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# VECTOR AND TENSOR GAUGE PARTICLES IN SL(6,c) THEORY

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## ABSTRACT

To the SL(6,c) gauge-invariant Lagrangian presented in an earlier paper, an additional SL(6,c) invariant term is added, which allows for the propagation of $1^-$ and $1^+$ massive nonets in addition to the nonet of $2^+$ particles.

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The Lagrangian recently proposed by us for the description of systems with local $SL(6,c)$ symmetry was so constructed that the gauge system reduced simply to a massive nonet of $2^+$ particles (at least in the free field approximation). Our purpose now is to suggest a modification of this Lagrangian which will allow massive octets of $1^+$ and $1^-$ particles as well. This can be achieved by a subtle rearrangement of terms without involving any new field variable.

Our gauge system comprises three distinct types of field, $B_\mu$, $S$ and $L_\mu$, which transform under local $SL(6,c)$ according to

$$\begin{aligned} B_\mu &\rightarrow \Omega B_\mu \Omega^{-1} + \frac{1}{i}\Omega\partial_\mu\Omega^{-1} \\ S &\rightarrow \Omega S \\ L_\mu &\rightarrow \Omega L_\mu \Omega^{-1} \end{aligned} \qquad . \qquad (1)$$

Their tensorial components can be exhibited in the Dirac basis as follows:

$$\begin{aligned} B_\mu &= \left(B^k_\mu + \frac{1}{2} B_{\mu[ab]}\,\sigma_{ab} + B^k_{\mu 5}\,\gamma_5\right)\frac{\lambda^k}{2} \\ S &= \exp i\left(P^k + \frac{1}{2} P^k_{[ab]}\,\sigma_{ab} + P^k_5\,\gamma_5\right)\frac{\lambda^k}{2} \\ L_\mu &= \left(L^k_{\mu a}\,\gamma_a + L^k_{\mu a 5}\, i\gamma_a\gamma_5\right)\lambda^k \end{aligned} \qquad . \qquad (2)$$

All tensorial components are real in this basis. Notice that $S$ is thereby constrained to be an $SL(6,c)$ matrix. Its components $P$ belong to a non-linear realization of the local symmetry.

We do not wish to regard all the fields which appear in expressions (2) as independent variables. For the sake of convenience, we shall impose a number of constraints among them, by means of which some of the components are to be eliminated. These constraints must, of course, be compatible with the local symmetry. Now, according to (1), the combinations

$$\begin{aligned} b_\mu &= S^{-1} B_\mu S + \frac{1}{i} S^{-1}\partial_\mu S \\ \ell_\mu &= S^{-1} L_\mu S \end{aligned} \qquad (3)$$

are local <u>invariants</u>. We are therefore entitled to impose the constraints

$$\ell^k_{\mu a5} = 0$$

$$\ell^k_{\mu a} - \ell^k_{a\mu} = 0 \quad , \tag{4}$$

which serve to eliminate 144 + 54 = 198 components. (It should be remarked that these constraints are covariant with respect to the global Poincaré and SU(3) symmetries, under which $\ell^k_{\mu a}$ and $\ell^k_{\mu a5}$ transform as tensor and pseudo-tensor nonets, respectively.) Which components are removed by means of (4) is to some extent a matter of choice. For example, the components $L^k_{\mu a5}$ and $L^k_{\mu a} - L^k_{a\mu}$ could be expressed in terms of P and the remaining L's . Alternatively, P and certain of the L's could be expressed in terms of the remaining L's . In this case, the elimination of P would leave the Lagrangian as a (highly non-linear) function of $L_\mu$ and $B_\mu$ where the components of $L_\mu$ are subject to 198 - 70 = 128 non-linear but covariant constraints.

In order for the above considerations to have any sense, it is important that the action should be minimized by a Poincaré and SU(3) invariant vacuum solution, in which both $\langle L_\mu \rangle$ and $\langle S \rangle$ are non-vanishing. We shall in fact require that the values

$$\langle B_\mu \rangle = 0 \ , \ \langle S \rangle = 1 \ , \ \langle L_\mu \rangle = \gamma_\mu \ , \tag{5}$$

be a solution of the classical equations of motion.

Comparing (5) with (3) we find

$$\langle \ell^k_{\mu a} \rangle = \eta_{\mu a}\, \delta^{ka} \quad . \tag{6}$$

This suggests that the 4 x 4 matrix field $\ell^0_{\mu a}$ is invertible. We therefore define the "vierbein" fields,

$$f_\mu = f_{\mu a}\, \gamma_a \quad \text{and} \quad f^{-1}_\mu = f^{-1}_{\mu a}\, \gamma_a \ , \tag{7a}$$

with

$$f_{\mu a} = \ell^0_{\mu a} = \ell^0_{a\mu} \quad . \tag{7b}$$

This definition is by no means unique. Equally acceptable would be the symmetric square root of the expression $\sum_0^8 \ell^k_{\mu a}\, \ell^k_{\nu a}$, which includes octet contributions. *) The basic requirements are that $f_{\mu a}$ should transform as a Poincaré tensor and global SU(3) singlet and be invertible, i.e.

$$f_{\mu a}\, f^{-1}_{\nu a} = \eta_{\mu\nu} \quad , \quad f_{\mu a}\, f^{-1}_{\mu b} = \eta_{ab} \; . \tag{8}$$

Both $f_\mu$ and $f^\mu$ are local invariants. They can be transformed, with the help of S, into local tensors, viz.,

$$F_\mu = S\, f_\mu\, S^{-1} \quad \text{and} \quad F^{-1}_\mu = S\, f^{-1}_\mu\, S^{-1} \; . \tag{9}$$

These fields, which we use to denote rather intricate non-linear combinations of the basic fields $L_\mu$ and P, will prove very useful in the construction of the Lagrangian. With the covariant quantities defined by

$$\begin{aligned}
\nabla_\mu L_\nu &= \partial_\mu L_\nu + i\, [B_\mu, L_\nu] \\
C_\mu &= \tfrac{i}{4} F_\lambda\, S\, \nabla_\mu\, S^{-1}\, F^{-1}_\lambda \left(= \tfrac{1}{4} F_\lambda \left(B_\mu - \tfrac{1}{i} S\, \partial_\mu\, S^{-1}\right) F^{-1}_\lambda\right) \\
C_{\mu\nu} &= \nabla_\mu\, C_\nu - \nabla_\nu\, C_\mu \; ,
\end{aligned} \tag{10}$$

we write the proposed Lagrangian in the form

$$\begin{aligned}
\mathcal{L} = \frac{1}{8}\, \mathrm{Tr} \Bigg[ &- \frac{1}{\kappa^2} \left(\nabla_\mu L_\nu \nabla_\nu L_\mu - \nabla_\mu L_\mu \nabla_\nu L_\nu\right) \\
&- \frac{1}{16\kappa'^2}\, F_\lambda\, C_{\mu\nu}\, F^{-1}_\lambda\, C_{\mu\nu} \\
&+ \frac{M'^2}{8\kappa'^2}\, F_\mu\, C_\nu\, F_\mu\, C_\nu \\
&- \frac{3M^2}{2\kappa^2}\, L_\mu\, L_\mu + \frac{M^2}{8\kappa^2}\, L_\mu\, [L_\mu, L_\nu]\, L_\nu \Bigg] \; ,
\end{aligned} \tag{11}$$

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*) Such definitions will probably turn out to be equivalent in the long run, when the possibility of making non-linear field transformations in the Lagrangian is taken into account.

where the constraints (4) and (7b) among $L_\mu$, $C_\mu$ and $F_\mu$ are tacitly understood. The parameters $\kappa$ and $\kappa'$ are coupling constants, while $M$ and $M'$ are masses.

Since the Lagrangian (11) is invariant with respect to local SL(6,c) transformations, we can impose 70 gauge conditions on the solutions without losing any information. In contrast to the covariant equations of constraint discussed above, the gauge conditions must, as usual, violate the SL(6,c) symmetry in order to fix a gauge. For example, one might require solutions to satisfy $\partial_\mu L_\mu = 0$. However, by far the most convenient conditions are simply

$$S = 1 \quad \text{or} \quad P = 0 \quad . \tag{12}$$

In this gauge we have $L_\mu = \ell_\mu$ and $F_\mu = f_\mu$, or

$$L_\mu = \gamma_\mu + \varphi^k_{(\mu a)}\, \gamma_a\, \lambda^k$$

$$F_\mu = f_{\mu a}\, \gamma_a$$

and

$$C_\mu = \frac{1}{4} F_\lambda\, B_\mu\, F^{-1}_\lambda \quad . \tag{13a}$$

The great advantages which follow from the introduction of $F_\mu$ and $F^{-1}_\mu$ into the Lagrangian can now be made apparent. Because of the identities (8) it follows that

$$C_\nu = \frac{1}{4} F_\mu\, B_\nu\, F^{-1}_\mu = \frac{1}{4} f_{\mu a}\, f^{-1}_{\mu b}\, \gamma_a \left(B^k_\nu + \frac{1}{2} B^k_{\nu[cd]}\, \sigma_{cd} + B^k_{\nu 5}\, \gamma_5\right) \frac{\lambda^k}{2}\, \gamma_b$$

$$= \left(B^k_\nu - B^k_{\nu 5}\, \gamma_5\right) \frac{\lambda^k}{2} ,$$

i.e. the tensor part of $B_\nu$ is extinguished. We have used here the basic identity

$$\gamma_a\, \sigma_{cd}\, \gamma_a = 0 \quad . \tag{13b}$$

The C-containing terms in (11) therefore reduce to the form

$$\frac{1}{\kappa^2} \left(- \frac{1}{4} B^k_{\mu\nu}\, B^k_{\mu\nu} + \frac{1}{2} M'^2\, B^k_\mu\, B^k_\mu - \frac{1}{4} B^k_{\mu\nu 5}\, B^k_{\mu\nu 5} + \frac{1}{2} M'^2\, B^k_{\mu 5}\, B^k_{\mu 5}\right) , \tag{14}$$

where

$$B^k_{\mu\nu} = \partial_\mu B^k_\nu - \partial_\nu B^k_\mu - 2\, f^{k\ell m}\left[B^\ell_\mu B^m_\nu + B^\ell_{\mu 5} B^m_{\nu 5}\right]$$

$$B^k_{\mu\nu 5} = \partial_\mu B^k_{\nu 5} - \partial_\nu B^k_{\mu 5} \quad . \tag{15}$$

Note that $B^k_{\mu\nu} = B^{\text{Yang-Mills}}_{\mu\nu} - 2f^{k\ell m} B^\ell_{\mu 5} B^m_{\nu 5}$, where $B^{\text{Yang-Mills}}_{\mu\nu}$ is the conventional Yang-Mills covariant field strength. Thus, in the Lagrangian we have constructed, the $(1^-)$ fields $B^k_\mu$ indeed act like Yang-Mills fields, while the $(1^+)$ fields $B^{k5}_\mu$ do not. The terms in (11) which do not contain C are the same as in Ref.1. They cause the propagation of a massive $2^+$ nonet.

Thus, our Lagrangian (11) describes the propagation of a $2^+$ nonet, a $1^-$ Yang-Mills octet together with a $1^+$ octet. The coupling strengths associated with the $(1^-)$ and $(1^+)$ fields, though equal among themselves, need not equal the coupling strength for the $2^+$ fields.

To conclude, the gauge interactions of quark matter are specified by the Lagrangian

$$\begin{aligned}
\mathcal{L}_{\text{matter}} &= \frac{i}{2}\left[\bar\psi\, L_\mu \nabla_\mu \psi - \nabla_\mu \bar\psi L_\mu \psi\right] - m\bar\psi\psi \\
&= \frac{i}{2}\left[\bar\psi L_\mu \partial_\mu \psi - \partial_\mu \bar\psi L_\mu \psi\right] - \frac{1}{2}\bar\psi\{L_\mu, B_\mu\}\psi - m\bar\psi\psi \\
&= L^k_{\mu a}\frac{i}{2}\bar\psi\, \gamma_a\, \lambda^k \overleftrightarrow{\partial}_\mu\, \psi - m\bar\psi\psi\; + \\
&+ L^k_{\mu a 5}\frac{i}{2}\bar\psi\, i\gamma_a\gamma_5\, \lambda^k \overleftrightarrow{\partial}_\mu\, \psi \\
&- \Big[d^{k\ell m}\left(L^k_{\mu d} B^\ell_\mu + \epsilon_{abcd}\, L^k_{\mu a 5} B^\ell_{\mu[bc]}\right) + \\
&\qquad + f^{k\ell m}\left(-L^k_{\mu a} B^\ell_{\mu[ad]} + L^k_{\mu d 5} B^\ell_{\mu 5}\right)\Big]\bar\psi\, \gamma_d \frac{\lambda^m}{2}\psi \\
&- \Big[d^{k\ell m}\left(L^k_{\mu d 5} B^\ell_\mu - \epsilon_{abcd}\, L^k_{\mu a} B^\ell_{\mu[bc]}\right) + \\
&\qquad + f^{k\ell m}\left(-L^k_{\mu a 5} B^\ell_{\mu[ad]} + L^k_{\mu d} B^\ell_{\mu 5}\right)\Big]\bar\psi\, i\gamma_d\gamma_5 \frac{\lambda^m}{2}\psi \quad .
\end{aligned}$$

Notice that half of these terms disappear in the gauge $S = 1$ where $L^k_{\mu a 5}$ vanishes. In linear approximation one is left with

$$\mathcal{L}_{\text{matter}} = \bar{\psi}\left(\frac{i}{2}\gamma_\mu \overleftrightarrow{\partial}_\mu - m\right)\psi + \varphi^k_{(\mu a)}\frac{i}{2}\bar{\psi}\gamma_a\lambda^k\overleftrightarrow{\partial}_\mu\psi$$

$$- B^k_\mu \bar{\psi}\gamma_\mu \frac{\lambda^k}{2}\psi + \epsilon_{\mu bcd} B^k_{\mu[bc]} \bar{\psi} i\gamma_d\gamma_5 \frac{\lambda^k}{2}\psi \quad .$$

As expected, vector field $B^k_\mu$ couples in this approximation like a true Yang-Mills field, while the axial vector $B^k_{\mu 5}$ does not. In fact, the axial vector field has no coupling to matter in this approximation. If we wish to construct a Yang-Mills theory for both $1^-$ and $1^+$ particles, we shall need to extend the considerations of this paper to the larger symmetry group SL(6,c) x SL(6,c) in the manner of Sec.IV of Ref.1a.

REFERENCE


1) a) C.J. Isham, Abdus Salam and J. Strathdee, ICTP, Trieste, preprint IC/72/155;

b) C.J. Isham, Abdus Salam and J. Strathdee, Lettere al Nuovo Cimento 5, 969 (1972).