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IMPACT OF QUANTUM GRAVITY THEORY ON PARTICLE PHYSICS

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IMPACT OF QUANTUM GRAVITY THEORY ON PARTICLE PHYSICS *

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ABSTRACT

Ideas of space-time curvature and torsion have influenced some recent thinking in particle physics. A review of these developments is presented.

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I. INTRODUCTION

When Roger Penrose and Dennis Sciama assigned me the task of reporting on the impact of quantized gravity theory on particle physics, I am sure they did this with a certain amount of positive anticipation. Sadly, the burden of my remarks will be - there has been very little impact. Principally this is occasioned by the belief - erroneous as I hope to show - held, by and large, by the particle physics community, that quantum-gravitational effects will manifest themselves only for energies in excess of 10^{19} BeV $\left(\sim G^{-\frac{1}{2}}\right.$, lengths $\left.< 10^{-33} \text{ cms}\right)$. This has meant that, though there has been some impact of quantized gravity theory on the development of field theories as such, particle physicists have, in general, made no attempt to enrich the concepts of their subject with the geometrical or topological notions which, after all, are - or should be - the crowning glory of Einstein's gravity theory, in its final quantized form. It is in fact these geometrical and topological notions towards which I wish to build in my remarks to-day. I shall, however, approach them, not by imposing them from outside - in the manner of a differential geometer - but as something emerging from the experience of the particle physicist with quantum fields with which he is familiar, and of which quantum gravity field should be but one - though of course the supreme - example.

II. THE PARTICLE PHYSICIST'S APPROACH TO QUANTUM GRAVITY

From the particle physicist's point of view, quantized gravity theory of Einstein is no more than the theory of spin 2^+ quantized massless gravitons. The physical graviton exists in two helicity states, but is described by a 10-component field $g^{\mu\nu}(x)$, which supports an extraordinarily elegant, group-theoretically simple, but (in the relativist's notation) fearsome set of gauge and space-time transformations.

After having said this, I must however repeat what Chris Isham has already stressed; there are the two problems absolutely peculiar to the quantization of gravity, which one does not encounter for other field theories.

1) In the canonical or in the LSZ approach, the concept of space-like separated points - so essential to the setting up of any sort of commutation relations - cannot be defined till after the theory is solved and the "light-cone" structure at each space-time point constructed and specified.

2) For the exotic - and often singularity-ridden - topological manifolds which may emerge as solutions of Einstein's classical equations, the defining of concepts of positive- and negative-frequency boundary

conditions, and the completeness of quantum-mechanical Hilbert-space description present new problems which one encounters for gravity theory alone.

One may approach these problems in one of the two ways: Either one starts by considering the most general geometrical and topologically complex classical situations likely to be encountered and set about quantizing (classical) geometry - somewhat in the manner of Dirac's attempts to solve the classical electron equations and to quantize them afterwards - or one proceeds in the reverse order, heuristically, as a particle physicist normally does.

a) Start with an (asymptotically flat) Minkowskian manifold and build a conventional field theory of a classical tensor field $g^{\mu\nu}(x)$ satisfying Einstein's equation, using Feynman's path-integral formulation.

b) In this approach, gravity theory is distinguished from other field theories the particle physicist encounters in that the theory employs two tensors $g^{\mu\nu}$ and $g_{\mu\nu}$, one of which is simply the inverse of the other. This makes Einstein's Lagrangian an intrinsically non-polynomial Lagrangian, when expressed in terms of one of the two tensors. To preserve quantum field-theoretic localizability in the sense of Jaffe, one adopts a convenient parametrization for the two fields *) $g^{\mu\nu}$ and $g_{\mu\nu}$:

$$g^{\mu\nu} = (\exp \kappa\phi)^{\mu\nu} ,$$

$$g_{\mu\nu} = (\exp -\kappa\phi)_{\mu\nu} ,$$

where ϕ is a 4×4 symmetric matrix and $\kappa^2 = (G/16\pi)$. Note that this demand for localizability, implying as it does that $\det g^{\mu\nu} \neq 0$ for finite $\phi(x)$, excludes space-time singularities associated with the vanishing of $\det g^{\mu\nu}$.

c) For this parametrization the Einstein Lagrangian has the form of an infinite power series in κ :

$$\mathcal{L}_{\text{Einstein}} = \partial\phi\,\partial\phi + \kappa\phi\,\partial\phi\,\partial\phi + \kappa^2\phi^2\,\partial\phi\,\partial\phi + \cdots$$

d) To implement the gauge conditions, Einstein's Lagrangian is supplemented by an appropriate gauge term. Using, for example [1], a "conformal"

*) A group theoretical significance has recently been accorded to the exponential parameterization in a beautiful paper by A.B. Borisov and V.I. Ogievetsky (Dubna Preprint E2-7684, 1974). They remark that general relativity is the simultaneous non-linear realization of affine and conformal symmetries. The exponential parameterization of $g^{\mu\nu}$ represents the standard manner in which non-linear realizations of $GL(4,R)$ are constructed.

gauge of "weight" ω (for details see Ref.1), and after imposing appropriate boundary conditions on Feynman's path integral, one can show that the free spin 2^+ graviton propagator for the incoming (or outgoing) field ϕ^f in this gauge is given by:

$$\langle T\phi^f_{\mu\lambda}\ \phi^f_{\mu\nu}\rangle = \frac{1}{2}\left[\eta_{\kappa\mu}\ \eta_{\lambda\nu} + \eta_{\kappa\nu}\ \eta_{\lambda\mu} - 2c\ \eta_{\mu\lambda}\ \eta_{\mu\nu}\right] D(x) \ , \qquad (1)$$

where $D(x)$ is the zero-mass scalar propagator and $c = \omega(\omega-1) + \frac{1}{2}$. ($\omega = 1$ corresponds to the well-known De-donder gauge.)

e) One can now make a perturbation expansion (in powers of κ) for the Green's functions of the theory in terms of Feynman diagrams. Besides the necessity to take account of Feynman-deWitt-Faddeev-Popov-Mandelstam unitarity-preserving ghosts, the peculiar feature of gravity theory (arising from the non-polynomial nature of $\mathcal{L}_{\text{Einstein}}$) is that at every Feynman vertex there impinge indefinite numbers of graviton lines. Mathematically this is seen by considering the example of the free Green's function

$$\langle g^{\alpha\beta}(x)\ g^{\gamma\delta}(0)\rangle_{\text{free}} = \langle \exp(\kappa\phi^f(x))^{\alpha\beta}\ \ \exp(\kappa\phi^f(0))^{\gamma\delta}\rangle \ . \qquad (2)$$

Using (1), and after some imaginative though ferocious algebra, Ashmore and Delbourgo[2] have shown that the exponentials appearing on the right-hand side of (2) can be evaluated to give the propagator in the form:

$$\frac{2}{9}\left[\eta^{\alpha\gamma}\ \eta^{\beta\delta} + \eta^{\alpha\delta}\ \eta^{\beta\gamma} - \frac{1}{2}\eta^{\alpha\beta}\ \eta^{\gamma\delta}\ \frac{d}{d(\kappa^2 D)} + \frac{4c-1}{18}\ \left(\eta^{\alpha\gamma}\ \eta^{\beta\delta} + \right.\right.$$
$$\left.\left. + \eta^{\alpha\delta}\ \eta^{\beta\gamma}\right) + \frac{5-2c}{18}\eta^{\alpha\beta}\ \eta^{\gamma\delta}\right] a(D) \ , \qquad (3)$$

where

$$a(D) = (2 - 3z + \frac{1}{2}z^2)\ \exp(-2cz) + \left[(2 + 3z - z^2 + \right.$$
$$\left. + \frac{1}{2}z(z+\frac{1}{2})\ 3\pi L_0(z) - \frac{1}{2}z_1{}^2\ \pi\ L_1(z)\right] \exp\left(z(1-2c)\right),\ z = \frac{\kappa^2 D}{2} \ , \quad (4)$$

Here L_0 and L_1 are the Struve functions (Fig.1). (For the definition of $\exp(\kappa^2 D)$ for $x^2 \to 0$, one uses a Euclidean ansatz explained in Sec. IV.)

f) So far no geometry has entered in the formalism developed. We have simply built up a perturbation representation of Green's functions in a Minkowskian manifold. This manifold may have nothing whatever to do with the space-time of general relativity. My contention is that with this humble apparatus available, the heuristic techniques of the particle physicist, like analytic continuation, renormalization, summation of series and the like - techniques which are so far foreign to the general relativist's experience - are sufficiently powerful to yield what may be recognized as geometrical aspects of gravity theory.

One example of this is the well-known discussion of Thirring [3] who starting with the "bare" unobservable metric of the Minkowskian manifold shows how one builds up the physical space-time metric through a renormalization of lengths and time intervals, following the very familiar procedures of renormalization theory in particle physics. A second example is the Duff-Sardelis [4] reproduction of the Schwarzschild and Reissner-Nordström classical metrics, including of course a description of their singularities, by a summation of infinite sets of tree diagrams in the "Minkowskian" version of Einstein-Maxwell theories (Fig.2).

The analogy of the Duff-Sardelis work is with the particle physicist's treatment of the bound state problem. By summing infinite sets of perturbation diagrams, the particle physicist builds up expressions for Green's functions which exhibit bound-state poles in the energy momentum plane - poles which, before the diagrams are summed, are naturally nowhere in evidence. In the Duff-Sardelis work, one is recovering space-time singularities of the metric without starting with these.*)

Finally to mention one more weapon in the armoury of the particle physicist. The perennial problem of classical vs. quantum aspects of the gravitational field - the difficulty which one continually encounters in reconciling $g^{\mu\nu}$ as a quantum operator and $g^{\mu\nu}$ as a classical function - may possibly be resolved through the use of Wilson's operator product expansions. For example, one may conjecture (from the non-polynomiality of gravity theory and as a generalization of expressions like (2), (3) and (4)) that a typical operator product for operators in gravity theory $\left(\text{like } g_{\mu\nu}(x)\, g_{\alpha\beta}(0)\right)$ has the form:

$$g_{\mu\nu}(x)\, g_{\alpha\beta}(0) = \left[\frac{1}{x^2} \exp(\kappa^2 D(x)) + \cdots \right] O_{\mu\nu\alpha\beta}(0) \,, \qquad (5)$$

where $O_{\mu\nu\alpha\beta}$ is operator-valued. This operator expansion is reminiscent of Newman-Penrose expansions for gravitational field components for large x^2. The difference with Newman-Penrose expansion, however, is that we are conjecturing its validity as an operator product expansion not only for large x^2 but for all x^2 including $x^2 \approx 0$.

*) To be sure, in the large, it is the class of asymptotically flat metrics only whose singularities one will determine, by starting with the Minkowskian manifold. My excuse for emphasising these is that firstly this class embraces most metrics of interest; secondly, I believe that, topology in the small, will also be recoverable by the use of the diagrammatic and heuristic techniques I am speaking about. (The universe and its (quantum) topology are determined by where gravitons are and what space-time interaction patterns they give rise to. But I shall not elaborate on this.)

To summarize, the particle physicist has, after years of experience, learnt that there are very few systems he can quantize exactly - very few cases where he can solve the equations of quantum field theory. His experience has taught him that a frontal attack on any problem in quantum field theory is pointless. He has learnt to proceed heuristically, conjecturing solutions, extending the domains of the solutions he knows by methods which may appear atrociously vile to those brought up in strict mathematical disciplines like differential geometry. His successes, however, have not been inconsiderable. I feel that the same procedures may help in making a dent on the problem of quantizing geometry.

III. f-g GRAVITY THEORY AND EFFECTS OF "GENERALIZED CARTAN TORSION" IN PARTICLE PHYSICS

I shall now turn attention to the subject proper and discuss a number of topics where one has sought for a physical - and not just a mathematical - impact of gravity on particle physics and vice versa. The first topic is f-g mixing, the two-tensor theory of gravity and "torsional" effects in particle physics.

If gravitons represent spin-2 parity-plus quantum states, these states must superpose with every other spin-2^+ quantum state, unless there is a rigorous selection rule excluding such a superposition from manifesting itself physically. The analogy is with the photon-ρ-meson mixing phenomena which, when elaborated into the so-called vector-dominance hypothesis, has played a considerable role in particle physics during the last decade.

Now 2^+ quantum states - indeed a 9-fold of 2^+ strongly-interacting SU(3) octet + SU(3) singlet states - were discovered some ten years ago. The members of this nonet are f^0, $f^{0'}$, $A_2^{0\pm}$, $K^{*\pm}$, K^{*0} and $\bar{K}^{0*}$, with masses ranging between 1200 and 1600 MeV. Concentrating on the neutral f^0 field, quantum-mechanical superposition implies that there must exist transition matrix elements between the field $f^{\mu\nu}(x)$ representing f^0 and the gravitational field $g^{\mu\nu}(x)$ (Fig.3). The physical eigenstates (the physical particles) would be obtained by diagonalizing the appropriate f-g mixing matrix.

So much for general principles. One can now elaborate this mixing of fields into the so-called two-tensor theory of gravity, proposed by Isham, Strathdee and myself, by Wess and Zumino [5] and by E. Lubkin. There are three basic postulates of this theory.

1) The dynamics of the f-meson is described by an Einstein Lagrangian $\mathcal{L}_E(f)$ (with the Christoffel symbol $\Gamma(f^0)$ occurring in this Lagrangian defined using the "metric" tensor $f^{\mu\nu}$ in complete analogy with $\Gamma(g)$ for the

gravitational tensor $g^{\mu\nu}$). The only difference from $\mathcal{L}_E(g)$ is that the Newtonian constant $G_N m_e^2$ ($\sim 10^{-44}$) is replaced by the constant $G_F m_N^2 (\approx 1)$, characteristic of strong interactions. The two "metric" tensors, $f^{\mu\nu}$, $g^{\mu\nu}$, are completely at par at this stage and, in fact, independent of each other.

2) To the Lagrangian

$$\mathcal{L}_E(g, G_N) + \mathcal{L}_E(f^0, G_f)$$

we add a generally covariant f-g mixing term

$$\mathcal{L}_{mixing} = \frac{M_f^2}{G_F} (f-g)^{\mu\nu} (f-g)_{\mu\nu} .$$

This has the effect that f and g fields mix. The eigenfields (in the approximation $O\left(\frac{G_N}{G_f}\right)$) are described by a massive field: $\tilde{f}^{\mu\nu} = f^{\mu\nu} - g^{\mu\nu}$ and the graviton field $\tilde{g}^{\mu\nu} = \left(1 + \frac{G_N}{G_f}\right)^{-1}\left(g^{\mu\nu} + \frac{G_N}{G_f} f^{\mu\nu}\right)$ which, to a good approximation, represents the massive physically observed f^0 particle and the massless graviton, respectively. (The field $\tilde{g}$ corresponds to the "true" metric.)

3) For interaction with matter we make a postulate (in analogy with the ρ photon case) which states that leptons interact with the g field (in the standard Einstein manner, with the coupling parameter*) G_N) while the f^0 field interacts with hadrons alone, with the coupling parameter G_f. Hadrons have no direct coupling with the g field. To the extent that f^0-g mixing can be neglected, the f^0-meson exchanges between hadrons will give rise to a strong (nuclear) force only. Insofar as the equations describing this interaction are Einsteinian in form, this force may be called strong gravity. When f-g mixing is taken into account, the mixing term describes (correctly) the conventional gravitational interaction of hadrons. (The effective gravitational potential involving hadrons arises from the f's transforming into g's through the mixing term. There may be differences between the conventional gravity theory where gravitons interact directly with hadrons rather than through the intermediacy of f mesons, but these differences will be of the order of $\left(\frac{G_N}{G_f}\right)^2$ (see Fig.4).)

*) Recent CEA and SLAC experiments which seem to indicate that electrons may also possess anomolously strong interactions when interacting with hadrons at sufficiently high energies may necessitate a reformulation of this postulate. The important and basic point is the assumption that space-time geometry at distances $\approx 10^{-14}$ is dominated by the f-tensor with its stronger coupling rather than the weakly coupled Einstein's g-tensor.

Now, what are the virtues of f gravity? To the extent that f-g mixing (and thus also the mass of f particles) can be neglected, we have a "two-metric" theory for two non-communicating worlds of hadrons and leptons. Concentrating on hadrons alone, if the geometrical aspects of Einstein's ideas are indeed relevant for quantum physics, here, in "f gravity", we have created a physical model, where a geometrical description should begin to manifest its virtues, around lengths of the order of $R_{Schwarzschild}$ for strong gravity, i.e. around lengths of the order of $2G_f m_f \approx (1/m_N^2)\, m_f \sim \sim 10^{-14}$ cms. One may inquire, for example, if it is profitable to describe normal hadrons with sizes around 10^{-14} cms, as "black holes" - or perhaps, more correctly, as "grey holes" (when f-g mixing and thus the massiveness of f mesons is taken into account) - in the strong f-gravity field. (The nomenclature "grey holes" is meant to convey that the absorptive characteristics of these objects must be accompanied by the peculiarly quantum-mechanical phenomena of diffractive shadow scattering.) Could this description and the attendant physics possibly be one way to resolve the dilemma of quark imprisonment inside normal hadrons? The surface of a black hole is the one surface which truly confines everything that is inside. Do "grey" holes lie on Regge trajectories? What are the collision mechanisms of "grey holes", their diffraction scattering, their absorption processes, their directional properties (for example, for charged Kerr black holes), and the like? The f-gravity hypothesis, i.e., the hypothesis which describes the strongly interacting f-meson-hadronic system, using Einstein's equation, with $f^{\mu\nu}(x)$ acting as a metric field (in the approximation that f^0-g-mixing term and thereby the mass of the f^0 mesons is neglected), clearly provides us with the most favourable laboratory system one could devise to observe quantum geometry at work. Of course, the entire hypothesis could be false; f-mesons may not be described by an Einstein-like Lagrangian, or alternatively the treatment of f-g mixing and the neglect of f-mass as a perturbation may be totally unjustified. Only further investigation can tell.

But besides the possible metrical aspects of the neutral $f^0(x)$ field, there is one other aspect of the f-meson system treated as an SU(3) nonet where one may possibly observe the impact of space-time ideas upon the internal symmetry notions of particle physics. As you are aware, one of the unresolved dilemmas of particle physics is the dilemma of internal symmetries of isospin (associated with an "internal" SU(2) group structure and its extension - the unitary spin (associated with "internal" SU(3))). The dilemma was deepened in 1964 when one discovered that the hadronic system appears to exhibit even higher symmetries like Wigner's SU(4) and its extension SU(6), which combine internal SU(2) (or SU(3)) with the spin group $SU(2)_S$, $(SU(4) \subset SU_I(2) \times SU_S(2)$, $SU(6) \subset SU(3) \times SU_S(2))$. Since f^0 , on the one hand, belongs to an SU(3) nonet and, on the other, may play the role of the strong graviton, the question is , what generalization of the gravitational space-time formalism is needed to accommodate the internal SU(3)? Isham, Strathdee and myself have tried to give one possible answer.

We start by generating the Einstein Lagrangian for the neutral SU(3)-singlet field $f^{\mu\nu}(x)$, using Weyl-Cartan-Fock-Ivenenko-Sciama-Kibble-Trautman vierbein formalism which has SL(2,C) gauge invariance as its key feature. We then generalize the SL(2,C) gauge structure to SL(6,C), which contains the internal SU(3) as a subgroup, thereby obtaining a Lagrangian which describes a nonet of strongly-interacting spin-2^+ fields and their interactions. The distinguishing geometrical feature of this Lagrangian is the emergence of a generalized torsion density, which we associate with the physically observed generators of the mysterious SU(6) spin-unitary-spin-containing symmetry observed in hadronic physics. From this point of view, besides the concept of $f^{\mu\nu}$ as the "strong metric", Cartan's torsion, suitably generalized, could be the second conceptual gift of gravity theory to particle theory insofar as this concept can embrace within it both the notion of internal symmetries like SU(3) as well as marry these symmetries with spin-symmetry to give rise to the particle physicist's SU(6).

As is well known the SL(2,C) Weyl-gauge formalism starts with the 16-component vierbein field $L^{\mu a}(x)$ which is related to the gravitational field $f^{\mu\nu}(x)$ through the relation

$$(\det f^{\mu\nu})^{-\frac{1}{2}} f^{\mu\nu}(x) = \eta_{ab} L^{\mu a}(x) L^{\nu b}(x) , \qquad \eta_{ab} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} .$$

Here, Greek μ are the space-time and Latin "a" the SL(2,C) indices. To make it possible for the particle physicist to see the elegant group theory behind Weyl's ideas, so totally obscured in standard treatments using clutters of indices, I shall use Dirac's γ and σ matrices as bases to exhibit the SL(2,C) transformations. Thus write

$$L^{\mu}(x) = L^{\mu a}(x) \gamma_a .$$

The SL(2,C) transformations are generated by the matrices

$$\Omega = \exp i [\sigma_{ab} \epsilon^{ab}] ,$$

and L^μ transforms as $L^\mu \to \Omega L^\mu \Omega^{-1}$. It is easy to show that if the SL(2,C) parameters ϵ^{ab} are functions of x, and if there exists an SL(2,C) "gauge" connection B_μ which transforms like

$$B_\mu \to \Omega B_\mu \Omega^{-1} - i \Omega \partial_\mu \Omega^{-1} \quad ,$$

with $B_\mu = B_\mu^{ab} \sigma_{ab}$, then one can define "Weyl-covariant" derivatives, using the "gauge-connection" B_μ, such that, e.g.

$$\nabla_\mu L^\nu = \partial_\mu L^\nu + i [B_\mu, L^\nu]$$

transforms as:

$$\nabla_\mu L^\nu \to \Omega(x) \overline{(\nabla_\mu L^\nu)} \Omega^{-1}(x) ,$$

and the "covariant curl"

$$B_{\mu\nu}(x) = \partial_\mu B_\nu - \partial_\nu B_\mu + i [B_\mu, B_\nu]$$

transforms as

$$B_{\mu\nu}(x) \to \Omega B_{\mu\nu} \Omega^{-1} \quad .$$

If matter is present, represented for example by the Dirac spinor $\psi(x)$, the covariant derivative for $\psi(x)$ is

$$\nabla_\mu \psi(x) = (\partial_\mu + iB_\mu) \psi(x) \quad .$$

With this preparation, one immediately sees that the generally covariant Weyl-Fock-Ivenenko Lagrangian

$$\mathcal{L}_{\text{Weyl}} = -i \, \text{Tr} \, [L^\mu, L^\nu] B_{\mu\nu} + i\bar{\psi} L^\mu (\partial_\mu + iB_\mu)\psi + m \bar{\psi}\psi$$

is also SL(2,C) <u>gauge invariant</u> describing the propagation of a 2^+ field $L^\mu(x)$. (The trace is taken over the Dirac matrices and I have incorporated $(\det \frac{\text{Tr}}{4} L^\mu L^\nu)^{-\frac{1}{2}}$ into the definitions of the fields ψ and L.) The equations of motion which result by varying L^μ and B_μ are:

$$i[L^\mu, B_{\mu\nu}] = T_\nu \to \text{Einstein's "curvature" equation},$$

$$\partial_\mu [L^\mu, L^\nu] + i[B_\mu [L^\mu, L^\nu]] = S^\nu \to \text{Cartan's "torsion" equation} ,$$

where T_ν and S^ν are the matter stress-tensor density and matter-spin densities, respectively.

Identifying

$$\mathcal{J}^{\nu} = i\ [B_{\mu}[L^{\mu},L^{\nu}]] - S^{\nu}$$

with Cartan's spin torsion density, the second equation of motion tells us that

$$\partial_{\nu}J^{\nu} = 0 \quad .$$

Consider now the generalization of this formalism to include SU(3). All one has to do is keep precisely the above Lagrangians as well as the field equations; simply change SL(2,C) generators σ^{ab} to SL(6,C) generators

$$\sigma_{ab}\,\lambda^{i}\ ,\ \gamma_5\lambda^{i}\ ,\ \lambda^{i}\quad ,$$

where the λ^{i}'s are the nine U(3)(3 x 3) Gell-Mann matrices, and the SL(2,C) ideals γ_a's are generalized to SL(6,C) ideals

$$\gamma_a\lambda^{i},\ i\gamma_a\gamma_5\lambda^{i}$$

with

$$L^{\mu} = L^{\mu a i}\ \gamma_a\ \lambda^{i} + L^{\mu a i 5}\ i\ \gamma_a\gamma_5\ \lambda^{i}$$

$$B_{\mu} = B_{\mu}^{abi}\ \sigma_{ab}\ \lambda^{i} + B_{\mu}^{i}\ \lambda^{i} + B_{\mu}^{5i}\ \lambda^{i}\ \gamma_5 \quad .$$

One can now show [6)] that the L's represent a nonet of 2^{+} fields which we identify with the f nonet.

I shall not go into the details of how one adds a mass term for the Lagrangian, and how the f-g mixing term introduced earlier to describe the neutral (f^{0}) sector of this SU(3) nonet becomes part of the enlarged formalism. This is described in Ref.7, where also following Trautman's ideas on spin-torsion, we consider how torsion affects large-scale collapse when f-gravitational effects are taken into account. Here I wish merely to stress the following:

a) As we anticipated, the SU(6) spin-internal-spin symmetry of particle physics does emerge as a generalization of spin-torsion concepts of Cartan and can be incorporated into a dynamical set of generally covariant equations describing an SU(3) nonet of strongly interacting particles.

b) One may conjecture that the combination of parameters

$$Q^2 + \frac{S^2}{m^2 G}$$

which occurs in the charged Kerr solution to Einstein's equations (where Q is electric charge, S the spin and m the mass) will generalize for the above $SL(4,C)$ or $SL(6,C)$ gauge-invariant generalization of Einstein's equation to the characteristic $SU(4)$ - or $SU(6)$ - combination:

$$I(I+1) + \frac{J(J+1)}{m^2 G_f} .$$

Since it is this type of combination which occurs in mass formulae for hadronic states, this lends support to the point of view that hadrons are indeed "grey" holes of the Kerr-Newman variety. Note that the charged Kerr black hole radius

$$r = r_+ = m + \sqrt{m^2 - Q^2 - S^2/m^2}$$

(here modified with $Q^2 \to I(I+1)$ and $S^2 \to J(J+1)$ is smaller (for same m) for particles carrying non-zero I-spin and J-spin. A confinement heirarchy can therefore be built up among I and J-spin carrying objects.

IV. EFFECTS OF SPACE-TIME CURVATURE: QUANTUM GRAVITY THEORY AND REGULARIZING OF SINGULARITIES AND INFINITIES OF QUANTUM FIELD THEORIES

So far I have dealt with possible effects of torsion and its generalizations for particle physics. Turn now to the possible effects of space-time curvature. It is an old conjecture of Landau, Pauli, Klein, deWitt, Deser and others that it is the neglect of space-time curvature which is responsible for the ultraviolet infinities of particle physics. I shall describe some of the work done at Trieste and in London in this connection [8]. Apparently, not only does quantum gravity regularize the ultraviolet infinities of other theories, it also regularizes its own - at the same time possibly quenching the space-time singularities. Further, gravity is perhaps the only theory with the property that there are no "ambiguities" in the regularized magnitudes we obtain at the end of the calculation [1].

This is a big claim. Part of what I have asserted has been rigorously proved; part is heuristic and part conjecture. The chain of arguments will go something like this; mathematically, quantum gravity (as we have formulated it in Sec. II) is a non-polynomial theory; it must therefore be treated non-perturbatively and contributions from millions and trillions of exchanged quantum gravitons must be summed. This summation (whose neglect physically amounts to neglect of space-time curvature) is responsible for quenching of infinities; the lack of "ambiguity" for the results in the summation procedure stems from the peculiar gauge and other invariances of gravity theory.

Not surprisingly, after the summation we shall see that the effective parameter for the crucial gravitational effects will not be the tiny number $G_N m_e^2 \approx 10^{-44}$ but instead more like $|\log G_N m_e^2| \approx 100$.

Consider the Dirac-Maxwell-Einstein Lagrangian describing an electron interacting with photons and gravitons. Its form is:

$$\mathcal{L}_{eff} \approx \partial(e^{\kappa\phi})\, \partial(e^{\kappa\phi}) + \bar{\psi}\, e^{\kappa\phi}\, [\gamma(\partial + iA) + m] \quad .$$

Here A is photon field, ϕ represents gravitons (I am ignoring spin), ψ is the electron field and κ equals $\sqrt{G_N/16\pi} \approx 0.25 \times 10^{-21}\, m_e^{-1}$. (Since I am concerned here with showing you what the basic ideas are, the formalism will be presented in its simplest form. For rigorous details refer to Ref.9.) The electron self-energy graphs are shown in Fig.5. The basic graph is the 1-photon·exchange graph; all the others simply show this process taking place in the environment of millions and trillions of gravitons. A neglect of these millions and trillions of gravitons is a neglect of space-time curvature (<u>"Space-Time (and its Curvature) is where Gravitons are"</u>).

Now

$$\langle\bar{\psi}\psi\rangle = -(i\gamma\partial + m)\,\frac{1}{x^2} + \text{less singular terms}$$

$$\langle AA\rangle = -\frac{1}{x^2}$$

$$\langle\phi\phi\rangle = -\frac{1}{x^2}$$

$$\langle e^{\kappa\phi}\, e^{\kappa\phi}\rangle = \exp(-\kappa^2/x^2) \quad .$$

The contribution of all graphs shown in Fig.5 to the self-mass of the electron is given by the Fourier transform (evaluated at $\not{p} = m$) of the function

$$F(x) = \alpha \quad \exp(-\kappa^2/x^2) \left(-\frac{1}{x^2}\right) (i\gamma\partial + m) \left(\frac{1}{x^2}\right) \quad .$$

If the gravitons are neglected ($\kappa = 0$),

$$\delta m = F(\tilde{p}) = \int F(x)\, e^{ipx}\Big|_{\not{p} = m} \approx \alpha m \int \frac{d^4x}{(x^2)^2}\, e^{ipx}\Big|_{\not{p} = m} \quad .$$

So that δm is logarithmically infinite (the well-known infinity of electron self-mass discovered first by Weisskopf in 1935). However, the inclusion of the quantum gravity factor $\exp(-\varkappa^2/x^2)$ acts as a regularizer. One can easily show (using a Euclidean ansatz to define this factor $\exp(-\varkappa^2/x^2)$, a topic to which I return) that the effect of this damping factor is to give:

$$\delta m \approx \frac{3\alpha}{4\pi} m \ \ell n \left(\frac{4\pi}{\varkappa m}\right)^2 \approx \frac{3\alpha}{4\pi} m \ \ell n \left(\frac{1}{R_s m}\right) +$$

$$+ \text{ terms involving } \alpha\kappa |\log \kappa m| .$$

Clearly the self-mass infinity of electron self-energy has gotten regularized at energies corresponding to the Schwarzschild radius R_s of the electron. A somewhat remarkable feature of this calculation is that when the value of κ is substituted in the formula, numerically $\frac{\delta m}{m} \approx \frac{2}{11}$. In other words, in the lowest order of α, gravity-modified electrodynamics gives the result that a substantial fraction of electron self-mass (δm) is electromagnetic in origin. Inclusion of gravity not only quenches the electromagnetic infinity of electron self-mass; it also appears to give support to Lorentz's conjecture that all of electron's self-mass may be a result of its interactions. Indeed, one may conjecture that if all the higher orders in α and multiphoton exchanges were included in the calculation (with the effective parameter for the problem being $\alpha|\log \kappa m|$), one may recover the result

$$\frac{\delta m}{m} = \sum_n a_n |\alpha \log \kappa m|^n = 1 ,$$

i.e. $\delta m = m$. In other words, there is possibly a fundamental relation between α, G_N and m_e, which has the form $1 = \frac{\delta m}{m} = F\left\{\alpha \log\left(\frac{4\pi}{\kappa m}\right)^2\right\}$ and where the function F is a power series with the first term given by*)

$$\frac{3\alpha}{4\pi} \log \left(\frac{4\pi}{\kappa m}\right)^2 .$$

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*) It is amusing that the notion of gravity acting as the final regularizer and the relation $\alpha \log G_N m_e^2 \approx 1$, have begun to be accepted as representing respectable physics by the orthodox opinion in particle physics (Summer 1974). See recent papers by G. Parisi, D. Gross, H. Fritsch and P. Minkowski, H. Georgi and S.L. Glashow and others. The motivation is as follows: Unify weak, electromagnetic and strong interactions using Yang-Mills vector (and axial-vector) fields gauging a simple Lie group descirибing internal

(cont...)

(continuation of footnote)

symmetries. The theory possesses one unique coupling constant α. Assume that the presently manifested, disparate strengths of weak versus electromagnetic versus strong forces is a manifestation (from spontaneous symmetry-breaking) of a heirarchy of differing masses for the Yang-Mills gauge mesons. Now for asymptotically free theories (e.g. a Yang-Mills theory) the effective strength of couplings decreases as energy increases. At what energy does the effective coupling of hadrons, (strong ≈ 1 at presently accessible Mev energies) become equal to α ? To answer this question , use renormalization group ideas, or more simply the conventional calculations of self-charge which give $\delta g^2/g^2 \approx \alpha \log \Lambda^2/M_N^2$ where Λ is the relevant energy. Assuming $\delta g^2/g^2 \approx 1$, we recover the result that when Λ approaches 10^{19} BeV the effective strong constant will equal the constant for electromagnetism and weak interactions.

Up till now we have considered gravity as quenching the ultraviolet infinities of quantum electrodynamics. What about gravity quenching its own ultraviolet infinities? So far as the appearance of damping factors like $\exp(-\kappa^2/x^2)$ are concerned, there is no difference between calculations in gravidynamics or electrodynamics and one may fully expect that gravity will act as its own regularizer. (The principal difficulty in making precise statements for gravidynamics comes about because of the problem's formal complexity. The major problem in all these summations is the arranging of the summation in such a manner as to secure the gauge invariances of gravidynamics and this is hard.)

As an illustration of quantum gravity quenching the space-time singularities of classical gravity, let me consider the quantum corrections to Duff's graphical generation of the Schwarzschild solutions for a massive static source. What Duff essentially did was to sum all tree graphs where a static source repeatedly emits <u>free</u> gravitons, with a propagator which has the essential form $\left(-\frac{1}{x^2}\right)$ for <u>each</u> graviton line (Fig.6). Basically - and simplifying things in order to illustrate the ideas involved - when all these graphs are summed and the static character of the source taken into account (i.e. we take the "static" approximation $\frac{1}{r}$ of $-\frac{1}{x^2}$), one obtains for the appropriate co-ordinate system an expression like

$$\frac{1}{r} + \frac{2M}{r^2} + \frac{(2M)^2}{r^3} + \cdots$$

This sums to the characteristic exterior Schwarzschild expression $1/(r-2M)$ for $r > 2M$. As is well known, the invariant quantity $R^{abcd} R_{abcd}$ for the metric thus obtained exhibits a singularity like $1/r^6$ for $r \to 0$ - the true Schwarzschild singularity. Does the singularity survive quantum corrections?

My conjecture would be No. If we replace each free propagator in the tree diagrams by the millions and trillions of quantum gravitons which are inevitably exchanged - and even if we do not take into account any other closed loop corrections which arise in quantum gravity - we shall replace the free propagator $-1/x^2$ by an expression like $-1/x^2 \exp(-\kappa^2/x^2)$. Very roughly speaking, the Duff series

$$\frac{1}{r} + \frac{2M}{r^2} + \frac{(2M)^2}{r^3} + \cdots$$

is likely to be replaced by a series of terms like

$$\frac{1}{r}\, e^{-|\kappa/r|} + \frac{2M}{r^2}\, e^{-|2\kappa/r|} + \cdots$$

The conjecture would be that this will modify the Schwarzschild metric to

$$\frac{1}{r\, e^{-\kappa/r} - 2M} .$$

The horizon would be shifted, but it is the true singularity at $r = 0$ which will finally disappear, and the mean value of $R^{abcd} R_{abcd}$ will be proportional to $1/G^3 \log GM^2$. I realize that this is a big conjecture. Even if it is plausible to replace $-1/x^2$ by $-1/x^2 \exp(-\kappa^2/x^2)$ for the propagator factors, in the Duff tree diagram approximation, the final result may get drastically modified when other quantum loops are taken into account. And such loops must be considered if the gauge invariancesof the theory are to be preserved. My own feeling is that such loops notwithstanding, the civilizing influence of damping "curvature" factors like $\exp(-\kappa^2/x^2)$ will persist and that gravity quenches its own infinities and its own space-time singularities. Just to be provocative, let me remind Professors Wheeler and Penrose of a wager we had in 1971 at a lunch in a Strand restaurant hosted <u>in absentia</u> by Professor Bondi. The bet from their side was that Duff could not possibly recover the Schwarzschild solution and its singularities by summing a perturbation series of Feynman diagrams. They were bound to lose their bet. May I now offer a wager that a further closed loop summation carried out in the appropriate co-ordinate system (gauge)in the manner I have indicated will quench the Schwarzschild-like space-time singularity itself?

Let me now turn to a technical matter; I wish to define the so-called Euclidean ansatz, which specifies the manner in which $x^2 \to 0$ in $\exp(-\kappa^2/x^2)$, and show that gravity theory is "ambiguity"-free. The Euclidean ansatz, which one uses for all non-polynomial theories, is the following. Given a Green's function $G(x_1,\ldots,x_n)$, we can consider its Fourier transform for the so-called Symanzik region in momentum space for which all momenta are space-like. Symanzik shows that such regions exist and that Green's functions are essentially discontinuity-free ("real") in this region of momentum space. We consider Fourier transforms of Green's functions,

$$\tilde{G}(p) = \int G(x)\, e^{ipx}\, d^4x$$

defined in the Symanzik region $(p = 0, \underline{p})$ where x, p, etc., stand for $(x_1,\ldots,x_n)$, $(p_1,\ldots,p_n)$. In the integral on the right, one may unambiguously continue $x_0 \to ix_4$, $x^2 \to -x_4^2 - \underline{x}^2 = -|R|^2$, so that in particular when $G(x) = G(x^2)$, $\tilde{G}(p)$ has the form:

$$\tilde{G}(p) = i \int G(R^2)\, e^{i\underline{p}\cdot\underline{x}}\, dx_4\, d^3x \quad .$$

The passage to x_4 from x_0 and x^2 to $-R^2$ is the <u>Euclidean ansatz</u> carried through (without ambiguities) in the Symanzik-region. The basic hypothesis of Symanzik's field theory is that once one knows the Fourier transforms of Green's functions in the Euclidean-Symanzik region, one may pass to all other regions of space- and time-like momenta by analytic continuations carried out in the momentum space *) p.

Now following a suggestion first made by Efimov, for computations with non-polynomial theories, one adopts Symanzik's Euclidean ansatz. (As you are aware, the Euclidean ansatz has recently been embraced by the Harvard-Princeton schools of fundamental field theorists, so that I do not have to apologize for it.) <u>However, even with the Euclidean ansatz, there are further problems for non-polynomial theories.</u> Consider

$$\langle e^{\kappa\phi(x)}\ e^{\kappa\phi(0)} \rangle = \exp(-\kappa^2/x^2) = \exp \frac{\kappa^2}{R^2} \quad .$$

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*) In all fairness to Symanzik, he has only claimed that for conventional renormalizable theories, such analytic continuation in momentum space yields results identical with those obtained without following this procedure.

Clearly the Euclidean limit $x^2 = -x_4^2 - \underline{x}^2 = -R^2 \to 0$ exists only for $\kappa^2 < 0$. For a hermitian Lagrangian, κ, however, is real and $\kappa^2 > 0$. Thus we must make a new - a second - ansatz; compute Fourier transforms of Green's functions, involving factors like $\exp(\kappa^2/R^2)$ for negative κ^2, and then continue in the κ^2 plane to positive value of κ^2. This continuation, even, if it can be justified, introduces ambiguities. As we saw before when we were computing electron self-mass, as a rule the Fourier transforms of Green's functions in these theories involve expressions like $\log \kappa^2 p^2$. Which branch of the logarithm represents physics? The likeliest conjecture would be - it is the principal branch. But why? This is the standard "ambiguity" problem of non-polynomial theories, which persists even after the Euclidean ansatz is accepted.

Our claim is, that there is one non-polynomial theory which is free of this problem - and this is quantum gravity. This remarkable distinction is a consequence of the gauge invariance of gravity theory. Let us go back to the propagator $\exp(\kappa^2/R^2)$. In order to compute its Fourier transform, κ^2 must be negative. This is indeed the case for theories with ghosts. Can we tolerate ghosts in quantum gravity?

As we well know, gravity theory is absolutely redolent with ghosts. Though the tensor $g^{\mu\nu}$ represents a helicity-2 physical object, it also describes a spin-1 as well as a spin-zero ghosts. It is the gauge invariance of the theory which keeps these ghosts from appearing in physical states - they are there, so far as Green's functions are concerned. Can we arrange to work in a gauge which emphasises the spin-zero ghosts and where the effective constant κ^2_{eff} is always negative? The Green's functions computed in this gauge would be ambiguity free. Naturally when we consider the S-matrix elements on mass shell, the results must be gauge-invariant, provided an appropriate gauge-invariance-preserving set of diagrams has been summed. It should, in the end, make no difference which gauge we worked in.

The idea of employing a special gauge to reduce the ambiguities or singularities of a gauge theory is not new. In quantum electrodynamics this idea has been used to obtain finiteness for wave function renormalization constants. In spontaneously broken Yang-Mills theories, the work of 't Hooft and others has shown that there exists the so-called "renormalization" gauge where the theory contains ghosts and which help to cut down the infinities of most Green's functions rendering the theory renormalizable. In the so-called "unitary gauge" where there are no ghosts, the Green's functions absolutely bristle with infinities which disappear only when one goes on to the mass shell. It is a well-known and well-tried procedure to take advantage of special gauges when working with gauge covariant field theories.

How do we show that for quantum gravity such desirable gauges do exist? Consider the "conformal" gauges of weight ω introduced in Sec.I, (see Eq.(1)) where $(\phi^f_{\kappa\lambda}\ \phi^f_{\mu\nu})$ exhibits propagation of spin-zero objects (in addition to spin-2 objects) through the gauge-dependent term $-2c\ \eta_{\kappa\lambda}\ \eta_{\mu\nu}\ D(x)$ with $c = \omega(\omega-1) + \frac{1}{2}$. The weight ω can be varied at will. In this gauge the complete 2-point propagator for gravity is exhibited in (3) and (4). Ashmore and Delbourgo show that for $R^2 \to 0$, the asymptotic behaviour of (3) is given by $\exp(\kappa^2(1-c)D)$. Clearly, so long as c is > 1, there will be no ambiguities so far as the 2-point non-polynomial Green's functions $(g^{\alpha\beta}(x)\ g^{\gamma\delta}(0))$ are concerned since $\kappa^2_{\text{effective}} = \kappa^2(1-c) < 0$. Can we prove that the same thing will go on happening for three- and higher point Green's functions? In Ref.1 we have given heuristic arguments for such a belief, but of course a rigorous proof awaits an extension of Ashmore-Delbourgo's formula (3) to n-point Green's functions. To summarize, we have shown that for the 2-point Green's functions, the ambiguity problem does not exist for quantum gravity, on account of its gauge invariance properties and, except for the Euclidean ansatz, no further assumptions are needed for non-perturbative computations in this theory.

V. THE OUTLOOK

For me the most fascinating contributions to this Conference have been Hawking's on the dissolution of small black holes, and Unruh's paper (not formally presented) on second quantization in the Kerr metric. I value these contributions for the results obtained, but perhaps, if I may say so, even more for their heuristic approach which is the only approach likely to be fruitful in the immediate future.

I do not believe a frontal differential-geometry inspired attack on quantum gravity is likely to lead to valid physics. In particle physics, after long experience, we have become conditioned to never asking what the theory can do for us; instead we humbly try to see what we can do for the theory. Let me give some examples of conjectured ideas - some of the most important ideas we have in the subject - which were not derived through any logical process of deduction from quantum field theories:

i) Regge poles;

ii) Dispersion theory;

iii) Duality and Veneziano form of physical amplitudes;

iv) Scaling and "Free Field" behaviour of "partons."

All that the theory was asked a posteriori was to confirm that these notions did not contradict any of its basic tenets. I believe such will also be the history of this extraordinarily rich theory of quantum gravity, to which, as the theory of space-time and its geometry, we all look for the eventual resolution of mysteries encountered in particle physics.

Just to conclude by giving two examples of ideas which I find exciting at present. One is the notion of multisheeted space-time and geodetic completion - an idea which has been considered in the past by Brill, Wheeler, Israel, Boyer and Lindquist. The idea of joining on of past and future patches was rightly rejected in the context of geodetic completion as too contemptuous of causality requirements. Must it also be rejected in the context of quantum fields with their multiple-valuedness properties, as Dr. Sarfatt has pointed out? And in any case, is the whole notion of multisheeted space-time likely to be replaced by something else if the true singularity at $r = 0$ (say the Kerr metric) disappears as a consequence of quantum corrections which - as I conjectured earlier - replace $\frac{1}{r}$ -like terms by singularity-free terms of the type $\frac{1}{r}\exp(-\kappa/r)$.

The second exciting idea concerns the recent discovery by Wess and Zumino of an exact supersymmetry between bosons and fermions of equal mass. Wess and Zumino show that two spin-zero particles of opposite parities form a supersymmetry-multiplet together with a Majorana fermion. The Klein-Gordon Lagrangian for the zero-spin particles can be transformed into the Dirac Lagrangian by what are basically space-time transformations involving anti-commuting parameters. A second example of a supermultiplet combines a spin-1 boson (e.g. the photon) with a Majorana fermion (e.g. the neutrino). Strathdee and Salam have shown that these space-time transformations provide an extension of the Poincaré algebra in the following manner. Define super-fields (of which the Fermi and Bose fields mentioned above are components) over the 8-dimensional space whose points are represented by the pair (x_μ, θ_α) where x_μ denotes the usual space-time co-ordinate and the angle θ_α is a constant Majorana spinor. The variables θ_α differ radically from co-ordinates of the usual sort in that they anticommute;

$$\theta_\alpha \theta'_\beta + \theta'_\beta \theta_\alpha = 0 \quad .$$

This has the important consequence that any local function $f(\theta)$ must be a polynomial. (All monomials $\theta_{\alpha_1}\theta_{\alpha_2}\cdots\theta_{\alpha_n}$ vanish identically for $n > 4$

We now postulate that the following space-time transformation (reminiscent of the twistor formalism of Penrose) and which leaves $|dx_\mu - \frac{1}{2} i\bar{\epsilon}\gamma_\mu d\theta|^2$ invariant)

$$x_\mu \to x_\mu + \frac{i}{2}\bar{\epsilon}\gamma_\mu \theta$$

$$\theta_\alpha \to \theta_\alpha + \epsilon_\alpha \quad , \; \epsilon_\alpha = \text{a constant Majorana spinor.}$$

For this transformation, a general superfield $\Phi(x,\theta)$ would transform in the following manner:

$$\Phi'(x',\theta') = e^{i\bar{\epsilon}S}\,\Phi(x,\theta)\,e^{-i\bar{\epsilon}S} \quad .$$

Here the transformation generators S_α satisfy

$$\{S_\alpha, S_\beta\} = -(\gamma_\mu C)_{\alpha\beta}\,P_\mu \quad ,$$

$$[S_\alpha, P_\mu] = 0 \quad .$$

(C is the charge conjugating matrix in Dirac γ-algebra and P_μ are the generators of space-time transformations.) A scalar superfield is naturally a field which transforms according to $\Phi'(x',\theta') = \Phi(x,\theta)$.

One can now show that this scalar superfield $\Phi(x,\theta)$ consists of the two irreducible representations mentioned above. One of these representations corresponds to a physical particle multiplet which contains two (opposite parity) spin-zero particles plus a Majorana spinor; the other contains a spin-1 particle together with another Majorana spinor.

The question which is exercising us is this; what supermultiplet, if any, does the Einstein graviton belong to? Is it accompanied by a massless spin-$\frac{3}{2}$ particle, coupling universally to matter with the same coupling parameter as gravity. Is the Universe full of such particles? How does general covariance come into this. Surely the graviton is not the absolute "Boson". If every other Bose particle can have a Fermi partner, the graviton cannot be the only particle not to share this symmetry principle. We know that the sypersymmetric interaction Lagrangians so far invented contain fewer infinities than one might naively have expected; e.g. there is no quadratic self-mass infinity for scalar particles; for some supersymmetric

Lagrangians studied there are no intrinsic vertex part infinities. What influence the Fermi partners of gravity would have on infinities in gravity theory and on singularities of space-time manifolds? And what exactly is the geometry of this new extended manifold constructed from the anticommuting angles θ_α together with x_μ ? The extended manifold is perhaps the first non-trivial and at the same time the first physically significant extension of space-time we possess. Can this manifold support internal symmetries? Physics is truly an unending, an everlasting, an absorbing quest, and the physics of quantum gravity even more so.

REFERENCES

1) C.J. Isham, Abdus Salam and J. Strathdee, Phys. Letters 46B, 407 (1973).

2) J. Ashmore and R. Delbourgo, J. Math. Phys. 14, 176 (1973).

3) W. Thirring, in Contemporary Physics, Vol.I (IAEA, Vienna 1968), p.458.

4) M.J. Duff, ICTP, Trieste, preprint IC/73/67 (published in Phys. Rev., C. & A.); D.A. Sardelis, ICTP, Trieste, preprint IC/73/186.

5) C.J. Isham, Abdus Salam and J. Strathdee, Phys. Rev. D3, 867 (1971); J. Wess and B. Zumino, Brandeis Lectures (1971).

6) C.J. Isham, Abdus Salam and J. Strathdee, Lettere al Nuovo Cimento 969 (1972); ICTP, Trieste, preprint IC/72/155 (Proceedings of the Coral Gables Conference, 1973).

7) C.J. Isham, Abdus Salam and J. Strathdee, Nature, 244, 82 (1973).

8) Abdus Salam and J. Strathdee,Lettere al Nuovo Cimento 4, 101 (1970).

9) C.J. Isham, Abdus Salam and J. Strathdee, Phys. Rev. D3, 1805 (1971); Phys. Rev. D5, 2548 (1972).

FIGURE CAPTIONS

Fig.1 Sums of diagrams with billions and trillions of gravitons exchanged build up "space-time curvature".

Fig.2 A sum of all tree diagrams reproduces the classical Schwarzschild expression for the metric tensor.

Fig.3 f-g transition diagram.

Fig.4 Lepton-lepton gravitational potential arises from an exchange of g-field quanta; while the lepton-hadron and hadron-hadron gravitational potential has its origin in the transformation of f-quanta to g-quanta.

Fig.5 The first graph shows 1-photon exchange; the others exhibit this graph in the sea of exchanged gravitons.

Fig.6 Graphs of Fig. 2 with millions and trillions of gravitons exchanged between space-time points. The sums of these graphs are expected to quench Schwarzschild (and other metrical) singularities.

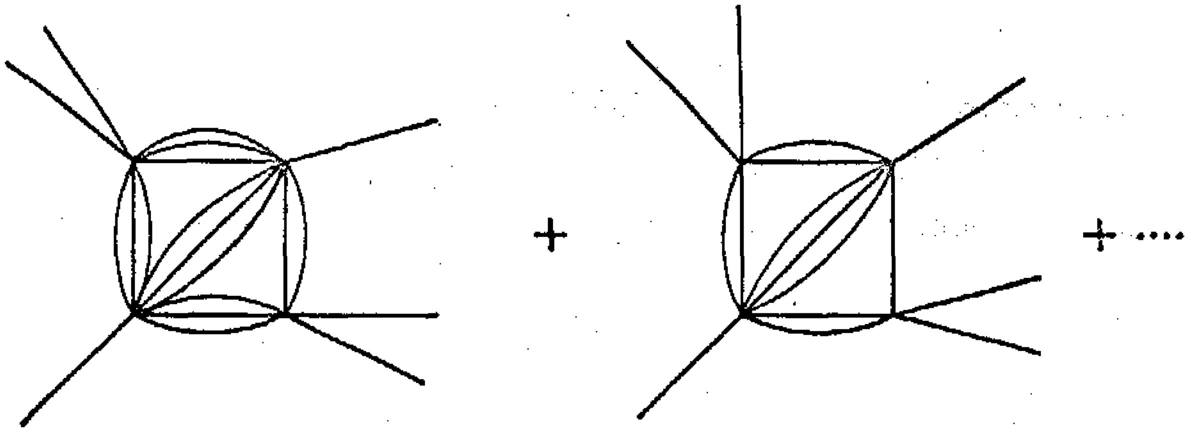

FIG. 1

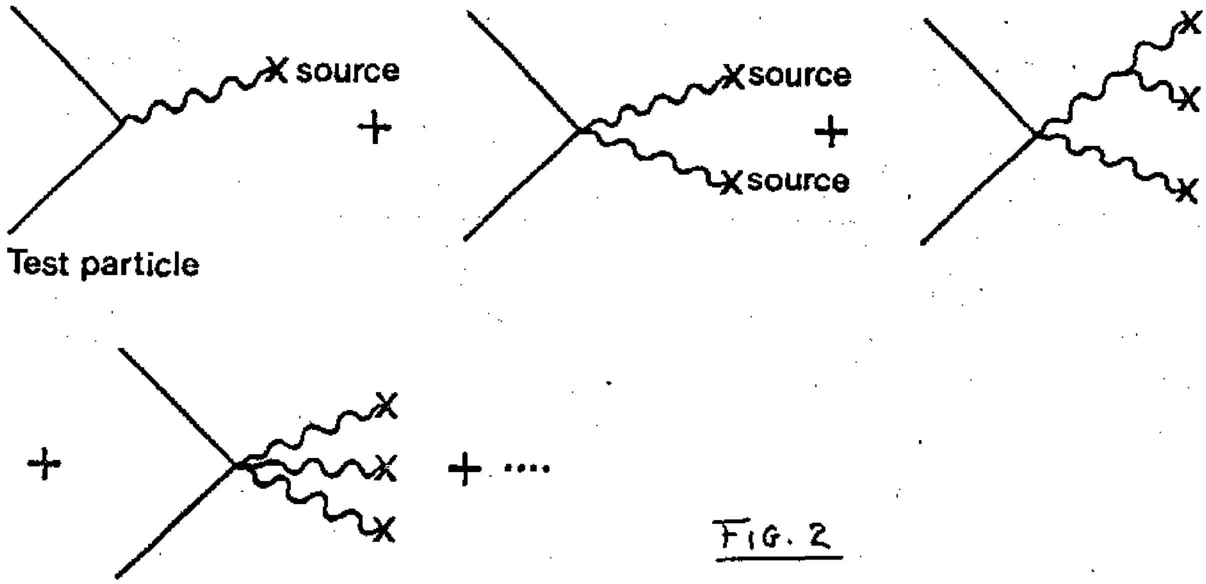

FIG. 2

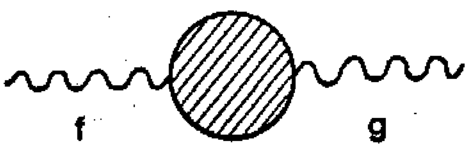

FIG. 3

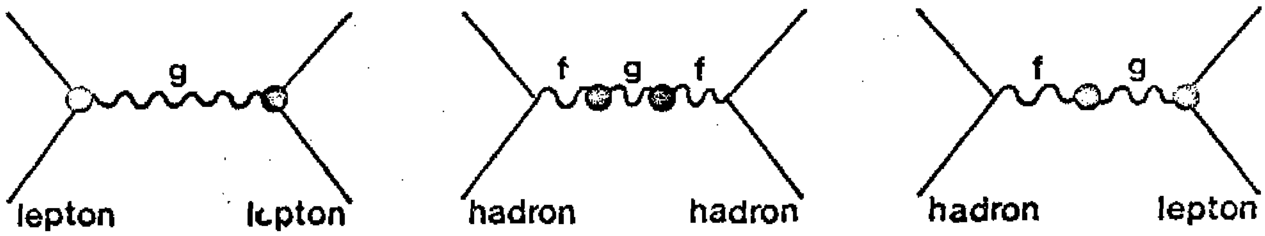

FIG. 4

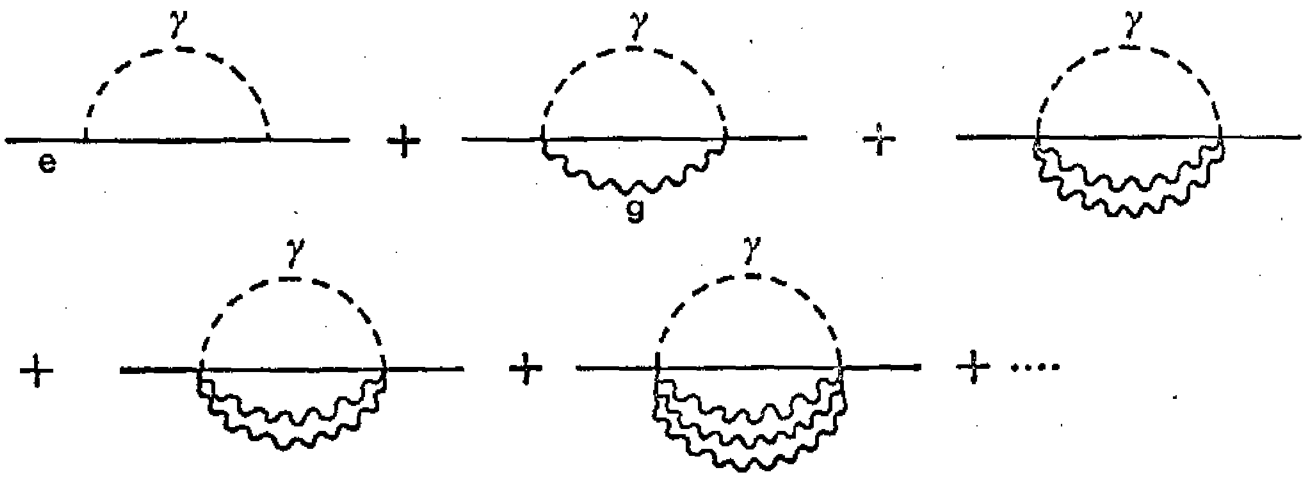

FIG. 5

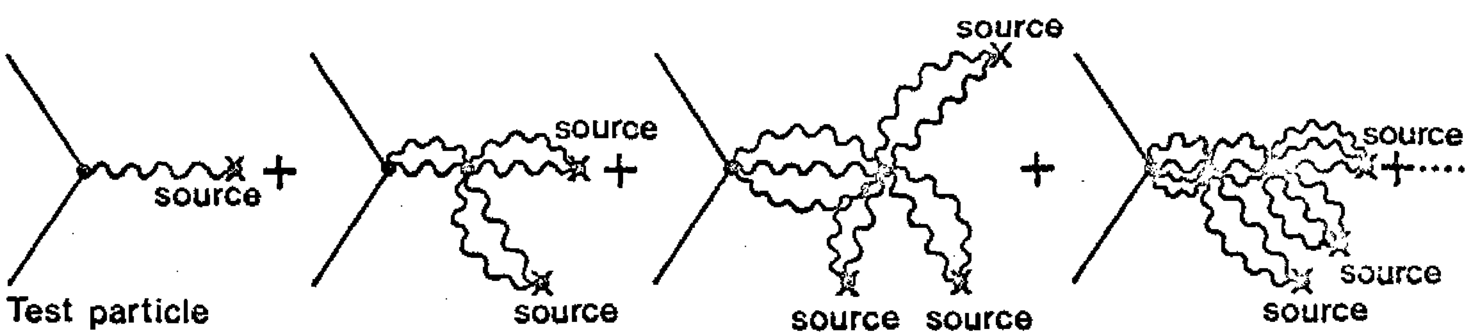

FIG. 6