A STUDY OF SHADOWING IN HEAVY NUCLEI

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A STUDY OF SHADOWING IN HEAVY NUCLEI *

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ABSTRACT

The theory for shadowing of real and virtual photons in heavy nuclei is presented in the context of the generalized vector-meson dominance (GVMD) model. In particular, the consequence of including off-diagonal elements in GVMD is investigated. It is found that including a positive real/imaginary part in the virtual photon-hadron amplitude considerably improves agreement with experiments. Such an important real part is consistent with the energy dependence of the vector-meson electroproduction data. These data also exhibit a feature which may be called the decoupling of the Pomaranchukon as a function of $q^2$, the photon mass. The relation to a similar feature of the pure hadronic interactions is discussed.

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I. INTRODUCTION

It is by now proven that a great deal of information, for example the phases of elementary particle reactions, is accessible only through interactions involving nuclei. Nuclei provide a unique target for observing effects which are of short time or space duration, such as the scattering after creation of short-lived particles, or the related energy fluxes recently discussed by Gottfried. This is in no small measure due to almost surprising success and accuracy of the Glauber theory and its modifications in describing the multiple scattering in nuclei.

Of particular interest has been, and is, the scattering of real and virtual photons off nuclei. From our knowledge of their interaction with hydrogen we are aware that an important component in the scattering involves the creation and then rescattering of vector mesons. Consequently, it was expected that the total cross-section of both real and virtual photons off nuclei would exhibit a continuous decrease from low energy, where only the photon propagation contributes (see Fig. 4b) giving a cross-section proportional to $A$ (due to the long mean-free-path of the photon in nuclear matter, which implies small effects coming from its multiple scattering), to the medium and high-energy regions, where there is a dominant contribution to the cross-section proportional to $A^{2/3}$ coming from the multiple scattering and thence strong absorption of photoproduced hadronic states.

Experimentally, the data proved somewhat surprising in that while shadowing (which we shall define precisely in Sec. III) is observed for real photons, almost no shadowing or even antishadowing is found for electron-nucleus scattering in which the square of the virtual photon four-momentum $q^2$ is less than $-0.25 \text{ (GeV/c)}^2$.

Unfortunately, the data contain large errors and a viable viewpoint is that many models such as GVMD can successfully describe the data with what one may call conventional parameters for the basic vector meson-nucleon interaction. In this paper we shall present one such analysis in the context of what we wish to call extended vector-meson dominance (EVMD). EVMD is a version of GVMD in which we explicitly include off-diagonal terms such as $\rho N \rightarrow \rho' N$ and we shall describe it briefly in the next section of this paper.

This modification of GVMD does not improve on the recent calculation of Schildknecht and, in the absence of any knowledge on the size of the systematic errors in the experiment, we might also claim satisfactory agreement with the shadowing data. However, there are two
important modifications which constitute the main raison d'être of this work. One is based on the phenomenological observation that many experimental features (such as the diffraction slope off hydrogen) show, like shadowing, a rather abrupt \( q^2 \) dependence between 0 and \(-0.5\) (GeV/c)^2. The other modification is that the effective energy behaviour of the \( \gamma^* p \rightarrow p p \) cross-section tends to exhibit a progressively smaller Pomeranchuk part as \( q^2 \) decreases; we shall refer to this effect as decoupling of the Pomeranchukon and describe it in more detail in Sec.IV. It has its counterpart in pure hadron-hadron interactions, as we shall see by examining some of the latest Serpukhov data on inclusive cross-sections.

Further, the generalized Glauber theory used has been found to be sensitive to the ratio of the real to imaginary (R/I) parts of the vector meson-nucleon amplitude; in such a way that if R/I is important, and positive for the dominant \( \gamma^* N \rightarrow \rho N \) amplitude (for virtual photons \( \gamma^* \)) then even antishadowing can be achieved for some finite range in the photon energy.

The presence of an important real part in \( \gamma^* N \rightarrow \rho N \), when \( q^2 \) decreases, is consistent with the previously mentioned change in the effective energy behaviour, since with the decoupling of the Pomeranchukon there is no compelling reason to have a predominantly imaginary amplitude.

In Sec.IV of this paper we shall present a phenomenological \( \gamma^* N \rightarrow \rho N \) amplitude which exhibits both of the above features while being consistent with real photon data. A discussion and interpretation are presented in Sec.V together with our conclusions.

II. EVMD

In this section we shall present the relevant features of the EVMD approach mentioned in the introduction. In such a model one considers two trios of vector mesons, the \( \rho, \omega, \phi \) and the "effective resonances" or "coherent states" \( \hat{\rho}, \hat{\omega}, \hat{\phi} \), allowing the presence of off-diagonal terms \( v \rightarrow \tilde{v} \). The photon thus couples to hadrons through the following current-field equivalence:

\[
J_\mu(x) = e \sum_{i} \frac{m_i^2}{f_i^2} v_i^\dagger(x) ,
\]
where \( i \) runs over the six \( \nu, \bar{\nu} \) "poles". We shall assume that the \( \mathcal{f}_\nu \) are in the same ratio among themselves as the \( f \).

The amplitude for the photo- and electroproduction of transversally polarized rhos is given by

\[
A_{\gamma^* \rho} = e \sum_i \frac{A_{\nu i}^{i \bar{\nu}}}{f_i(v_i)} = \frac{e A_{\rho \rho}^{i \bar{\nu}}}{f_{\rho}} \left[ \frac{1}{(\rho)} + \frac{\delta_{\rho \rho}}{(\bar{\rho})} \right], \tag{2.1}
\]

where \((v_i) = 1 - \frac{q^2}{m_i^2} \) and \( \delta_{\rho \rho} = (A_{\rho \rho}^{i \bar{\nu}} f_{\rho})/(A_{\rho \rho}^{i \bar{\nu}} f_{\bar{\rho}}) \) which is in general complex, represents the off-diagonal contribution. In GVMD \( \delta_{\rho \rho} = 0 \) and, indeed, the predictions of GVMD for the production of rhos reduce to those of VMD because of this diagonal ansatz. For longitudinally polarized rhos we have

\[
A_{\gamma^* \rho} = \frac{e}{f_{\rho}} \frac{A_{\rho \rho}^{i \bar{\nu}}}{(\rho)} \sqrt{- \frac{2}{m_\rho}} + \frac{e}{f_{\rho}} \frac{A_{\rho \rho}^{i \bar{\nu}}}{(\bar{\rho})} \sqrt{- \frac{2}{m_{\bar{\rho}}}}. \tag{2.2}
\]

Assuming that asymptotically \( R_{\gamma^* \rho} = \frac{q_{\gamma^* \rho}}{q_{\gamma^* \rho}} \) tends to zero, we demand the maximum interference between the \( \rho \) and \( \bar{\rho} \) contribution to Eq.(2.2), i.e.

\[
A_{\rho \rho}^{i \bar{\nu}} m_{\rho} = -A_{\rho \rho}^{i \bar{\nu}} m_{\bar{\rho}}. \tag{2.3}
\]

At high energies we may parametrize \( A_{\rho \rho}^{i \bar{\nu}} \) by the factorized form

\[
A_{\rho \rho}^{i \bar{\nu}} = i k \, c_{\rho \rho}^{i \bar{\nu}}(s) \exp \left[ \frac{b_{\rho}^{i \bar{\nu}}}{2} \right] (1 - \eta_{\rho}^{i \bar{\nu}}) \tag{2.4}
\]

\((k \) is the magnitude of the three-momentum and \( s \) is the square of the energy in the centre-of-mass frame) where \( \eta_{\rho}^{i \bar{\nu}} \) is the ratio of the real to imaginary parts of the amplitudes. We shall take \( b_{\rho}^{i \bar{\nu}} = b^{i \bar{\nu}} = b \) since there is insufficient data to determine these separately and we do not consider longitudinal-to-transverse conversion. Thus we may write

\[
\frac{\partial \sigma(\gamma^* \rho)}{\partial t} \bigg|_{\theta = 0} \frac{1 + \epsilon R_{\gamma^* \rho}}{16\pi} \frac{\rho^2}{f_\rho^2} \sigma_{\rho \rho}^{12} (1 + \eta_{\rho}^{2}) \times \left[ \frac{1}{(\rho)} + \frac{\delta_{\rho \rho}}{(\bar{\rho})} \right]^2 \exp \left[ b_{\rho}^\min \right], \tag{2.5}
\]

where \( \epsilon \) represents the amount of longitudinal polarization of the incoming \( \gamma^* \) beam.
In Fig. 1 we show a fit to the data for \( \frac{d\sigma(\gamma^* + p)}{dt} \) at \( E_{\gamma^*} = 10.5 \text{ GeV} \). We have normalized the data at \( q^2 = 0 \) so as to be consistent with a forward differential cross-section of 102 \( \mu b/(\text{GeV}/c)^2 \) (consistent with that quoted by Lefrançois and Benaksas et al.) and \( m_{\rho'}^2 = 3.5 \text{ GeV}^2 \) (somewhat larger than the square of \( \rho' \), i.e. \( m_{\rho'}^2 \approx 2.56 \text{ GeV}^2 \)), and \( \eta_{\rho} = -0.2 \). The slope \( b \) has been parametrized as \( b = 7.5/(1 + 4 \frac{m_{\rho'}^2 + q^2}{2k}) \) (see Secs. IV and V). The fits shown are for \( \delta_{\rho'} = -0.14 \) (with this value, a forward differential cross-section of 102 \( \mu b/(\text{GeV}/c)^2 \) implies \( \sigma_{\rho p}^\prime = 30 \text{ mb} \)) and \( \xi_{pp} = \xi_{pp}^\prime = 0.5 \) and 0.7. Taking into account that the data of Dakin et al. are subject to a 16\% systematic uncertainty, we may conclude that both fits are acceptable theoretical fits, although the fit is better when \( \xi_{pp} = 0.5 \), and both fits are a little too high with respect to the experimental point of \( q^2 = -1.27 \text{ (GeV}/c)^2 \).

In Fig. 2 are shown the experimental values of \( R_{\gamma^* p} \) with the two fits. Again both fits are acceptable, but the value of \( \xi_{pp} = 0.7 \) seems to be preferred.

For the forward virtual Compton scattering off hydrogen we have the expression

\[
\sigma(\gamma^* p + \gamma^* p) = \sigma(\gamma p + \gamma p) \left\{ \left( (\rho) - (\rho) - (\delta) - 1 \right) \right\} \times \left\{ \left( (\rho) - (\rho) - (\delta) - 1 \right) \right\}
\]

and (assuming maximal interference)

\[
\sigma_{\gamma^* p + \gamma^* p} = \sigma(\gamma p + \gamma p) \left[ \frac{m_{\rho}^2}{m_{\rho}^2} \right] \left\{ \left( (\rho) - (\rho) - (\delta) - 1 \right) \right\} \times \left\{ \left( (\rho) - (\rho) - (\delta) - 1 \right) \right\}
\]

In Eqs. (2.6) and (2.7) we have made the simplifying assumption that \( \delta_{\rho'} = \delta_{\omega} = \delta_{\phi} = \delta \) and \( \xi_{pp} = \xi_{\omega p} = \xi_{\phi p} = \xi \). We have also neglected off-diagonal terms such as \( \omega - \phi \), \( \omega - \phi \), etc. The \( r_i \) are fixed by the relations.
\[(r_\rho + r_\omega + r_\phi) (1 + 2\delta + \Delta) = 1\]

\[
\frac{\sigma_{pp}}{f_{2p}^2} = \frac{\sigma_{ww}}{f_{2w}^2} = \frac{\sigma_{\phi\phi}}{f_{2\phi}^2}
\]

and

\[
\Delta \equiv \frac{(A_{pp} f_\rho^2)}{(A_{pp} f_\rho^2)}.
\]

In Fig. 3 we show fits to the data for \(\sqrt{s} = 3,4,4.4\) GeV for a value \(\Delta = 0.40\). (Then, if we take the values \(\frac{f_{2p}}{4\pi} = 19.2\), \(\frac{f_{2\phi}}{4\pi} = 11.2\), \(\phi\)'s implies \(\sigma_{ww} = 29.4\) mb and \(\sigma_{\phi\phi} = 10.6\) mb at \(\nu = 10.5\) GeV, and \(r_\rho = 0.742\), \(r_\omega = 0.093\), \(r_\phi = 0.058\) giving the value 

\[\sigma_{(\gamma^*p \rightarrow \gamma^*p)}(\nu = 10.5) = 119\, \text{mb}.\]

In the fits we have taken \(m_\phi^2 = m_\rho^2\), \(m_\omega^2 - m_\phi^2 = m_\rho^2 - m_\omega^2\) and 

\[\sigma_{\gamma p \rightarrow \gamma p}(\nu) \sim \left(1 + \frac{0.7}{\nu^\alpha}\right).\]

The fits are acceptable.

In the next section we shall compute the nuclear shadowing using the above elementary amplitudes as an input. The energy dependence and phase will be given by

\[\sigma_{\gamma A}(1 - i\eta) \sim \left[1 + \frac{0.7}{\nu^\alpha} + i \frac{0.7}{\nu^\beta}\right], \nu = \rho, \omega,\]

normalized at \(\nu = 10.5\) GeV to \(\sigma_{\gamma p}^+ = 30\) mb and \(\sigma_{\omega p}^+ = 29.4\) mb.

\(\sigma_{\phi p}^+ = 10.6\) mb will be energy independent, and \(\eta_\phi = 0\).

### III. PHOTON NUCLEUS SCATTERING

In this section a calculation is made of the nuclear shadow by combining the eikonalized particle-nucleus scattering theory with the EVM results for the basic particle-nucleon events. We shall use the simpler model of nuclear matter (e.g., a constant density distribution) and shall not consider mixing between transverse and longitudinal components inside the nucleus.

We shall be concerned with the inclusive reaction (Fig. 4)

\[e + A \rightarrow e + X.\]

In the one-photon exchange approximation the cross-section is given by

\[
\frac{d\sigma}{d^2p d\Omega} = \Gamma_A \left\{ \sigma^{\gamma A\gamma A}_\gamma(v, q^2) + \epsilon \sigma^{\gamma A\gamma A}_\gamma(v, q^2) \right\},
\]

\[-6-\]
where $\sigma_{y* A}^t$, $\sigma_{y* A}^l$ are the total cross-sections for transverse and longitudinal (scalar) virtual photon scattering off nuclei, respectively. \( \Gamma_A \) is the photon flux factor and $\epsilon$ the polarization parameter,

$$
\Gamma_A = \frac{\alpha}{2\pi^2(-q^2) E_0} \frac{E_0}{E} \left[ \frac{W - m_A^2}{2m_A} \right] \frac{1}{1-\epsilon},
$$

$$
\epsilon = \frac{1}{1 + 2 \left[ 1 + \frac{\alpha^2}{(-q^2)} \right] \tan^2(\theta/2)} \quad (3.2)
$$

To see the shadowing effects, we define the effective nucleon number $A_{\text{eff}}$ or the shadow factor $S$ in the following manner:

$$
\left[ \sigma_{y* A}^t(v, q^2) + \epsilon \sigma_{y* A}^l(v, q^2) \right] = A_{\text{eff}}(v, q^2) \left[ \sigma_{y* N}^t(v, q^2) + \epsilon \sigma_{y* N}^l(v, q^2) \right]
$$

$$
S(v, q^2) = A_{\text{eff}}(v, q^2)/A \quad (3.3)
$$

Shadowing, no shadowing or antishadowing correspond to $s < 1$, $s = 1$ or $s > 1$, respectively.

By using the optical theorem one obtains the total cross-section in terms of the forward amplitude for coherent and elastic photon-nucleus scattering. In the high-energy and small-angle limit one can study the coherent photon-nucleus interactions using the Feynman graph technique (assuming that the motion of the nucleons in the nucleus is non-relativistic). The amplitude obtained accounts for the propagation through the nucleus of photoproduced hadronic states on their mass shells.

For heavy nuclei one arrives at the expressions of the eikonalized coupled channel optical model by using a factorized nuclear wave function (which neglects the centre-of-mass constraint and correlations among nucleons). The optical model describes the coherent particle-nucleus scattering by means of a matrix pseudopotential closely related to the basic particle-nucleon scattering events. The optical model equations are

$$
(v^2 + v^2 - m_i^2) \psi_i(x) = \sum_j U_{ij}(x) \psi_j(x), \quad (3.4)
$$

where an index labels the channels $i,j,k = y*, y, \nu$. $\psi_i(x)$ is the wave function in channel $i$ with incident state in channel $k$, $U_{ij}(x)$ is
the optical potential which allows transitions from channels \( j \) to \( i \)
(in first approximation \( U_{ij}(\vec{x}) = -4\pi \rho(\vec{x}) A_{ij}(t' = 0) \), \( \rho \) being
the nuclear density). The moment in channel \( i \) is \( k_i^2 = v_i^2 - m_i^2 \) and, for
photons in the initial channel, it is \( k_{\gamma i}^2 = v_i^2 - q_i^2 \). The amplitude
for the transition from channels \( i \) to \( j \) with transverse momentum transfer \( \Delta \) is in
the eikonal version (see Appendix)

\[
T_{ji}^{\Delta,\Delta}(\vec{b},z) = \int d^2 b \exp[-i\Delta \vec{b}] \int_{-\infty}^{\infty} dz \rho(\vec{b},z) \exp[-iq_{ji}z] \Gamma_{ji}^{\Delta,\Delta} (\vec{b},z),
\]

(3.5)

where \( q_{ji} = k_j - k_i \) and the profile functions \( \Gamma_{ji}^{\Delta,\Delta} \) are given by
the integral equations

\[
\Gamma_{ji}^{\Delta,\Delta} (\vec{b},z) = A_{ji}^{\Delta,\Delta} (0) - \int_{-\infty}^{\infty} dz' \sum_{k} \frac{U_{ji}^{\Delta,\Delta}(\vec{b},z')}{2i k_{k}} \exp[-q_{kk}(z-z')] \Gamma_{kk}^{\Delta,\Delta} (\vec{b},z').
\]

(3.6)

For heavy nuclei we can use the simplest nuclear model given by a constant
density

\[
\rho(\vec{b},z) = \begin{cases} 
\frac{1}{v_0} & |z| < z_0 \\
0 & |z| > z_0
\end{cases}, \quad z_0 = \sqrt{2 - b^2},
\]

(3.7)

the nuclear radius is \( R = 1.13 A^{1/3} \) fm, and \( v_0 = 6.044 \) (fm)\(^3\) is the
volume per nucleon.

With this model and using a Laplace transformation,

\[
\Gamma_{ji}^{\Delta,\Delta} (\vec{b},z) = \frac{k_i}{2\pi i} \int_{a-i\infty}^{a+i\infty} \tilde{\xi} \exp\left[\xi (z+z_0)\right] \Gamma_{ji}^{\Delta,\Delta} (\xi),
\]

(3.8)

in Eq. (3.6), a set of matrix equations is obtained:

\[
\Gamma_{ji}^{\Delta,\Delta} (\xi) = \frac{t_{ji}}{\xi} + i \lambda \sum_{k} \frac{t_{kj}^{\Delta,\Delta}}{\xi - i q_{ki}},
\]

(3.9)

with \( t_{ji} = \frac{1}{k_i} A_{ji} (0) \) and \( \lambda = \frac{2\pi}{v_0} \).

The use of the optical theorem together with Eqs. (3.9), (3.8)
and (3.5) leads (to first order in \( q \)) to a total cross-section (see
Appendix)

\[
\sigma_{Y_{\gamma A}}(v,q^2) = A \left[ \sigma_{Y_{\gamma N}}(v,q^2) - \Gamma_{\gamma}^{\Delta,\Delta}(v,q^2) \right] + 2S_0 A^{2/3} \Gamma_{\gamma}^{\Delta,\Delta}(v,q^2) + \theta(\frac{1}{A}) + \theta(\alpha^2) + ...
\]

(3.10)

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The last two terms are negligible in numerical effects, $S_0 \Delta^{2/3} = \pi R^2$, and $\sigma_{\gamma^*N}$ is the virtual photon-nucleon total cross-section. The $T$'s account for the propagation, after production, of the hadronic states considered. (see Figs. 4b and Eqs. (3.15), (3.16) and (3.17) for their definition). The inclusion of longitudinal components gives a shadow factor

$$S(v,q^2) = 1 - \frac{(r^+ + \epsilon r^+)}{\sigma_{\gamma^*N} + \epsilon \sigma_{\gamma^*N}}.$$ (3.11)

Of course, for real photons, $\epsilon = 0$. At low energy, because of the nuclear form factor, $(\Gamma^+ \Gamma^0) \rightarrow 0$ and $s \rightarrow 1$, giving the so-called volume effect. However, with increasing photon energy the momentum transfer necessary for hadron photoproduction becomes smaller than the average momentum of the nucleons in the nucleus and there is an appreciable hadronic contribution to the shadow factor which separates from 1. Once the $\sigma_{\gamma^*N}$ is given, the energy behaviour depends on the considered hadronic spectrum and on the amplitudes for photoproduction and scattering off the nucleons.

From Sec. II we can obtain the relevant quantities for the coherent states $\psi$; in particular we have

$$\frac{A^\perp}{A^l} = \delta \left( \frac{f_{\psi}}{f_{\psi}} \right), \quad \frac{A^\perp}{A^l} = \delta \frac{f_{\psi}}{f_{\psi}}.$$ (3.12)

The values of $\Delta$ and $\delta$ were fixed in the previous section from the total photon cross-section and rho electroproduction off hydrogen, respectively.

In accordance with the analysis of the experiments about diffraction dissociation of pions into 3- and 5-pion systems on heavy nuclei where equal pion-nucleon and several pion-nucleon total cross-sections were found, we should expect $A^\perp_{VV} \approx A^l_{VV}$. Then from Eqs. (3.12) (with $\Delta = 0.40$) we obtain $(f_{\psi}/f_{\psi})^2 \approx 2.5$. This ratio is different from that observed for the $\rho'$ (e.g., $(f_{\rho'}/f_{\rho'})^2 \approx 2$). This suggests that the state $\rho'$ contains more than the $\rho'$; being more like the proper mode that describes the coherent propagation of the whole set of states $\rho'$, $\rho''$, etc. The condition of maximum interference (Eq. (2.3)), on the other hand, fixes the amplitudes for the longitudinal components,

$$\frac{A^\perp_{VV}}{A^l_{VV}} = \frac{m_{\psi}}{\Delta m_{\psi}^2} \frac{A^\perp_{VV}}{A^l_{VV}} \quad \frac{A^\perp_{VV}}{A^l_{VV}} = \frac{m_{\psi}}{\Delta m_{\psi}^2} \frac{A^\perp_{VV}}{A^l_{VV}}.$$ (3.13)
The photoproduction amplitudes are now:

\[ A^\Gamma_{\gamma\gamma} = iK_{\gamma} \gamma^\Gamma \frac{f_{\gamma}}{f_{\gamma}} \sigma^\Gamma_{\gamma\gamma} (1-i\eta_\gamma) \left[ \frac{\delta}{(\gamma)} + \frac{\Delta}{(\gamma)} \right] \exp \left[ \frac{1}{2} b_\gamma t_{\text{min}} \right] \]

\[ A^\Gamma_{\gamma\gamma} = iK_{\gamma} \gamma^\Gamma \frac{f_{\gamma}}{f_{\gamma}} \sigma^\Gamma_{\gamma\gamma} (1-i\eta_\gamma) \left[ 1 - \frac{m_\gamma^2}{m_\gamma^2} \right] \frac{1}{(\gamma)(\gamma)} \sqrt{\frac{2}{m_\gamma^2}} \exp \left[ \frac{1}{2} b_\gamma t_{\text{min}} \right] . \]

(3.14)

With the above parametrization, we have at hand all the elements required for calculating the shadow factor. In fact, from the arguments presented in the Appendix we obtain the following expressions for the \( \Gamma \)'s:

\[ \Gamma_\gamma = 4\pi \sum_{\gamma = \rho, \omega, \phi} \text{Im} \left[ \frac{R_{\gamma\gamma}(0)}{D_{\gamma\gamma}(0)} \right] - \Gamma_\gamma \]

\[ \Gamma_\gamma = 4\pi \sum_{\gamma = \rho, \omega, \phi} \text{Re} \left[ \frac{R_{\gamma\gamma}(x_\gamma) - R_{\gamma\gamma}(x_\gamma*)}{x_\gamma - x_\gamma*} \right] , \]

where

\[ R_{\gamma\gamma}(x) = (\eta_\gamma + id_\gamma) t_{\gamma\gamma} + (\eta_\gamma + id_\gamma) t_{\gamma\gamma} - 2t_{\gamma\gamma} t_{\gamma\gamma} \]

\[ D_{\gamma\gamma}(x) = (\eta_\gamma + id_\gamma) (\eta_\gamma + id_\gamma) - t_{\gamma\gamma}^2 \]

(3.15)

(3.16)

with

\[ \eta_\gamma = t_{\gamma\gamma} + d q_{\gamma\gamma} \quad , \quad d = \lambda^{-1} = \frac{V_0}{2m} \]

The \( x_\gamma \) are the positions of the poles of \( D_{\gamma\gamma}(x) \) placed at

\[ x_\gamma = i d_\gamma = \frac{\eta_\gamma + \eta_\gamma*}{2} - i \sqrt{\left( \eta_\gamma - \eta_\gamma^* \right)^2 + t_{\gamma\gamma}^2} . \]

(3.17)

If we place \( t_{\gamma\gamma} = t_{\gamma\gamma}^* = 0 \) in the above expressions, we obtain the more simple version describing \( \rho \), \( \omega \), \( \phi \) propagation alone. For low energies (\( \nu < 5 \text{ GeV} \)) both models give the same shadowing. This is due to the higher mass of the \( \gamma \) states, which implies relatively high \( q_{\gamma\gamma} \) for those energies. However, with increasing \( \nu \), the \( q_{\gamma\gamma} \to 0 \) and more shadowing appears then predicted by \( \rho \), \( \omega \), \( \phi \) propagation.

The results obtained with our model for incident real photons are shown in Fig.5. We include, for comparison, the fits obtained with VMD, 2) and by Schindlbeck 7) using GVMD. We can see that, while the VMD prediction gives too much shadowing, both the EVMD and GVMD models agree quite well with the data. 3)
In Fig. 6 we plot the expected \( (v, q^2) \) dependence for the three models. Again VMD appears to give too much shadowing. The results of GNMD seem slightly better than those of EVMD. Taking into account the absence of information about the systematic errors and the long error bars, we should conclude that both models describe the \( q^2 \) behaviour of the shadow factor reasonably well. However, further experimental evidence for the basic events, analysed and discussed in the next section, suggests that there are other mechanisms playing a dominant role in the description of the hadronic data.

Two further remarks to close this section. The first concerns the small contribution coming from the off-diagonal terms \( t_{\gamma\gamma} \) which are not presented in the GVMD description of the hadronic spectrum. The second is that in both VMD and Schildknecht GVMD calculations the contribution from longitudinal photons was neglected at the nuclear and hydrogen levels. This corresponds to putting \( \epsilon = 0 \) in Eq. (3.11). However, the longitudinal components again give a small contribution to the shadowing and do not appreciably modify the \( q^2 \) behaviour.

IV. PHENOMENOLOGICAL ANALYSIS OF THE \( p_0, \omega \) ELECTROPRODUCTION DATA

In this section we shall discuss the main features of the differential cross-section for electroproduction of rhos as a function of the virtual photon energy \( v \) and the square mass of the photon \( q^2 \). In particular we shall analyse the Cornell experiment of Ahrens et al. as this experiment was carried out at different values of \( v \) and \( q^2 \). We shall also compare the results with other experiments.

The data of the Cornell experiment are given in Table I. We must remark that in the Cornell experiment the electroproductions of \( p_0 \) and \( \omega \) were not separated and it was assumed that the \( \omega \) contribution was 15% of the \( p_0 \). Each distribution was fitted with an exponential form \( \frac{d\sigma_{p_0}}{dt} = \frac{d\sigma_{\omega}}{dt} \bigg|_{t=0} e^{3t} \), with \( t \) being the virtual photon-vector meson invariant momentum transfer.

In column 3 of Table I the experimental values of the slope parameter are given. One observes that the slope parameter \( B \) decreases as \(-q^2\) increases. This kind of behaviour is also present in pure hadronic processes and will be discussed in some detail in the next section.
In the table we have 4 pairs of points with different energy \( \nu \) but almost equal \( q^2 \) (i.e. \( -q^2 = 0.59 , 0.61 ; -q^2 = 0.78 , 0.80 ; -q^2 = 0.99 , 0.79 ; -q^2 = 1.36 , 1.37 \)), which allows us to study the energy behaviour in different fixed values of \( q^2 \). Some information on the energy behaviour may be obtained by writing

\[
\left. \frac{d\sigma}{dt} \right|_{t=0} \sim \nu^{-n} \quad \text{with} \quad n = -\nu \frac{d}{d\nu} \left[ \ln \left( \frac{d\sigma}{dt} \right) \right]_{t=0} , \quad (4.1)
\]

In general \( n \) will not depend only on \( q^2 \) but also on \( \nu \).

For each pair of points of almost equal \( q^2 \) we obtained

\[
n(\nu, q^2) \text{ from } \frac{d\sigma}{dt} \bigg|_{t=0} = \frac{\nu_1}{\nu_2} \bigg( \frac{d\sigma}{dt} \bigg|_{t=0} \bigg) = \frac{\nu_1 + \nu_2}{2} , \quad (4.2)
\]

where \( \nu_1 \) and \( \nu_2 \) are the values of \( \nu \) for the pair of points. The values of \( n \) are plotted in Fig.7. At the top of each point is given the value of \( \overline{\nu} \). All 4 values are near \( \overline{\nu} = 5 \) GeV (\( \overline{\nu} = 5.1 \) on average).

The value at \( q^2 = 0 \) is obtained from the experimental data for \( \rho \) and \( \omega \) photoproduction (see Ref.24 for a compilation and analysis). We observe that \( n \) increases with \( -q^2 \) and that \( n \approx 1 \) for \( -q^2 = 1 \) GeV.

This feature suggests a larger relative importance of the non-Pomeranchukon Regge poles when \( -q^2 \) increases. We also see that the value of \( \frac{d\sigma}{dt} \overline{\nu} \) decreases when \( -q^2 \) increases. This decrease is not explained by the \( \rho \) propagator, as can be seen by comparing column 5 with the data for \( \rho \) and \( \omega \) photoproduction (Ref.10,17) \( \left( \frac{d\sigma}{dt} \omega (q^2 = 0) \right|_{t=0} = 145 \pm 10 \) \( \mu \)b for \( \nu = 4 \) GeV, and \( \approx 128 \pm 10 \) \( \mu \)b for \( \nu = 7 \) GeV). This decreasing looks more remarkable if one remembers that the data includes the contribution of longitudinal photons, which is important and rising with \( -q^2 \). Although systematic errors (in the Cornell experiment only statistical errors are quoted) might modify this picture, it seems that we may talk of a rather drastic dependence on \( q^2 \).

Both this rather rapid decrease of \( \frac{d\sigma}{dt} \overline{\nu} \) with \( q^2 \) and the increasing importance of non-Pomeranchukon Regge poles lead us to call this phenomena the rapid decoupling of the Pomeranchukon when \( -q^2 \) increases.

On the other hand, this rapid diminution of the Pomeranchukon's importance does not justify the assumption that the electroproduction amplitude is predominantly imaginary; since from a term \( \sim \nu^3 \) one would expect real and imaginary parts of the same order of magnitude and a constant term to be mainly real. A fit which includes the previous features is given by
These formulae differ from Eqs. (2.1) and (2.2) by the presence of the cut-off mass $m_\Lambda$ and the additional real part $D \sqrt{-q^2}/\nu$, and if $m_\Lambda = \infty$ and $D = 0$ we have the fit of Sec.II. The functional form $D \sqrt{-q^2}$ was taken arbitrarily, and other functional forms (i.e., $D = \frac{0.6 - q^2}{2\nu}$) fit equally well the data in the range $-2 < q < -0.4$. The values of $(1 - q^2/m_\sigma^2) \frac{\partial \rho_\omega}{\partial q^2}$ for $m_\Lambda = 1.16$ GeV/c$^2$ and $D = 7$ and all the other parameters, as in Sec.II ($F_\rho = 0.7$), are shown in the last column of Table I. Agreement with the experimental data is rather good. However, a value of $m_\Lambda = 1.9$ GeV/c$^2$ is preferred if the data of the SLAC experiment of Dakin et al. is fitted with the previous formulae. The value obtained for $n$ (from Eq. (4.1)) at $\Delta = 5.15$ is also shown in Fig.7.

For the time being we have no model for the value and functional dependence of the additional real part. In Eqs. (4.3) and (4.4) we have taken the same R/I ratio for both the longitudinal and transversal contributions. Another possibility is to replace the additional real part only in the longitudinal amplitude, as in such a case the $\sqrt{-q^2}$ would appear in a natural way. However, this would imply that $R_{\gamma\varphi}(q^2)$ grows rather quickly when $q^2$ decreases, which, even if the errors are large, does not seem to be supported by the data (see Fig.2). The presence of restrictions due to gauge invariance or the possible existence of fixed poles might lead to possible approaches. Remarks on the connection with pure hadronic reactions will be the topic of the next section. However, our present ignorance does not preclude us from testing our phenomenological amplitude in the nuclear laboratory.

In Fig.8 we show the fit to the shadowing obtained using Eqs. (4.3) and (4.4) for the elementary amplitudes for $\rho$ and $\omega$ photoproduction (with unmodified $\phi$ photoproduction). We also show the fit obtained with modified $\rho$ and $\omega$ mesons. In both cases we obtain an excellent fit to the data. The best fit is obtained when both $\rho$, $\omega$ and $\rho'$, $\omega'$ have additional real parts.
In Fig. 9 we show the shadow obtained by only $\rho$, $\omega$, $\phi$ propagation in the nucleus. The contribution to shadowing due to $\bar{\rho}$, $\bar{\omega}$, $\bar{\phi}$ will be the difference between fits in Fig. 8 and the dashed lines of Fig. 9.

V. DISCUSSION AND CONCLUSION

As we have seen in the previous section, a considerable improvement in the agreement between theory and experiment can be achieved by decoupling the Pomeronchukon as $-q^2$ increases and consequently increase the R/I ratio to a positive value. We can try to understand this decoupling by concentrating on the $\gamma^* \rightarrow \rho$ transition ($\gamma^*$ virtual or real photon) inside the nucleus. Forgetting for the moment about GVMD or EVMD, we notice that this potential diffractive process appears to become less diffractive as the difference in masses of the incoming ($q^2$) and outgoing ($p^2$) particles increases. The slope parameter $B$ also decreases when that difference increases. Both features have previously been seen, even in pure hadronic processes.\(^{(24,25)}\) For example, in the Serpukhov data at 25 and 40 GeV the process $\tau p \rightarrow pX$ has been studied for forward-produced bosonic states $X$ ($0.15 < |t| < 0.32$).\(^{(24)}\) It has been noted that as the invariant square of the momentum of $X$ increases ($M^2$) the slope parameter $B$ decreases. As a matter of fact, the data at 25 and 40 GeV are consistent with $B$ being a function of the scaling variable

$$\frac{M^2}{s} \simeq \frac{M^2 - M^2_{min}}{2M}.\quad \text{In Fig. 10 we plot the values of the slope parameter for both electroproduction data of Sec.IV and the $\tau p \rightarrow pX$ data of Ref.24, with respect to the variable } z = \frac{M^2_{out} - M^2_{in}}{2v} (\text{i.e. } z = \frac{m^2_p - q^2}{2v} \text{ for rho electroproduction}).$$

Even if the range of energies is rather different ($\bar{\nu} \simeq 5$ GeV for the electroproduction experiment, $\bar{\nu} \simeq 32.5$ GeV for the pure hadronic case), the data show the same behaviour and, as a matter of fact, the fits given by Eq. (4.4) agree very well with $\tau p \rightarrow pX$ data. In Ref.24 the energy dependence of the differential cross-section was parametrized by

$$\left. \frac{d\sigma(\tau p \rightarrow pX)}{dM^2} \right|_{0.15 < |t| < 0.32} \sim \nu^{-n} \quad \text{(5.1)}$$

and it was found that $n$ increases with $M$, the mass of the outgoing bosonic system $X$. In Fig.11 we plot $n$ in front of $z = \frac{m^2_p - m^2_{min}}{2v}$ for the average value $\bar{\nu} \simeq 32.5$ GeV. We also plot the value of $n$ obtained for the vector-meson electroproduction data from Eqs. (4.3) and (4.4) at $t = 0.25$ GeV\(^2\) and $\bar{\nu} = 5.15$ GeV.
Taking into account the systematic errors of the pure hadronic data (and the even larger errors of the electroproduction data), there is good agreement between the electroproduction data and the pure hadronic data. The similar behaviour for both phenomena, with respect to the scaling variable $z$, may be summarized by saying that the data are consistent with

$$
\frac{d\sigma}{dt \, dq^2} \propto f(t, z) \, F(t, z)
$$

(5.2)

$$
\frac{d\sigma}{dt \, dq^2} \propto g(q^2, m^2) \, F(t, z)
$$

(5.3)

with the same function $F$ for both cases. The scaling variable $z$ appears in a natural way by using the Regge pole model for the amplitudes and assuming approximate factorization (see Fig. 12).

$$
A_{\pi p + Xp} \sim \sum_i \beta_i(t) \left( \cos \theta_t \right)^{2\alpha_i} G_i(t, m^2, M^2)
$$

(5.4)

with $\cos \theta_t \rightarrow \frac{-t^2}{4m^2} \frac{m^2 - m^2}{M^2 - m^2}$

$$
A_{\gamma^* p + pp} \sim \sum_i \beta_i(t) \left( \cos \theta_t \right)^{2\alpha_i} G_i(t, m^2, q^2)
$$

(5.5)

with $\cos \theta_t \rightarrow \frac{-t^2}{4m^2} \frac{m^2 - q^2}{m^2 - q^2}$

where $\beta_i$ is the residuum at the NNR$_i$ vertex and $m$ is the proton mass. The $G_i$ and $G_i(t, m^2, M^2)$ are the residua at the upper vertices and $\theta_t$ is the scattering angle in the cross channel. If

$$
G_i(t, m^2, M^2) \propto G_i(t) \, F(m^2, M^2)
$$

(5.6)

$$
G_i(t, m^2, q^2) \propto G_i(t) \, F(m^2, q^2)
$$

(5.7)

we obtain Eqs. (5.2) and (5.3).

The kind of behaviour illustrated by Eq. (5.6) appears in the triple Regge limit if the sum over the whole set of states with mass $M^2$ is dominated by the Pomeron contribution. On the other hand, Eq. (5.7) would be natural if, as suggested by some authors, the hadronic resonance electromagnetic transition form factors had the same asymptotic behaviour (e.g., $FF \sim \left( \frac{-t}{m^2_R} \right)^2$ with $m^2_R$ being the mass of the resonance).
The presence, in both cases, of the same functions $g_1(t)$ (if true) may be an important clue to dynamics. Needless to say this feature should be further tested experimentally. Therefore, not only do both E-M and pure hadronic processes show the effects which we refer to as decoupling of the Pomeranchukon, but also the functional forms might be similar. A word of warning is needed here. There is no detailed reliable model of the Pomeranchukon, and this object has often surprised us in the past few years, not least by its rising energy behaviour in the ISR pp total cross-section experiments. This is combined with the present ambiguity in defining the diffractive part of multiparticle processes. It is, therefore, just conceivable that even the large $N^2$ part of Eq. (5.5) is not a Regge but a Pomeranchukon effect. We therefore emphasise that by Pomeranchukon we mean those parts of the amplitude which are energy independent (or, at most, slightly rising) over the energy ranges we discuss.

In conclusion, we have found that EWMD is as satisfactory as a framework but no better than CWMD itself. That is to say that, if the $\tilde{\omega}$ is treated as a coherent state and not broken up into $\tilde{\omega}'$, $\tilde{\omega}''$, etc. (and the same goes for $\tilde{\phi}$), then the shadowing is not very sensitive to the off-diagonal elements. However, it is sensitive on the $\Lambda/\pi$ ratio, and there is improved agreement with experiment when this ratio is positive. We noted the apparent decoupling of the Pomeranchukon as a function of $m_{out}^2 - m_{in}^2$ and found evidence of the same or a similar effect in purely hadronic processes. In nuclei this phenomenon also helps to improve the agreement between experiment and theory. The most important need now is to have accurate additional data on the shadowing of real and virtual photons in nuclei. The situation at present is that the theory is ahead of the data and in danger of taking off - perhaps in the wrong direction - unless constrained by experiment. Apart from greater accuracy, it should be noted that for finite $q^2$ and high enough $\nu$, the phenomenological fit degenerates into the simpler Pomeranchukon-dominated parametrisation of Sec. II. In which case the features of shadowing should once more appear clearly.

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The optical model equations (Eqs.(3.4)) give an amplitude for transitions from channel $i$ (momentum $\vec{k}_i$) to channel $j$ (momentum $\vec{k}_j$) (we do not consider polarization here),

$$T_{ji}(\vec{k}_i, \vec{k}_j) = -\frac{1}{4\pi} (\vec{k}_j, j|\hat{U}|\vec{k}_i, i +) =$$

$$= -\frac{1}{4\pi} \int d\vec{k} \exp[-i\vec{k} \cdot \vec{x}] \sum_{\ell} U_{j\ell}(\vec{x}) \psi_{\ell i}(\vec{x}). \quad (A.1)$$

The eikonalization corresponds to putting $\vec{k}_i = (k_i, \vec{0})$ with $k_i = \sqrt{\nu - m_i^2}$ and $\vec{k}_j = (k_j, \vec{0})$ with $k_j = \sqrt{\nu - m_j^2}$. Writing the wave function in the form

$$\psi_{\ell i}(\vec{x}) = \exp[i\vec{k}_i \cdot \vec{x}] S_{\ell i}(\vec{x}), \quad (A.2)$$

we obtain, instead of the left-hand side of Eqs.(3.4),

$$(\nu^2 + k_i^2) \psi_{\ell i}(\vec{x}) = (\vec{\nu} + i\vec{k}_i) (\vec{\nu} - i\vec{k}_i) \exp[i\vec{k}_i \cdot \vec{x}] S_{\ell i}(\vec{x}) =$$

$$= 2i\vec{k}_i \exp[i\vec{k}_i \cdot \vec{x}] \psi_{\ell i}(\vec{x}) + \exp[i\vec{k}_i \cdot \vec{x}] \nabla^2 S_{\ell i}(\vec{x}). \quad (A.3)$$

If we suppose $S_{\ell i}(\vec{x})$ to be slowly varying in front of $\exp[i\vec{k}_i \cdot \vec{x}]$ and take into account that for small angles $\Delta \ll k_j$, we can approximate (A.3) by

$$(\nu^2 + k_i^2) \psi_{\ell i}(\vec{x}) \approx 2i\vec{k}_i \exp[i\vec{k}_i \cdot \vec{x}] \frac{\nabla^2}{\nabla^2} S_{\ell i}(\vec{x})$$

with the $\hat{z}$ axis parallel to $\vec{k}_i$.

Eqs.(3.4) are now

$$2i\vec{k}_j \frac{\partial}{\partial z} S_{ji}(\vec{z}) = \exp[-ik_j \vec{z}] \sum_{\ell} U_{j\ell}(\vec{z}) \psi_{\ell i}(\vec{z}), \quad (A.4)$$

and (A.1) is

$$T_{ji}(\vec{b}) = \frac{ik_j}{2\pi} \int d^2 b \exp[-ib \cdot \vec{b}] \left\{ \delta_{ji} - S_{ji}(\vec{0}, z = \infty) \right\} \quad (A.5)$$

which is the impact parameter representation for the amplitude. $S_{ji}(\vec{b}, \infty)$ is the $S$-matrix element for the impact parameter $\vec{b}$.

Eq.(3.5) is obtained by defining

$$S_{ji}(\vec{b}, z) = \delta_{ji} - \frac{2\pi}{ik_j} \int_{-\infty}^{\infty} dz' \rho(\vec{b}, z') \exp[-iq_{ji} \cdot z'] T_{ji}(\vec{b}, z'). \quad (A.6)$$

By substitution of (A.6) into (A.4) and using the definition of the optical
potential $U_{ji}(x) = -4\pi \phi(x) A_{ji}(t' = 0)$, we obtain for $\Gamma$ the integral equations (3.6). For constant density distribution we perform the Laplace transformation Eq. (3.8), and arrive at Eqs. (3.9),

$$
\Gamma_{\gamma \gamma}(\xi) = \frac{t_{\gamma \gamma}}{\xi} + i \lambda t_{\gamma \gamma} \frac{\Gamma_{\gamma \gamma}(\xi)}{\xi} + i \lambda \sum_{\nu} t_{\nu \gamma} \frac{\Gamma_{\nu \gamma}(\xi)}{\xi - i \alpha_{\nu \gamma}}
$$

$$
\Gamma_{\nu \gamma}(\xi) = \frac{t_{\nu \gamma}}{\xi} + i \lambda t_{\nu \gamma} \frac{\Gamma_{\nu \gamma}(\xi)}{\xi} + i \lambda \sum_{\nu'} t_{\nu' \nu} \frac{\Gamma_{\nu' \nu}(\xi)}{\xi - i \alpha_{\nu' \nu}}.
$$

To first order in $\alpha(e)$ for $\Gamma_{\gamma \gamma}(\nu \gamma)$, we neglect the second terms on the right-hand sides of (A.7) and, considering only the inelastic transitions $\gamma \leftrightarrow (\nu, \overline{\nu})$ and $\nu \leftrightarrow \overline{\nu}$, we obtain:

$$
\Gamma_{\nu \gamma}(\xi) = \left\{ \frac{t_{\nu \gamma}}{\xi} - \frac{R_{\nu \gamma}(\xi)}{D_{\nu \gamma}(\xi)} \right\},
$$

with $R_{\nu \gamma}$ and $D_{\nu \gamma}$ given by Eqs. (3.16).

The elastic photon nucleus amplitude is now:

$$
T_{\nu \gamma}(p) = \frac{k_{\nu}}{v_0} \int d^2b \exp[-iA\beta] \int_{-\infty}^{\infty} dz \frac{1}{2\pi i} \frac{d}{\xi} \exp[i(\xi(z + z_0))] \Gamma_{\gamma \gamma}(\xi) = \frac{k_{\nu}}{v_0} \int d^2b \exp[-iA\beta] \int_{-\infty}^{\infty} dz \sum_{\nu} \exp[i \nu (z + z_0)] \text{Res}_{\nu \gamma}(\xi) = \frac{k_{\nu}}{v_0} \int d^2b \exp[-iA\beta] \left\{ \sum_{\nu} \frac{\exp[2\nu (z + z_0)]}{\xi^{\nu}} \text{Res}_{\nu \gamma}(\xi) + 2z_0 \text{Res}_{\nu \gamma}(0) \right\}.
$$

From (A.8), the poles of $\Gamma_{\nu \gamma}(\xi)$ are located at $\xi = 0$ and at the zeros of $D_{\nu \gamma}(\xi)$ placed at

$$
\xi_{\nu \gamma} = i \lambda \left\{ \frac{n_{\nu} + n_{\gamma}}{2} \right\} \sqrt{\left( \frac{n_{\nu} + n_{\gamma}}{2} \right)^2 + 4t_{\nu \gamma}^2} = -i \lambda x_{\nu \gamma}.
$$

The residua are

$$
\text{Res}_{\nu \gamma}(0) = \frac{t_{\nu \gamma}}{\xi_{\nu \gamma}} - \sum_{\nu} \frac{R_{\nu \gamma}(0)}{D_{\nu \gamma}(0)}
$$

$$
\text{Res}_{\nu \gamma}(\xi_{\nu \gamma}) = -\frac{1}{x_{\nu \gamma}} \frac{R_{\nu \gamma}(\xi_{\nu \gamma})}{x_{\nu \gamma} - x_{\nu \gamma}}, \quad \text{Res}_{\nu \gamma}(\xi_{\nu \gamma}) = \frac{1}{x_{\nu \gamma}} \frac{R_{\nu \gamma}(\xi_{\nu \gamma})}{x_{\nu \gamma} - x_{\nu \gamma}}.
$$

(A.11)
By using the optical theorem we obtain the total cross-section

$$\sigma_{\gamma A}(\nu, q^2) = \frac{4\pi}{k_Y} T_{\gamma\gamma} (A = 0) -$$

$$= 4\pi A \text{ Im } \text{Res} T_{\gamma\gamma}(0) - \frac{4\pi S_0 A^{2/3}}{V_0} \sum_{VV} \text{ Im } \left\{ \text{Res} T_{\gamma\gamma}(\xi_V) + \frac{\text{Res} T_{\gamma\gamma}(\xi_{\nu})}{\xi_V} \right\} +$$

$$+ \frac{4\pi}{V_0} \int d^2 b \sum_{P = V, \nu} \text{ Im } \left\{ \frac{\exp\left[i 2\xi_{P} q_{0}\right]}{\xi_P} \text{Res} T_{\gamma\gamma}(\xi_P) \right\} . \quad (A.12)$$

The last term, usually called modulated term, is negligible for numerical effects. It gives the contribution from the nuclear rim (badly described by our constant density), so that we neglect it. The first and second terms, proportional to the volume and surface of the nucleus, respectively, are called volume and surface terms.

Taking into account the relation between $\xi_V$ and $x_V$ and using (A.11) we obtain:

$$\sigma_{\gamma A}(\nu, q^2) = 4\pi A \text{ Im } \left[ t_{\gamma\gamma} - \sum_{V} \frac{R_{VV}(0)}{D_{VV}(0)} \right] +$$

$$+ 2S_0 A^{2/3} \sum_{V} \text{ Re } \left\{ \frac{R_{VV}(\xi_V)}{x_V^2} - \frac{R_{\nu\nu}(\xi_{\nu})}{x_{\nu}^2} \right\} . \quad (A.13)$$

And the shadow factor will be

$$S(\nu, q^2) = 1 - \frac{\Gamma_{\nu}(\nu, q^2)}{\sigma_{\gamma N}(\nu, q^2)} + 2S_0 A^{-1/3} \frac{\Gamma_{\nu}(\nu, q^2)}{\sigma_{\gamma N}(\nu, q^2)} . \quad (A.14)$$

with $\Gamma_{\nu}$ and $\Gamma_{\gamma}$ given by Eqs.(3.15), (3.16) and (3.17).
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Cross-sections and slopes for $\gamma^* + p \rightarrow p + (p_0 \text{ or } \omega)$.
Fig. 3

\[
\frac{\sigma_t}{R = \sigma_t} = \frac{1}{W^2} \sum_{n=1}^{A-1} \sum_{\gamma=\nu,\bar{\nu}} \langle \gamma q^2 \rangle
\]

\[= \text{Im} \left[ \frac{k \gamma^*}{4 \pi i} \right] \]

(a)

(b)

-24-
Fig. 5

\[ S = \frac{\text{A}_{\text{eff}}(\gamma)}{A} \]

![Graph with energy-loss versus \( \nu \) GeV showing different models (VMD, GVMD, EVMD) and reactions (Pb(A=207), Au(A=197)).]

Fig. 6

![Graph with energy-loss versus \( \nu \) GeV showing different models (VMD, GVMD, EVMD) and reactions (SLAC-MIT).]
Fig. 7

Fig. 8
(q, ω, φ) propagation

- EVMD Q(ω) modified by a cut-off m_A = 1.23 GeV/c^2 D=0
- New Q(ω) m_A = 1.23 GeV/c^2 D=7

--- Fig. 9 ---

--- Fig. 10 ---
Fig. 11

\[ \frac{1}{2} \pi p \rightarrow X p \ \text{Ref. 25} \]

\[ \gamma^* p \rightarrow Q p \ \text{Eqs. (4, 3, 4)} \]

with \[ \overline{\nu} = 5.1 \ \text{GeV} \]


Fig. 12

\[ \cos \theta_t \rightarrow \sqrt{-t} \ \frac{s}{4m^2 - m_{\pi}^2} \]

\[ \cos \theta_t \rightarrow \sqrt{-t} \ \frac{s}{4m^2 - q^2} \]
**FIGURE CAPTIONS**

**Fig.1** Differential cross-section for the production of rhos by real and virtual photons in the forward direction.

**Fig.2** The ratio of longitudinal to transverse $\rho^0$ production.

**Fig.3** The total photon-proton cross-section and the ratio of longitudinal to transverse cross-sections $R$ as a function of $q^2$ (with $U_0 = \frac{W^2 - m^2}{2m}$). The $W$ values for $R$ experimental points are shown in parentheses.

**Fig.4** a) The optical theorem for virtual photon-nucleus Compton scattering.

b) One and multistep contribution to the photon-nucleus amplitude.

**Fig.5** The shadow factor for real photons.

**Fig.6** The shadow factor for virtual photons.

**Fig.7** Energy exponent of $\frac{\Delta \sigma}{\Delta t}$ (Eqs. (4.1) and (4.2)). At the top of each point is given the value of $\bar{U} = \frac{\bar{U}_1 + \bar{U}_2}{2}$. The fit is from Eq. (4.3) with $D = 7$ and $\bar{U} = 5.15$ GeV.

**Fig.8** Our fit to the shadow factor for virtual photons using Eqs. (4.3) and (4.4) with $D = 7$ and $m_\Lambda = 1.23$ GeV/c$^2$.

**Fig.9** $\rho, \omega, \phi$ contribution to the shadow factor.

**Fig.10** The slope parameter versus the scaling variable

$$z = \frac{M^2_{\text{out}} - M^2_{\text{in}}}{2\nu}.$$  

**Fig.11** The energy exponent versus the scaling variable

$$z = \frac{M^2_{\text{out}} - M^2_{\text{in}}}{2\nu}.$$  

The dashed lines show systematic errors quoted in Ref.25.

**Fig.12** Regge pole expansion of $\pi p \rightarrow pX$ and $\gamma^* p \rightarrow p\rho$ scattering amplitudes.
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