the moment) of $SU(2)_{\mathcal{J}}$. One finds the following $(\mathcal{J}, \mathcal{J})^{P}$ combinations:

$$\begin{aligned} 64 = \quad & (0,\ 0)^{+} \\ & + (1,\ \tfrac{1}{2})^{i} \\ & + (0,\ 0)^{-} + (1,\ 1)^{-} + (2,\ 0)^{-} \\ & + (0,\ \tfrac{3}{2})^{-i} + (1,\ \tfrac{1}{2})^{-i} + (2,\ \tfrac{1}{2})^{-i} \\ & + (0,\ 0)^{+} + (1,\ 1)^{+} + (2,\ 0)^{+} \\ & + (1,\ \tfrac{1}{2})^{i} \\ & + (0,\ 0)^{-} \quad . \end{aligned} \qquad (7)$$

As stated before, these 64 states act as "ground states" on each one of which one erects an SU(6)-containing irreducible 64-dimensional representation of the centre-of-mass Clifford algebra. The SU(6) thus persists in the sense mentioned above.

To complete the discussion of decomposition of the direct product, the formula (7) essentially gives the Clebsch-Gordan series for the product of two supersymmetric $\underset{\sim}{64}$'s, at threshold. Above threshold the relative momentum must be taken into account. One way to do this is to expand the product states in partial waves and combine the orbital angular momentum $\vec{\ell}$ with the spin values $\vec{\mathcal{J}}_{\text{spin}}$ (appearing in (8)) to define a total

$$\vec{\mathcal{J}} = \vec{\ell} + \vec{\mathcal{J}}_{\text{spin}} \quad .$$

For mere amplitude counting, however, it is not necessary to do this. One simply decomposes the multiplets (7) relative to the subalgebra $O(3)_{\mathcal{J}} \times O(2)_{\mathcal{J}}$ which preserves the direction of the relative momentum. Forward amplitudes are classified by the representations $(I, \mathcal{J}_3)^P$ of this algebra. One then finds a total of 32 forward amplitudes describing the elastic scattering of two $\underset{\sim}{64}$ supermultiplets. For the off-forward scattering, the $\mathcal{J}_3$ quantum number is not relevant and one finds a total of 68 amplitudes. This remarkably small set of 68 amplitudes describes the scattering of $2^{12} = 4096$ states, thanks to supersymmetry.

III. SU(6) AND NON-LINEAR REALIZATIONS OF SUPERSYMMETRY

In view of the foregoing, we face two problems if we wish to relate the SU(6) found in the supersymmetry context to the SU(6) which generalizes Wigner's SU(4).

1. The first problem is one of large multiplets. The quark, for example, has come accompanied by diquarks, triquarks and so on. Clearly this is supersymmetry's way to preserve both unitarity as well as SU(6) but it is disconcerting for physical applications.

2. The second problem is the persistence among products of representations of a particular symmetry-type of SU(6) representations. The Clifford algebras appear to set up a sort of trap; we start with a tower of a certain type of SU(6) rest-representations (the antisymmetric SU(6) tensors for the algebra considered in the last section) and this tower is repeated in the product states.

One way to avoid large towers of SU(6) multiplets would be to break the symmetry by the addition of a simple non-supersymmetric but SU(6)-symmetric piece to the Lagrangian. This would result in a lifting of the mass degeneracy of the SU(6) multiplets contained in the tower, through a mass formula of the following type:

$$M_r = a_0 + ra_1 + \frac{r(r-1)}{2!}a_2 + \frac{r(r-1)(r-2)}{3!}a_3 + \dots \quad (r = 0, 1, \dots, 6) , \tag{10}$$

where r denotes the rank (quark number) of the SU(6) components in the fundamental supersymmetric 64-fold tower. If the symmetry-breaking term has been judiciously chosen, it is possible that all states except the quark $(r = 1)$ can be made supermassive.

But even if this idea can be made to work, the addition of an explicit symmetry-breaking term, however "soft" in a field-theoretic sense, is likely to be disastrous. This is because the basic algebra (1) can no longer be sustained, since supersymmetry is no longer the symmetry of the theory. The $S_{\alpha i}$'s no longer commute with the total Hamiltonian, nor do the generators

$$[S_{ai}, S^+_{bj}]$$

close on a simple algebra. The SU(6) rest-symmetry would be lost.

It may be possible to achieve the objective of lifting the mass-degeneracy of the supersymmetric 64-fold by breaking the supersymmetry spontaneously while SU(6) is still preserved. This would of course result in the appearance of a Goldstone zero-mass multiplet. However, for other multiplets one may succeed in achieving a mass formula of the type (10). We have not carried out this programme in a straightforward manner but its feasibility appears to be assured on account of a parallel development of realizing supersymmetry non-linearly, initiated by Volkov and Akulov [2].

In this development, supersymmetry is realized by means of a single Majorana (12-component if we are considering SU(6)) spinor $\psi_{\alpha i}(x)$. The action of the supersymmetric group (1) on x and ψ is given by

$$x_\mu \to x'_\mu = x_\mu + \frac{i}{2}\bar{\epsilon}^\alpha_i (\gamma_\mu)^\beta_\alpha \psi_{\beta i}(x)$$

$$\psi_{\alpha i}(x) \to \psi'_{\alpha i}(x') = \psi_{\alpha i}(x) + \epsilon_{\alpha i} \quad .$$

Volkov and Akulov show that the quantity

$$\omega_{\nu\mu} = \eta_{\nu\mu} - \frac{i}{2}\bar{\psi}\gamma_\mu \partial_\nu \psi$$

can be used to set up invariant integrals. In particular, the appropriate Lagrangian for the mass Goldstone particles represented by the spinor $\psi_{\alpha i}$ is

$$\det \omega_{\lambda\mu} \quad ,$$

which of course contains the free Lagrangian $-\frac{i}{2}\bar{\psi}\gamma_\mu\partial_\mu\psi$ plus terms up to products of four ψ's (and $\partial\psi$'s). Insofar as the relation (3) implies that the Hamiltonian of the theory P_0 can still be written as a bilinear (hermitian) sum of products of "charges" $S_{\alpha i}$, we note that the SU(6) represented by the generators (5) is a symmetry of the theory (since we are dealing with zero-mass particles, these statements are only heuristic, in the sense that on account of zero mass the integrals representing charges presumably diverge). The symmetry is however realized non-linearly in terms of just the 12-component spinors $\psi_{\alpha i}$. We have cut down on large multiplets of the linearly-realized symmetry, but what exactly the present "SU(6)" signifies physically is not clear. In some sense, while the linearly-realized symmetry gives a theory of constituent quarks, the non-linearly-realized supersymmetry framework appears to provide an "SU(6)" representation for "current-quarks".

REFERENCES

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2) D.V. Volkov and V.P. Akulov, Phys. Letters 46B, 109 (1973).