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E R R A T A

The following sign changes should be made:

1) In Eq.(9) replace $g_2$ by $-g_2$ .

2) In the sentence following Eq.(9) read: "..., provided $g_1 = -g_2$ , etc."

3) In Eq.(10) replace $\zeta_2$ by $-\zeta_2$ .

4) In Eqs.(12) replace $A_\mu$ by $-A_\mu$ .

5) In Eq.(15) replace $g\Psi_2$ by $-g\Psi_2$ .

We wish to emphasise the fact that the opposite sign of the axial vector couplings of $\chi$ and $\psi$ causes this model to be anomaly-free.

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International Atomic Energy Agency
and
United Nations Educational Scientific and Cultural Organization

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

# SUPER-SYMMETRIC V-A GAUGES AND FERMION NUMBER *

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ABSTRACT

The problem of fermion-number conservation can be solved in a super-symmetric gauge theory provided both vector and axial-vector interactions are admitted.

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The discovery of super-symmetric Lagrangians which admit local internal symmetries [1] leads one to expect that super-symmetry may play a role in unified theories of weak, electromagnetic and strong interactions. An outstanding problem in the search for realistic models has been the difficulty of incorporating a purely-fermionic phase invariance which could be associated with the conservation of lepton or baryon number. In this note we wish to show that one solution of the fermion-number conservation problem can be achieved in the context of models which carry a local vector as well as axial vector symmetry like $SU(n) \times SU(n)$ .

We use the method of super-fields [2a),2b)] to construct the Lagrangian. Firstly, consider the matter super-fields $\Phi_\pm(x,\theta)$ which take the form of n-component columns (for example) and transform under the local group according to

$$\Phi_\pm(x,\theta) \to \exp[i\Lambda_\pm(x,\theta)] \; \Phi_\pm(x,\theta)$$

$$\Phi_\pm(x,\theta)^\dagger \to \Phi_\pm(x,\theta)^\dagger \exp[-i\Lambda_\pm(x,\theta)^\dagger] \quad , \tag{1}$$

where $\Lambda_\pm$ are $n \times n$ matrices made up of complex super-fields. They are not hermitian (and, in particular, we do not identify $\Lambda_-$ and $\Lambda_+^\dagger$ as in Ref. 1). The super-fields $\Phi_\pm$ can be represented in the form:

$$\Phi_\pm(x,\theta) = \exp\left[\mp \frac{1}{4}\bar\theta\partial\!\!\!/\gamma_5\theta\right]\left(A_\pm(x) + \bar\theta\,\frac{1 \pm i\gamma_5}{2}\,\psi(x) + \frac{1}{2}\bar\theta\,\frac{1 \pm i\gamma_5}{2}\,\theta\,F_\pm(x)\right), \tag{2}$$

where $A_\pm$ and $F_\pm$ are spin-zero fields, and $\psi$ is a Dirac spinor.

To make a kinetic term for $\Phi_\pm$ which is locally invariant and super-symmetric we need a pair of gauge super-fields, $\Psi_1$ and $\Psi_2$, which transform according to

$$\exp[g_1\Psi_1] \to \exp[i\Lambda_+^\dagger] \; \exp[g_1\Psi_1] \; \exp[-i\Lambda_+] \; .$$

$$\exp[g_2\Psi_2] \to \exp[i\Lambda_-^\dagger] \; \exp[g_2\Psi_2] \; \exp[-i\Lambda_-] \quad , \tag{3}$$

where $\Psi_1$ and $\Psi_2$ are hermitian and traceless $n \times n$ matrix super-fields. The invariant matter Lagrangian is given by

$$\mathcal{L}_{matt} = \frac{1}{8}(\bar{D}D)^2 \left(\Phi_+^\dagger \exp[g_1\Psi_1]\,\Phi_+ + \Phi_-^\dagger \exp[g_2\Psi_2]\,\Phi_-\right) , \tag{4}$$

while, using the methods of Ref. 2a, one can show that the Lagrangian for the gauge fields is:

$$\mathcal{L}_{\text{gauge}} = \frac{1}{128} (\bar{D}D)^2 \sum_{j=1,2} \frac{1}{2} \mathrm{Tr}\left(X_{j\mu} X_{j\mu} + X_{j\mu}^{\dagger} X_{j\mu}^{\dagger}\right) , \tag{5}$$

where

$$X_{1\mu} = -\frac{1}{g_1} \left(C^{-1} \gamma_\mu \frac{1 + i\gamma_5}{2}\right)^{\alpha\beta} D_\alpha \left(\exp[-g_1\Psi_1] D_\beta \exp[g_1\Psi_1]\right)$$

$$X_{2\mu} = -\frac{1}{g_2} \left(C^{-1} \gamma_\mu \frac{1 - i\gamma_5}{2}\right)^{\alpha\beta} D_\alpha \left(\exp[-g_2\Psi_2] D_\beta \exp[g_2\Psi_2]\right) . \tag{6}$$

These fields transform according to

$$X_{1\mu} \to \exp[i\Lambda_+] X_{1\mu} \exp[-i\Lambda_+] + \frac{2i}{g_1} \exp[i\Lambda_+] \partial_\mu \exp[-i\Lambda_+] ,$$

$$X_{2\mu} \to \exp[i\Lambda_-] X_{2\mu} \exp[-i\Lambda_-] + \frac{2i}{g_2} \exp[i\Lambda_-] \partial_\mu \exp[-i\Lambda_-] , \tag{7}$$

i.e. like Yang-Mills fields. One can show that the Lagrangian (5) is invariant (up to a variationally insignificant surface term).

One can choose a special super-gauge in which the super-fields $\Psi_j$ $(j = 1,2)$ take the form:

$$\Psi_j(x,\theta) = \left(\frac{1}{4} \bar{\theta} i\gamma_\nu\gamma_5\theta \, W_{j\nu}^{\alpha}(x) + \frac{1}{2\sqrt{2}} \bar{\theta}\theta \, \bar{\theta} \, \zeta_j^{\alpha}(x) + \frac{1}{16} (\bar{\theta}\theta)^2 \, D_j^{\alpha}(x)\right) \frac{\lambda^\alpha}{2} , \tag{8}$$

where $\lambda^\alpha$ denote the $n \times n$ hermitian generating matrices of $SU(n)$. The fields $W$ and $D$ are real and the $\zeta$'s are Majorana spinors. The explicit form of the Lagrangian (5) in this gauge is

$$\mathcal{L}_{\text{gauge}} = \sum_{j=1,2} \Bigg[ -\frac{1}{4} \left(\partial_\mu W_\nu^{\alpha} - \partial_\nu W_\mu^{\alpha} + g f^{\alpha\beta\gamma} W_\mu^{\beta} W_\nu^{\gamma}\right)^2 + \frac{i}{2} \bar{\zeta}^\alpha \gamma_\mu \left(\partial_\mu \zeta^\alpha + g f^{\alpha\beta\gamma} W_\mu^{\beta} \zeta^\gamma\right) + \frac{1}{2} (D^\alpha)^2 \Bigg]_j . \tag{9}$$

We now remark that, provided $g_1 = g_2$, the Majorana spinors $\zeta_1$ and $\zeta_2$ can be combined to form a complex Dirac spinor

$$\chi = \frac{1 - i\gamma_5}{2} \zeta_1 + \frac{1 + i\gamma_5}{2} \zeta_2 \tag{10}$$

and observe that it is possible to write

$$\sum_{j=1,2} \frac{i}{2} \bar{\zeta}_j^\alpha \gamma_\mu \left(\partial_\mu \zeta_j^\alpha + g f^{\alpha\beta\gamma} W_{j\mu}^{\beta} \zeta_j^\gamma\right) =$$

$$= i \bar{\chi}^\alpha \gamma_\mu \left(\partial_\mu \chi^\alpha + \frac{g}{\sqrt{2}} f^{\alpha\beta\gamma}\left(V_\mu^\beta \chi^\gamma + A_\mu^\beta i\gamma_5 \chi^\gamma\right)\right) , \tag{11}$$

where V and A are defined by

$$W_{1\mu} = \frac{1}{\sqrt{2}} (V_\mu + A_\mu) ,$$
$$W_{2\mu} = \frac{1}{\sqrt{2}} (V_\mu - A_\mu) . \tag{12}$$

The fermion part of the Lagrangian (9) when expressed in the form (11) is manifestly invariant under the fermion-number-defining phase transformation

$$\chi \to \exp[i\alpha]\, \chi \quad , \quad \bar{\chi} \to \exp[-i\alpha]\, \bar{\chi} . \tag{13}$$

This can be viewed as a $\gamma_5$-transformation on the Majorana spinors,

$$\zeta_1 \to \exp[\alpha\gamma_5]\, \zeta_1 \quad , \quad \zeta_2 \to \exp[-\alpha\gamma_5]\, \zeta_2 . \tag{13'}$$

(Contrary to appearances, these transformations commute with space reflections which must interchange $\zeta_1$ and $\zeta_2$ .)

For the matter fields $\Xi_\pm$ we choose, as illustration, a super-multiplet which transforms according to

$$\Xi_\pm \to \exp[i\Lambda_\pm]\, \Xi_\pm \exp[-i\Lambda_\pm] . \tag{14}$$

The invariant Lagrangian for matter is given by:

$$\mathcal{L}_{matt} = \frac{1}{8} (\bar{D}D)^2 \frac{1}{2} \mathrm{Tr}\left(\Xi_+^{\dagger} \exp[g\, \Psi_1]\, \Xi_+ \exp[-g\, \Psi_1] + \Xi_-^{\dagger} \exp[g\, \Psi_2]\, \Xi_- \exp[-g\, \Psi_2]\right) , \tag{15}$$

which reduces in the special super-gauge to the form:

$$\mathcal{L}_{matt} = \left(\partial_\mu A_+^* + g\, W_{1\mu} \times A_+^*\right) \cdot \left(\partial_\mu A_+ + g\, W_{1\mu} \times A_+\right)$$
$$+ \left(\partial_\mu A_-^* + g\, W_{2\mu} \times A_-^*\right) \cdot \left(\partial_\mu A_- + g\, W_{2\mu} \times A_-\right)$$
$$+ i\bar{\psi}\gamma_\mu \cdot \left(\partial_\mu \psi + \frac{g}{\sqrt{2}} V_\mu \times \psi - \frac{g}{\sqrt{2}} A_\mu \times i\gamma_5 \psi\right)$$
$$+ F_+^* \cdot F_+ + F_-^* \cdot F_- +$$

$$- \sqrt{2}\, gi \left\{ A_+^* \cdot \bar{\chi} \times \frac{1 + i\gamma_5}{2} \psi + A_+ \cdot \bar{\psi} \times \frac{1 - i\gamma_5}{2} \chi \right.$$

$$\left. + A_-^* \cdot \bar{\chi} \times \frac{1 - i\gamma_5}{2} \psi + A_- \cdot \bar{\psi} \times \frac{1 + i\gamma_5}{2} \chi \right\}$$

$$+ ig \left\{ A_+^* \times A_+ \cdot D_1 + A_-^* \times A_- \cdot D_2 \right\} \qquad (16)$$

{where the SU(n) contractions are implicit, e.g.

$$A_+ \cdot \bar{\psi} \times \frac{1 - i\gamma_5}{2} \chi = f^{\alpha\beta\gamma} A_+^{\alpha} \bar{\psi}^{\beta} \frac{1 - i\gamma_5}{2} \chi^{\gamma} \Big\} .$$

Note that the sign of the axial-vector coupling is opposite in (11) and (16). This Lagrangian clearly admits the phase symmetry (13) with the matter spinor transforming as:

$$\psi \rightarrow \exp[i\alpha]\, \psi \quad , \quad \bar{\psi} \rightarrow \exp[-i\alpha]\, \bar{\psi} \quad . \qquad (17)$$

To conclude, we make a number of remarks.

1) Since the phase transformation (13) (and thus fermion-number conservation for the gauge Lagrangian) necessitates $g_1 = g_2$, and since this is also the condition that parity is conserved ($\Psi_1 \rightarrow -\Psi_2$, $\Psi_2 \rightarrow -\Psi_1$), both V+A and V−A couplings must be present in equal strengths so far as the pure gauge Lagrangian (9) is concerned.

2) For the matter system there is no need to conserve parity. One could set $\Xi_+$ or $\Xi_- = 0$ in the Lagrangian without destroying the phase symmetry. One would then obtain pure (V+A) or (V−A) matter interactions.

3) There are no mass terms in the Lagrangian at present. (None are possible for the gauge Lagrangian because of the local symmetry.) One could add, however, a super-symmetric mass term for the matter fields, but this would violate the phase symmetry. To see this more explicitly, remark that whereas the kinetic energy term (15) for matter consists of the phase-invariant bilinears $\Xi_+^{\dagger} \Xi_+$, the bilinears which occur for the mass term are of the non-phase-invariant form $\Xi_+ \Xi_+$. The same remark would seem to apply to the renormalizable self-interaction terms of the type [3] $\bar{D}D\, \mathrm{Tr}(\Xi_+)^3$ for the matter fields, so that inclusion of such terms may violate fermion-number conservation.

4) Masses may be generated from the (fermion-number conserving) Lagrangian (9) plus (16) as it stands, provided we admit of spontaneous violation of internal symmetries [2a], with non-zero vacuum expectation values for the "Higgs" fields $A_\pm$ , $D_1$ , $D_2$ (and possibly $F_\pm$) generated through the radiative-mechanism of Coleman and Weinberg [4].

To summarize, the conclusion of this argument appears to be that spontaneous violation of symmetries for generation of masses is compatible with super-symmetry, local gauge-invariance and fermion-number conservation only if the spontaneous breakdown is radiative in origin.

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