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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

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THE INFLUENCE OF f GRAVITY ON GRAVITATIONAL COLLAPSE *

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ABSTRACT

In Trautman's model of a universe with 10^{80} aligned neutrons, collapse is averted if the effects of Cartan's torsion are taken into account. We investigate the effect of taking f gravity into account and show that spin-aligned hadronic matter will not collapse to densities higher than 10^{17} gms/cm^{-3}. Perfect spin alignment necessary for this result remains a problem here as in Trautmen's original work.

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I. INTRODUCTION

In a recent note Trautman [1] (following the work of W. Kopczynki) has considered the influence of spin on the affine geometry of space time, this influence being taken into account through Cartan's [2] generalization of Einstein's gravitational theory. In this approach the action integral is written in first-order (Palatini) form and the metric and connection fields are varied independently. In the absence of matter the equations of motion for the connection reduce into the usual equations for the definition of the Christoffel symbol in terms of the metric. However, if matter is included then the resulting equations will not necessarily lead to a Riemannian connection but imply the presence of an additional torsion term.
If matter is made up of spin-$\frac{1}{2}$ particles, the torsional effect manifests itself by the appearance in the Lagrangian of an effective spin-spin contact interaction term proportional in magnitude to the Newtonian constant.

Trautman [1] considered the effect of this term on the collapse of a pressure-free gas of spin-$\frac{1}{2}$ hadrons in a Friedmann metric and showed that the usual space-time singularity does not occur but, instead, a universe of 10^{80} neutrons reaches a minimum radius of 1 cm, provided that some mechanism can be found for aligning the nuclear spins.

The essential point is that this potentially important effect is a direct result of the assumed first-order form of the Einstein-Cartan Lagrangian of (spinning) matter and gravity. On the other hand, the authors [3] (and, independently, Wess and Zumino [4]) suggested some time ago that the natural vehicle for taking hadronic short-range forces into consideration in gravitational physics was through a two tensor (f-g) theory of gravity, where the physical (infinite range and hence massless) graviton field must be considered as a function of Einstein's g field and the strongly interacting 2^+ massive singlet f-meson field. We wish to investigate in this note the effect on Trautman's work of the interposition of f gravity, when Cartan's formulation is taken into account for both f and g fields. We shall show, in particular, that in our formulation f gravity has a profound effect and the minimum radius of the Trautman universe appears to be $\simeq 10^{13}$ cms rather than 1 cm. Stated differently, our main result is that the maximum matter density attained in Trautman's universe of 10^{80} neutrons with their spins aligned is $\simeq 10^{17}$ gms cm^{-3} rather than 10^{55} gms cm^{-3}. Thus hadronic matter may collapse to densities a few orders of magnitude higher than known nuclear densities and no further, provided of course one can find, with Trautman, a suitable mechanism for aligning nuclear spins.

II. THE TRAUTMAN METHOD

It is worth indicating how Trautman's result can be derived from the view point of Lagrangian field theory. Consider the interaction of a Dirac spinor field $\psi(x)$ with the vierbein gravitational field $L^{\mu a}(x)$ and spinor connection $B_{\mu ab}(x)$. The usual Einstein Lagrangian can be succinctly written, using Dirac matrices, as [5] (apart from a four divergence)

$$\mathcal{L} = \frac{(\det L)^{-1}}{8\, K_g^2} \, \mathrm{Tr}\left\{ [L^\mu, L^\nu]\, B_{\mu\nu} \right\} , \tag{2.1}$$

where

$$L^\mu \equiv L^{\mu a}\, \gamma_a \tag{2.2}$$

$$B_{\mu\nu} \equiv \partial_\mu B_\nu - \partial_\nu B_\mu + i\, [B_\mu , B_\nu] \tag{2.3}$$

$$B_\mu \equiv \frac{1}{4} B_{\mu ab}\, \sigma^{ab} \tag{2.4}$$

and where K_g^2 is the Newtonian constant. The trace is over the Dirac algebra. This Lagrangian is indeed a scalar density under the general co-ordinate group, since the matrix fields L^μ and B_μ are both genuine vectors and $B_{\mu\nu}$ is defined in Eq.(2.3) as the "covariant curl" of B_μ. What is perhaps equally important for our present investigation is that this Lagrangian is also manifestly invariant under the gauge group of local Lorentz transformations which rigidly rotate the tetrad of vectors $L^{\mu a}(x)$, $a = 0,1,2,3$, independently at every space-time point. Under this group the matrix fields transform as

$$L^\mu \to \Omega\, L^\mu\, \Omega^{-1} \tag{2.5}$$

$$B_\mu \to \Omega\, B_\mu\, \Omega^{-1} - \frac{1}{i}\, \Omega\, \partial_\mu\, \Omega^{-1} \tag{2.6}$$

$$B_{\mu\nu} \to \Omega\, B_{\mu\nu}\, \Omega^{-1} ,$$

where $\Omega(x) \in SO(3,1)$. From (2.5) and (2.6) the invariance of the Lagrangian in Eq.(2.1) follows at once.

The minimal Dirac Lagrangian which exhibits invariance under both these groups is

$$\mathcal{L}_{\text{matter}} = (\det L)^{-1} \left\{ \frac{i}{2} \bar{\psi} L^{\mu} \overleftrightarrow{\nabla}_{\mu} \psi - m\bar{\psi}\psi \right\} \quad . \tag{2.7}$$

It is the explicit appearance of B_μ in the covariant derivative in Eq.(2.7) that gives rise to a torsional term in the affine/spin connection and hence finally to an effective "spin-spin" interaction term

$$\mathcal{L}_{\text{eff}} = \frac{2}{32} K_g^2 \ (i\,\bar{\psi}\,\gamma_a\,\gamma_5\,\psi)\ (i\,\bar{\psi}\,\gamma^a\,\gamma_5\,\psi)\ (\det L')^{-1} \quad . \tag{2.8}$$

The appearance of this term has been emphasised by various authors including Weyl [6], Sciama [7] and Kibble [8]. The origin of the phrase "spin-spin" can be seen by looking at Eq.(2.8) in the non-relativistic limit when only the space parts of the axial currents contribute, in which case it becomes

$$\mathcal{L}_{\text{eff}} = -\frac{3}{32} K_g^2 \ (\bar{\chi}\,\underline{\sigma}\,\chi)\cdot(\bar{\chi}\,\underline{\sigma}\,\chi)\ (\det L)^{-1} \tag{2.9}$$

in terms of Pauli spinors.

Trautman essentially starts with such a spin-spin interaction in a classical system of a collapsing pressure-free dust of spin-half nucleons with the usual stress tensor

$$T_{\mu\nu} = \rho\, u_\mu\, u_\nu \quad . \tag{2.10}$$

He investigates the system in the Friedmann-Robertson-Walker metric,

$$(ds)^2 = (dt)^2 - (R(t))^2\ g_{ij}(\underline{x})\ dx^i\, dx^j \ , \tag{2.11}$$

and shows that the usual Friedmann equation is modified to

$$\frac{1}{2}\ddot{R} - \frac{GM}{R} + \frac{3G^2S^2}{2C^4R^4} = 0 \ , \tag{2.12}$$

where G is the Newtonian constant $(G = \frac{K^2}{4\pi})$, M is the total mass of the cloud of nucleons and S is the total spin assuming that all the individual spins are lined up in some way. This equation solves to give Trautman's result of a minimum value of $R(t)$ as 1 cm. from the formula:

$$R \approx N^{1/3} \left(\frac{G\hbar}{c^3}\right)^{1/3} \left(\frac{\hbar}{m_N c}\right)^{1/3} \quad . \tag{2.13}$$

One way of understanding this result is that the Hawking-Penrose [9] singularity theorems for the Einstein system of equations,

$$G_{\mu\nu} = -K_g^2 \ (T_{\mu\nu} + \Lambda\ g_{\mu\nu}) \ ,$$

which are derived from the Lagrangian

$$\mathcal{L} = \sqrt{-\det g} \left[\frac{R(g)}{2K_g^2} + \mathcal{L}_{matter} - \Lambda \right] \tag{2.14}$$

do not necessarily hold if the cosmological constant Λ is positive. However, the effective contact term Lagrangian in Eq.(2.9) can be regarded as being of the form

$$\mathcal{L}_{eff} = -\sqrt{-\det g}\ \Lambda_{eff}(x) \ , \tag{2.15}$$

where $\Lambda_{eff}(x)$ represents a field dependent, but, for spins aligned in the same direction, a positive effective "cosmological" contribution to the Lagrangian. We are thus permitted a possible bypass of the singularity theorems which would otherwise make collapse inevitable.

III. THE EFFECT OF A TWO-TENSOR f-g THEORY OF GRAVITY

The two-tensor theory of gravity starts by postulating two independent Lagrangians of the type given by the sum of Eqs.(2.1) and (2.7) - one for the g field interacting with leptons and one for the f field interacting with hadrons - with respective coupling constants K_g and K_f. To this system one then adds a generally covariant mixing term, involving the f and g fields only, which serves to couple gravity into the hadronic world, through the intermediacy of the f meson, in a manner which does no violence to the equivalence principle as applied to macroscopic hadronic matter. Although there is no absolute uniqueness about such a term, the effect to order K_g^2/K_f^2 is to define the diagonalized physical particle fields $\tilde{f}^{\mu\nu}$ and $\tilde{g}^{\mu\nu}$ as

$$\tilde{f}^{\mu\nu} = f^{\mu\nu} - g^{\mu\nu} \tag{3.1}$$

$$\tilde{g}^{\mu\nu} = \left(1 + \frac{K_g^2}{K_f^2}\right)^{-1} \left(g^{\mu\nu} + \frac{K_g^2}{K_f^2} f^{\mu\nu}\right) , \qquad (3.2)$$

in which $\tilde{f}^{\mu\nu}$ and $\tilde{g}^{\mu\nu}$ are, respectively, the massive and massless combinations.

We are now ready to investigate the effect of this mixing of fields on the collapse problem of the cloud of nucleons. It is clear that the relevant effective Lagrangian (cf. Eq.(2.8)) is

$$\mathcal{L}_{eff} = \sqrt{-f}\ \frac{3K_f^2}{32}\ (i\,\overline{\psi}\,\gamma_a\,\gamma_5\,\psi)\ (i\,\overline{\psi}\,\gamma^a\,\gamma_5\,\psi) , \qquad (3.3)$$

in which (since this term comes from the hadronic part of the total Lagrangian) the coupling constant is the hadronic K_f^2 (rather than K_g^2) and the multiplying scalar density is $\sqrt{-\det f}$.

Now by inserting the relation (Eq.(3.1)) between $f^{\mu\nu}$ and the physical lowest-order field $\tilde{f}^{\mu\nu}$ and $\tilde{g}^{\mu\nu}$ into Eq.(3.3) we see that the effective coupling to the "true" gravitational field $\tilde{g}^{\mu\nu}$ is

$$\mathcal{L}_{eff} = \sqrt{-\tilde{g}}\ \frac{3K_f^2}{32}\ (i\,\overline{\psi}\,\gamma_a\,\gamma_5\,\psi)\ (i\,\overline{\psi}\,\gamma^a\,\gamma_5\,\psi) \qquad (3.4)$$

to the lowest order in the expansion of $\det(\tilde{f} + \tilde{g})$. This term - the "longest range" term in the expansion in powers of $1/R$ - is the same as Trautman's except that the coupling constant is K_f^2 rather than K_g^2 . Substituting in (2.13) for R , we obtain an extra factor $(K_f^2/K_g^2)^{1/3}$. This is the justification for the claim that the ultimate size of the Trautman universe in our theory is 10^{13} cms - roughly the size of the solar system - rather than 1 cm, and that the ultimate density of hadronic matter in the universe is no higher than one or two orders of magnitude higher than density of nuclear matter.

There is an interesting extension possible of the above result. We have recently constructed [10] a theory which is similar in form to Einstein's except that it is invariant under an SL(6,C) rather than an SL(2,C) gauge group. Some of these extra generators correspond to the internal symmetry group SU(3) and, therefore, this new Lagrangian is a natural candidate for describing the hadronic side of an f-g theory in which a nonet of f-meson fields appear rather than just an SU(3) singlet. This of course is much more desirable from a physical point of view, as the massive 2^+ particles which occur in nature tend to appear in nonets. The main difference which this new theory would make to the above results is that the effective interaction in Eq.(3.4) would read:

$$\mathcal{L}_{eff} = \sqrt{-g}\,\frac{3K_f^2}{32}\,(i\,\bar{\psi}\,\lambda_j\,\gamma_a\,\gamma_5\,\psi)\,(i\,\bar{\psi}\,\lambda^j\,\gamma^a\,\gamma_5\,\psi)\,, \qquad (3.5)$$

in which ψ is a SL(6,C) quark field and the λ matrices are summed from $j = 0,\ldots,8$. Clearly in such a theory one would need to discuss mechanisms for aligning both internal (unitary) and spatial spins.

It should perhaps be emphasised that in all of these Lagrangian quantum field theory schemes the arguments concerning the occurrence or otherwise of collapse are really heuristic being based on the somewhat imprecise notion of the static limit of the Lagrangians. Essentially one is hoping to be able to interpret the quantum version of the Einstein equations of motion in the form

$$G_{\mu\nu}(g) = -K^2 \langle\varphi|T^{eff}_{\mu\nu}|\varphi\rangle \ , \qquad (3.6)$$

in which $|\varphi\rangle$ is the relevant state of the system and $T^{eff}_{\mu\nu}$ is the effective source of the gravitational field which is obtained after "integrating out" all those other fields which do not directly contribute to the specification of the particles in the state $|\varphi\rangle$. Thus there are three levels at which the Trautman treatment must be critically examined.

1) Trautman and, following him, we have treated matter as if it consisted of freely moving hadrons. We have neglected the effect of Cartan's torsional term on the motion of matter itself - a neglect which is probably justified for $R \approx 10^{13}$ cm at densities which are essentially those of nuclear matter, but may be questionable for smaller R .

2) There is the difficult problem of alignment of spins and the difficulties associated with the exclusion principle, which must be examined further. Clearly, ultimate matter density will increase from the value given above if spin alignment is not perfect.

3) And finally, one must recognize that, in the present theory, collapse has been averted essentially at a classical level without the necessity of invoking "true" quantum-mechanical effects. This may be an oversimplification.

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