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WILSON OPERATOR PRODUCTS IN NON-LINEARLY REALIZED THEORIES

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INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

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WILSON OPERATOR PRODUCTS IN NON-LINEARLY REALIZED THEORIES *

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ABSTRACT

A conjectured Wilson operator-product formula is presented for a class of non-polynomial Lagrangian theories. The formula would give the behaviour of the product at short distances near the light cone.

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It is the purpose of this note to present a conjectured Wilson operator-product ansatz for localizable non-linearly realized field theories which contain a dimensional parameter (the minor coupling constant). We have in mind theories of the following variety.

I. Self-interacting non-linear theories, of which an example is provided by the chiral SU(2) × SU(2) Lagrangian which describes massless pions:

$$\mathcal{L}_s = \frac{1}{16\kappa^2} \, \mathrm{Tr} \, \partial_\mu S^{-1} \, \partial_\mu S \quad , \tag{1}$$

where

$$S = \exp(\kappa \, \tau\cdot\phi \, \gamma_5) \quad . \tag{2}$$

The chief characteristic of these theories from the present point of view is that S is an entire function of $(\kappa\tau\cdot\phi)$ and the Lagrangian reduces to that for free particles in the limit $\kappa \to 0$.

II. Additionally, matter Lagrangians of the type [1]:

$$\mathcal{L}_{matter} = g \, \bar{\psi} \, \gamma_\mu \, S \, \psi \, A_\mu + \text{free field Lagrangians for } A_\mu \text{ and } \psi \, . \tag{3}$$

The characteristics of $\mathcal{L}_{matter}$ are:

i) All interaction terms contain an entire function S of $(\kappa\tau\cdot\phi)$.

ii) In the limit $\kappa \to 0$, $\mathcal{L}_{matter}$ reduces to a renormalizable Lagrangian, where the (major) coupling constant g is dimensionless.

iii) Besides κ , the theory contains no other dimensional parameter.

For Heisenberg operators in this class of theories, we conjecture that a Wilson product expansion exists, of the form:

$$A(x) \, B(0) = \sum_n F_n\left(\frac{\kappa^2}{x^2}\right) \frac{1}{(x^2)^{\frac{1}{2}(d_A + d_B - d_n)}} \, C_n(x) \quad , \tag{4}$$

where the parameters d_A, d_B, etc., are numbers to be computed, $C_n(x)$ is operator valued, and $F_n(\kappa^2/x^2)$ are entire functions of κ^2/x^2 with the typical form $\exp\left[\alpha_n(\kappa^2/x^2)\right]$. The conjecture is based on perturbation computations using non-polynomial techniques, which stem from the relations:

$$\left(T \; \exp[\kappa\phi(x)] \; \exp[\kappa\phi(0)]\right) = \exp\left[-\frac{\kappa^2}{4\pi^2}\frac{1}{x^2}\right] \tag{5}$$

and

$$\left(T \; \exp[\kappa\phi(x)] \; \exp[-\kappa\phi(0)]\right) = \exp\left[\frac{\kappa^2}{4\pi^2}\;\frac{1}{x^2}\right] \quad . \tag{6}$$

The important remark is that, since there is no dimensional parameter in the theory except κ^2 , dimensional considerations force the appearance of the combination (κ^2/x^2) . Furthermore, $F_n(\kappa^2/x^2)$ must be an entire function of κ^2/x^2 from the twin requirements of localizability [2] of the theory and the existence of a smooth limit when $\kappa \to 0$.

Relations (4), (5) and (6) imply a specific form of short-distance behaviour, which is controlled for $|x^2| \leq \kappa^2/4\pi^2$ by the entire functions $F_n(\kappa^2/x^2)$. In particular (5) (or (6)) imply zeros of infinite order for $x^2 \to 0$ whenever the light cone is approached from time-like (or space-like) distances, respectively. For a space-like (or time-like) approach to $x^2 \to 0$, one must make an analytic continuation.

It is at this stage of analytic continuation that the expressions (4), (5) and (6) develop ambiguities. For localizable[3] theories where the non-linear function $S(\kappa\phi)$ in (2) is entire, Lehmann and Pohlmeyer [4] have shown, using momentum space methods of Volkov, that (at least to the third order in an expansion in the major coupling parameter g) there exists a unique minimally singular continuation. It is here, however, that one hopes for a more fundamental and axiomatic approach to be developed, which may have the ansatz (4) in x-space as its starting point.

As is well known from the work of Zimmerman [5], the insistence on x-space methods in Wilson expansions confers greater power in field theoretic method, particularly in respect of problems of normal ordering.

The value of the theories we have considered (and this includes interaction of matter and gravity) cannot be overstressed. These offer the only known local and unitary theories [4] without indefinite metrics which possess an inbuilt ultra-violet cut-off $\approx 1/\kappa$.

REFERENCES AND FOOTNOTES

1) $\mathcal{L}_{matter}$ is not chiral invariant. For the purposes of this note, chirality if of no primary significance; the chiral invariant Lagrangian $\mathcal{L}_S$ simply provides an example in a localizable formulation of non-linear theories we wish to discuss. (See H. Lehmann and H. Trute, preprint DESY/8 (1972).)

2) M.K. Volkov, Ann. Phys. (N.Y.) 49, 202 (1968), who elucidates the connection between localizability and the entire function character of the non-polynomiality in the Lagrangian.

3) It is desirable to concentrate on localizable theories, not only because of the appearance of entire functions but also because of a theorem of Epstein, Glaser and Martin (Commun. Math. Phys. 13, 257 (1969)) about localizable theories in general, which we feel deserves to be better known. The theorem states that on-shell scattering amplitudes in all localizable theories are Froissart bounded. Their axiomatic proof, unfortunately, does not give a calculational procedure to guarantee the attaining of such a bound.

4) H. Lehmann and K. Pohlmeyer, in Nonpolynomial Lagrangians, Renormalization and Gravity, 1971 Coral Gables Conference, Vol.1, p.60;
J.G. Taylor, ibid, p.42;
K. Pohlmeyer, Commun. Math. Phys. 20, 101 (1971).

5) W. Zimmerman, in Lectures on Elementary Particles and Quantum Field Theory, Vol.I, eds. S. Deser, M. Grisaru and H. Pendleton, (Proc. Conf. Brandeis, 1970) MIT Press, Cambridge, Mass., 1970.