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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

PROGRESS IN RENORMALIZATION THEORY SINCE 1949

Abdus Salam

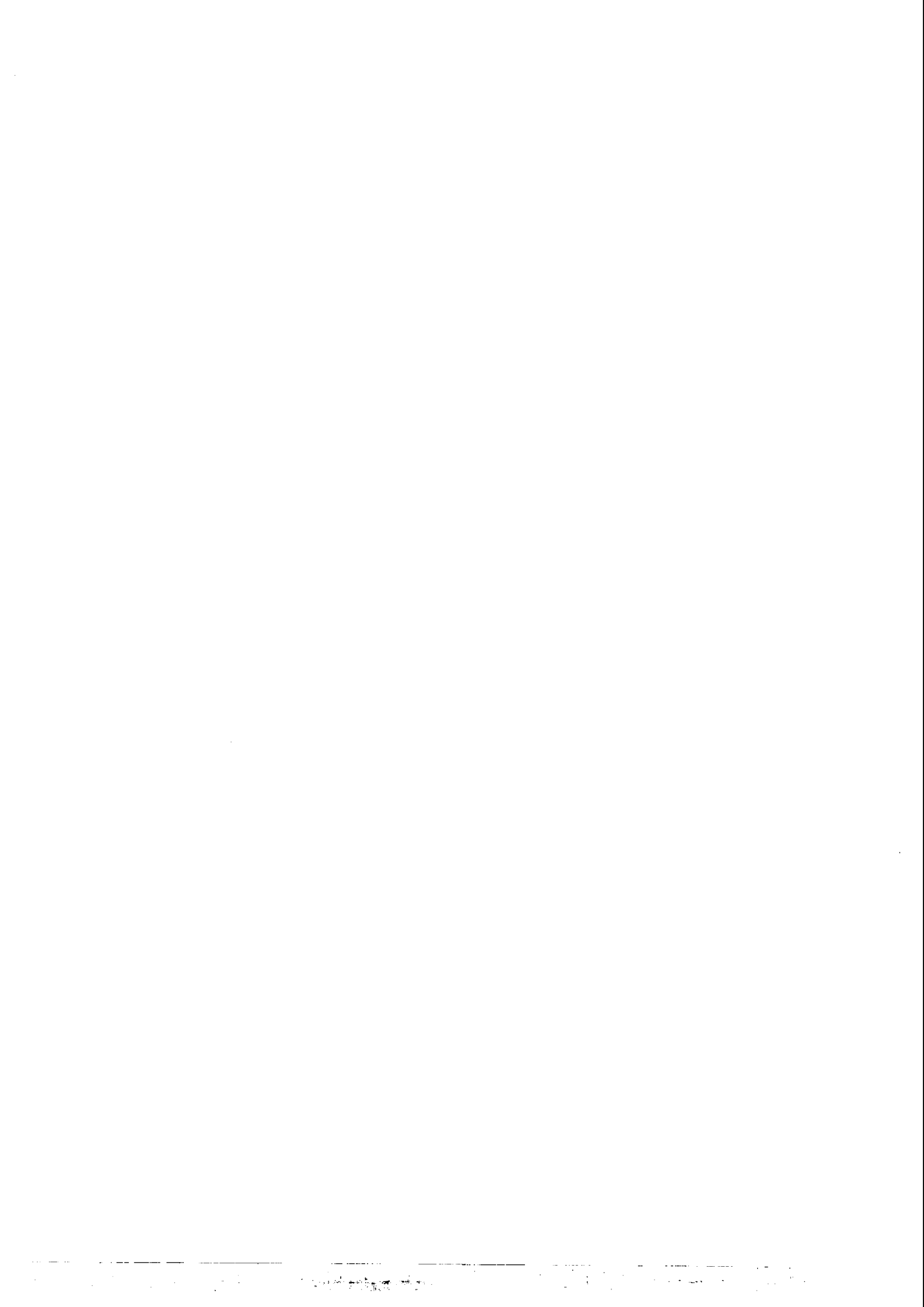International Atomic Energy Agency

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PROGRESS IN RENORMALIZATION THEORY SINCE 1949 *

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I.   INTRODUCTION

Julian Schwinger has just given us a brilliant exposition of the renormalization ideas associated with his, Tomonaga's and Dyson's name. My task is to take the story up from the exciting days of 1949 to the present.

As Schwinger has reminded us, even though renormalization and infinities are logically distinct, it was the persistent infinities encountered in field theories which brought the necessity of renormalization of mass and charge to the fore. The infinities arise mathematically from undefined products of distributions like

$$\frac{1}{x^2}\otimes\frac{1}{x^2} \quad,\quad \frac{1}{x^2}\otimes\partial^2\frac{1}{x^2} \approx \frac{1}{x^2}\otimes\delta(x) \quad\text{and}\quad \partial^2\frac{1}{x^2}\otimes\partial^2\frac{1}{x^2}\approx\delta(x)\otimes\delta(x)$$

when $x \to 0$. These undefined products are encountered when we solve a set of quantum field equations using a perturbation expansion. The first question which arises is this:

Are the infinities – or equivalently the undefined products – a consequence of the bad mathematics of a perturbation expansion? Would they appear even in exact solutions of field equations? In this latter event, is it the type of Lagrangian we are using which is at fault? And if the Lagrangian is to be modified in some essential manner, is there some missing physics which when supplied would circumvent the appearance of the infinities?

A suggestive answer to those questions has been given by Glimm and Jaffe in a series of brilliant papers published between 1969 and 1972. These authors have solved Yukawa-like field theories with the Lagrangians, ϕ^3, ϕ^4 and $\bar{\psi}(x)\psi(x)\phi(x)$ without using a perturbation expansion, in a space of two-space and one-time dimensions. The infinities (expected from naive perturbation theory) duly make their appearance in the exact solutions also. If one may extrapolate from three to four dimensions of space and time, it is not the perturbation expansion that is at fault. One needs to alter basically the type of Lagrangian one has been using in physics. One would still like the new Lagrangians to be "local" in the technical sense, in order to preserve causality and unitarity. However, as I will show later, it is in determining the correct local modification to the types of Lagrangian we have been using that some of the physics we have been missing out – the physics of quantum

gravity – which will serve as a guide.　　Before I proceed, let me summarise what Schwinger has told us.

(1)　Consider the class of Lagrangians which have the simple form of polynomials in field variables.　These are of the type:
$$\phi^3, \phi^4, \phi^5, \cdots, \bar{\psi}\psi\phi, \cdots, (\bar{\psi}\psi)^2, \cdots$$
Why we should ever restrict to the polynomial class of Lagrangians is not clear, but this is an assumption we have inherited from the history of the subject.

(2)　Compute the scattering-matrix elements in a perturbation expansion, using standard Feynman rules.

(3)　There is among this class of Lagrangians a small subset which we call <u>renormalizable</u>, with the property that only a limited few of the S-matrix elements for these theories are intrinsically infinite. This small sub-class of Lagrangians includes ϕ^3, ϕ^4, Yukawa $\bar{\psi}\psi\phi$ and the Maxwell-Dirac Lagrangian $\bar{\psi}\gamma_\mu\psi A_\mu$.　　A theory is <u>non-renormalizable</u> if the class of matrix elements which are infinite is itself limitless.

(4)　For the <u>renormalizable</u> class of Lagrangians we have the famous Dyson theorem on the possible types of infinities which can occur.　The theorem states that for the Maxwell-Dirac Lagrangian only two matrix elements are infinite.　These correspond to the self-mass δm and self-charge δe of the electron.

(5)　Explicitly Dyson's theorem may be stated thus:
　　Write the Maxwell-Dirac Lagrangian in the form:
$$\bar{\psi}\gamma\partial\psi + m_0\bar{\psi}\psi + e_0 \bar{\psi}\gamma_\mu\psi A_\mu .$$

Here m_0 and e_0 are the so-called "bare" constants of mass and charge.

(6)　Compute the self-mass and self-charge δm & δe.
Both turn out to be (logarithmically) infinite – i.e. have the form of undefined integrals $\int d^4x \quad \frac{1}{x^2} \otimes \frac{1}{x^2}$.

(7)　Dyson's renormalization theorem states that if we replace consistently in the computation of all <u>other</u> matrix elements bare mass m_0 and bare charge e_0 by the physical mass m and physical charge e
where
$$m = m_0 + \delta m$$
$$e = e_0 + \delta e$$
then all infinities in the theory are completely absorbed (renormalized) in these redefinitions.　The amazing thing is that this renormalized theory agrees spectacularly with experiment.　As Lamb and Telegdi will tell you, one measure of this agreement is the comparison of the anomalous

magnetic moment $\frac{g-2}{2}$ for the electron with theory.

      Experiment $\frac{1}{2}(g-2)_e = .001169644(7)$

      Theory $\frac{1}{2}(g-2)_e = .001169642$

The agreement of 1 part in 10^9 is a quantitative agreement unmatched anywhere else in Physics - except possibly in the Eötvös-Dicke experiment. Apart from a rather heuristic treatment of what are called overlapping infinities, Dyson's work was (almost) complete for quantum electrodynamics. For other theories it left open two problems, which will form the subject of my talk today.

Problem 1

      Which theories are renormalizable?

Problem 2

      What modification in the theory (missing physics) is needed to compute δm and δe finitely? I shall call this second problem the Lorentz problem because Lorentz was the first person to attack the problem of the computation of electron's self-mass δm in the classical theory.

One must emphasise that so long as one dealt with pure quantum electrodynamics, one could hide behind the inaccessibility of δm to experiment and therefore consider the Lorentz problem to be a pseudo-problem. It is only m - the physical mass-that experiment can determine. There is no way of measuring - within pure quantum electrodynamics - the bare mass m_o . However, going beyond electrodynamics in a higher symmetry theory where,for example, the electron and the electronic neutrino form a doublet, we do know the bare mass of the electron. It must equal (from symmetry considerations) the bare mass of the neutrino $(m_o) = o$. The Lorentz problem - the finite computation of δm (and δe) - must be solved in a complete theory of particles. One half of my talk will be devoted to indicating what I consider a natural solution to the Lorentz problem. I shall suggest that Maxwell-Dirac theory is incomplete in an essential respect. I shall suggest that if one considers the complete theory of photons, electrons and quantum gravitons, both δm and δe turn out to be finite and of the correct order of magnitude.

II. THE CLASS OF RENORMALIZABLE THEORIES

      For the second development I wish to highlight, we consider an extension to the class of renormalizable theories, recently discovered. This promises to give us a lot of new physics. But before we consider this extension, let us look at the situation as it obtained before these recent developments. Throughout we shall concentrate on fermions of spin-$\frac{1}{2}$ interacting with mesons. The fermions include electrons (e), muons (μ), and the neutrino; the octet of nucleons (N), and the triplet of quarks (q) .   For mesons

we shall consider multiplets of spin-zero (e.g. the nonet containing π, κ, η) of spin-1 (with ρ, K^*, φ, ω particles) and spin-2 (nonet containing A_2, ···, f, f'). Let these particles interact through polynomial three-field, and wherever necessary SU(3)-symmetric, Yukawa-like interactions.

The problem of which among these theories are renormalizable was solved early during 1950 - 1951 (Salam and Ward), following Dyson's method. This proof of renormalizability was sharpened and made mathematically more rigorous by Boguliubov, Parasuik, Hepp and Speer during the last decade. (During the last year a beautiful method of regularization has been developed by 't Hooft and Veltman at Utrecht, Bollini and Giambiagi in the Argentine, and Ashmore in Trieste. I shall, however, not speak about these somewhat technical developments.)

The following table summarises the situation of meson-fermion Yukawa-type interactions, so far as developments up to 1967 are concerned.

TABLE I

Force	Gravity	Weak	Electro-dynamics	Strong
Coupling parameters	$G_N\, m_e^2 \approx 10^{-44}$	$G_F\, m_N^2 \approx 10^{-5}$	$\dfrac{e^2}{4\pi} = \alpha = \dfrac{1}{137}$	$g^2 \approx 1$
Meson spin J = 0				SU(3) octet π, κ, η R
J = 1		$W^{\pm}$ NR	γ R	SU(3) octet ρ, K^*, φ R if mesons massless NR if mesons massive
J = 2	Graviton NR			SU(3) octet A_2, f, K^{**} NR

R stands for a renormalizable theory; NR is non-renormalizable.

The most interesting entry in this table is the Yang-Mills gauge theory of spin-one nonet (for example, the nonet of (ρ, K^*, φ, ω) interacting with the octet of physical nucleons (N, Σ, Λ, Ξ)). Yang and Mills argued that in analogy with the case of the photon for zero-mass gauge particles the theory is likely to be renormalizable. Now the nonet of the physical particles (ρ, K^*, φ, ω) consists of massive particles. One could demonstrate that for arbitrary mass values and arbitrary coupling of these particles to

nucleons, the theory is non-renormalizable.

This was the situation up to around 1967. The new developments about which I shall speak in the second part of the talk concern the renormalizability of the massive gauge-meson theories. As I said before, previous to the new developments one had considered arbitrary masses and arbitrary couplings for the gauge particles. Provided that these masses and the couplings are related in a specific manner, through a set of eigenvalue equations, the gauge theories appear to be renormalizable. Among the spectacular special results is the one which states that in order to achieve the renormalizability of weak interactions, weak and electromagnetic interactions must unite as aspects of the same internal symmetry structure. Further, there must exist either new heavy leptons or the internal symmetry group of strong interactions must be extended to $SU(4)$ rather than $SU(3)$ or $SU(3) \times SU(3)$ And all this to secure renormalizability! Surely this must be one of the very rare occasions when a seemingly mathematical criterion (like renormalizability) appears to be leading to new and exciting Physics.

III. THE LORENTZ PROBLEM

Of these two developments, consider the solution of the Lorentz Problem first. I wish to show that whereas Maxwell-Dirac theory of the electron is no more than renormalizable, the Maxwell-Dirac-Einstein theory is actually finite. The conjecture that this may indeed be the case was made long ago by Klein, Landau, Pauli, Deser, DeWitt and others. Recently Strathdee, Isham and Salam have shown that the conjecture is correct provided – and this is important – quantum gravitational effects are treated non-perturbatively.

Gravity is distinguished among other forces of nature by its universality, by its small coupling ($G_N \, m_e^2 \simeq 10^{-44}$ to be compared to $\frac{e^2}{4\pi} = \frac{1}{137}$) and finally by the fact that it is described by an incredibly beautiful Lagrangian.
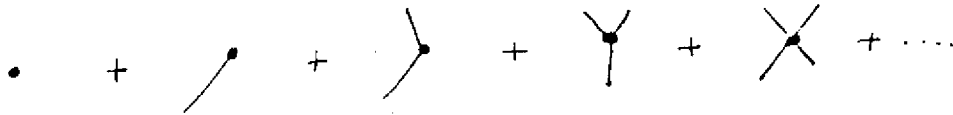
For my purposes, the feature which distinguishes this Lagrangian from all others discussed so far, is its essential non-polynomiality. Before I demonstrate the non-polynomiality of gravity, let me say a few words about non-polynomial Lagrangians in general. A polynomial Lagrangian like $\phi^4(x)$ describes the point-interaction of four ϕ-particles at one given space-time point

A non-polynomial Lagrangian like

$$\sum_{n=3}^{\infty} C_n \, \phi^n(x)$$

- 5 -

describes a whole set of terms, with arbitrarily large numbers of
ϕ-particles being created and annihilated at the same space-time point.
With certain further restrictions which I will not state, such Lagrangians can
be as <u>local</u> (and causal) in a technical sense as the polynomial Lagrangians
themselves; $L = g\, e^{\kappa\phi}$ is an example of a <u>local</u> non-polynomial Lagrangian.
The theory of such Lagrangians was developed mainly in the USSR by Efimov,
Fradkin, Volkov and others during 1963, with further developments being made
by Strathdee, Delbourgo and Salam in Trieste (1969), Lee and Zumino at CERN
(1969) ,   Lehmann and Pohlmeyer in Hamburg (1970) ,   J.G. Taylor in
Southampton (1970) and recently by Honerkamp at CERN (1972).

Graphs represented by $e^{\kappa\phi} = 1 + \kappa\phi + \dfrac{(\kappa\phi)^2}{2!} + \dfrac{(\kappa\phi)^3}{3!} + \cdots$

Now Einstein's gravitational Lagrangian, though often deceptively
written in a polynomial form, is really non-polynomial – it is an infinite
series in the field variable.. I shall write it in a form discussed
by Isham, Strathdee and Salam, which uses a Dirac γ-basis, to exhibit its
elegant structure.

$$\mathcal{L}_{\text{Einstein}} \;=\; \frac{\text{Tr.}\; L^\mu L^\nu \left(\partial_\mu B_\nu - \partial_\nu B_\mu + i\,[B_\mu, B_\nu]\right)}{\left(-\det\; \text{Tr.}\; L^\mu L^\nu\right)^{1/2}}$$

$$\mathcal{L}_{\text{Einstein-Dirac}} \;=\; \frac{\overline{\psi}\, L^\mu \left(\partial_\mu + i\,B_\mu + i\,\epsilon_0 A_\mu\right)\psi + m_0 \overline{\psi}\psi}{\left(-\det\; \text{Tr.}\; L^\mu L^\nu\right)^{\frac{1}{2}}}$$

Here $L^\mu = \gamma_a\, L^{\mu a}$

are the sixteen vierbein fields ; $B_\mu = B_\mu^{ab}\, \sigma_{ab}$ is the affine-
connection   (which, as a consequence of the equations of motion following
from the Lagrangian, can be shown to be proportional to the derivative of the
vierbein fields $L^{\mu a}$).   The familiar Einstein metric field $g^{\mu\nu}$ is
given by $\text{Tr.}\; L^\mu L^\nu$ ("tr." stands for the Dirac-matrix trace).   The
form above emphasises the SL(2,C) gauge-invariance of the theory.

One further remark before the theory can be used to describe quantum gravitons and their scattering. For the Feynman quantisation procedure (for example) to apply it is essential that the field $L^{\mu a}$ (or equivalently the field $g^{\mu\nu}$) should exhibit asymptotic flatness, and the theory should be set up in a world with the Minkowskian background. This, together with the localisability condition on the field theory, can be shown to imply that one must parametrise $L^{\mu a}$ in the form:

$$L^{\mu a} = exp\left(\kappa\ \gamma^{\rho\ell}\ \phi_{\rho\ell}\right)^{\mu a}$$

where $\gamma^{\rho\ell}$ are ten 4 x 4 symmetric matrices, K is the square root of the Newtonian constant

$$\kappa\, m_e \simeq \sqrt{G_N\, m_e^2} \approx 10^{-22}$$

and $\phi_{\rho\ell}$ is the <u>physical field</u> describing the creation and annihilation of gravitons. Note that the above expression for $L^{\mu a}$ $\left(= \eta^{\mu a} + \kappa\ \phi^{\mu a} + \cdots\right)$

implies that $\left\langle L^{\mu a}\right\rangle_0 = \eta^{\mu a}$, in accordance with our demand for asymptotic flatness.
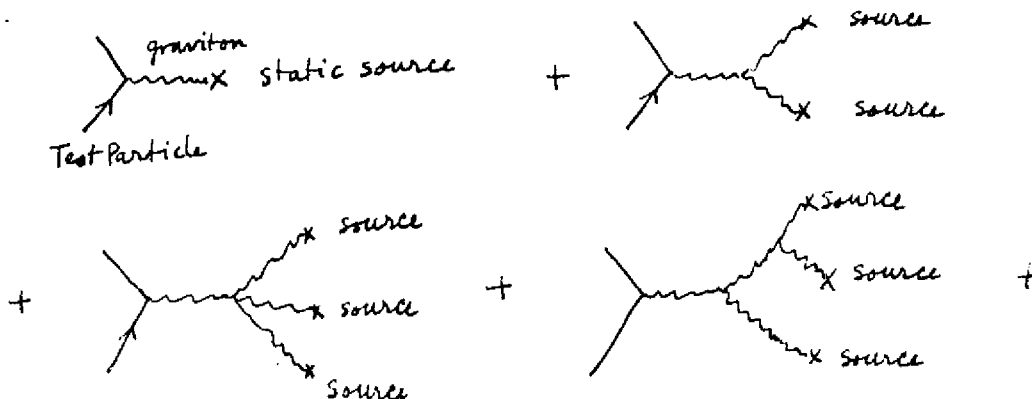
We are now ready to exhibit the infinity-suppressing effects of quantum gravity on the Maxwell-Dirac Lagrangian. Suppressing the tensor indices, the non-polynomial character of Maxwell-Dirac-Einstein Lagrangian is exhibited by the factor L^{a}_{μ}

in the numerator of the Dirac-Maxwell term. In its essentials then, $L_{Einstein}$ has the form $\partial_\mu\left(e^{\kappa\phi}\right)\partial_\mu\left(e^{\kappa\phi}\right)$ while $L_{Maxwell-Dirac-Einstein} \simeq \ell_0\ exp(\kappa\phi)\ \bar{\psi}\,\delta_\mu\psi A_\mu$. The non-polynomiality of this Lagrangian implies that in addition to the direct electron-photon interaction at a space-time point x, the theory describes the emission and absorption of millions and trillions of gravitons through terms like $\frac{1}{n!}(\kappa\phi)^n$ in the exponential factor $exp(\kappa\phi)$. It is this "atmosphere" of virtual gravitons, into which the theory immerses electrons and photons, which is the decisive element in the infinity suppression. Before we go on, one remark about the quantisation procedure we have adopted. Professional relativists – some of the most illustrious of them are present here today – feel somewhat embarrassed about Feynman quantisation. They suspect that, starting with the smooth boundary condition $L^{\mu a} \approx \eta^{\mu a}$, one has unduly restricted the theory and the type of space-time manifold in which one can operate. For example, they would feel that one would never, in this manner, operate in a Schwarzschild space-time manifold with its characteristic singularities.

My personal feeling is that the professional relativist ignores two important insights from the particle physicist's experience. These are:

(1) The power of the analytic continuation method.

(2) The possibility of renormalizing the bare Minkowskian metric $\eta^{\mu\nu}$ to the physical metric $g^{\mu\nu}$ - so beautifully demonstrated in Thirring's classic paper through the inevitable renormalization in general relativity of lengths and time intervals.

I am labouring this point because one of the aims of this meeting is to bring closer together the general relativist's and particle-theorist's points of view. To exhibit the power of analytic continuation techniques of the particle physicist, consider the following problem. Reconstruct the exact Schwarzschild solution around a source-singularity, starting with quantised theory of gravitons of Feynman. In graphical terms, consider all (tree) graphs of the following variety:

In the static limit, the first graph gives a contribution proportional to $\frac{2MG}{r}N$ where M is the mass of the static source.

In the next approximation the graphs contribute $+ \ldots \left(\frac{2MG}{r}\right)^2$ and so on.

The challenge - and I took a bet on this last year with Carter and Penrose -· is this. Can we take the exact series with all graphs of the above variety -· with millions and trillions of gravitons exchanged -· sum the series, continue the sum beyond its radius of convergence AND recover the exact Schwarzschild solution, manifesting all its customary space-time singularities. If we succeed, we would have recovered the singular Schwarzschild manifold, though we started our quantisation procedure with a non-singular Minkowskian one. For the particle physicist, the situation is very familiar. It is similar to summing perturbation graphs for scattering to give the Bethe-Salpeter equation and then using the equation (by continuing in the energy variable) to get bound states.

The relativistic problem was considered by M. Duff for his Ph.D. thesis at Imperial College last year. And Duff has shown that the analytic continuation

technique does indeed work.   Starting with Feynman's quantisation built onto
a non-singular Minkowskian manifold, one can indeed recover the Schwarzschild
singular manifold, provided that the contribution of all gravitons is taken
into account in the calculation and no approximation made.   Our quantisation
procedure starting with the "bare" Minkowskian metric ensures that all metrics
with asymptotic flatness will emerge after appropriate analytic continuations.

Let us now turn back to the Lorentz problem of computing δm and δe,
taking into account the relevant contributions of the sea of gravitons in
which the theory (and physics) places electrons and photons.

As I said earlier, all infinities in field theory come about as a
result of the consonance of singularities of propagators like $(-\frac{1}{x^2})$ at $x = 0$.
In the conventional Maxwell-Dirac theory with its Lagrangian

$e_o \bar{\psi} \gamma_\mu \psi A_\mu$ the photon propagator $(A_\mu A_\nu)$ equals $\delta_{\mu\nu}/x^2$,
the electron propagator ($\bar{\psi}\psi$) equals $\gamma\partial\frac{1}{x^2} + \frac{m}{x^2}$ + terms which
are less singular, while the contribution of the second-order graph

(where an electron emits and reabsorbs a
photon) is essentially given by:

$$\delta m \propto e_o^2 \int d^4x \ \frac{1}{x^2} (\gamma\partial + m) \frac{1}{x^2} \ . \quad \text{Since} \int \frac{1}{x^2} \gamma\partial \frac{1}{x^2} d^4x \equiv 0 ,$$

$$\text{therefore} \quad \frac{\delta m}{m} \propto e_o^2 \int \frac{d^4x}{(x^2)^2} \ .$$

This integral is logarithmically infinite at the lower limit of integration
(at small distances $x = 0$, or from the uncertainty relation for large energies).

Consider now what happens when the quantum ocean of gravitons is taken
into account.

Since $\langle \phi \phi \rangle = -\frac{1}{x^2}$, $\langle \phi^n \phi^n \rangle = n! \left(-\frac{1}{x^2}\right)^n$,

we obtain for the propagator, $\langle e^{\kappa\phi} e^{\kappa\phi} \rangle = \sum_{n=0}^{\infty} \frac{1}{n!} \left(-\frac{\kappa^2}{x^2}\right)^n$
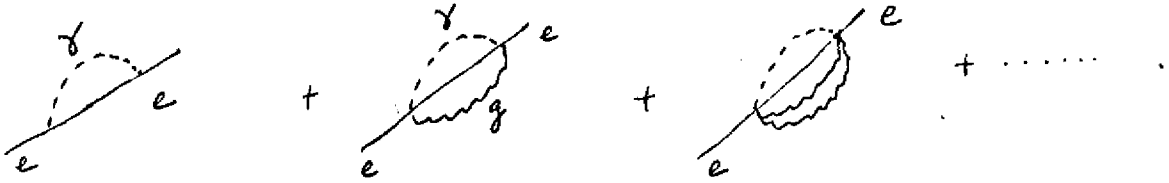
$$= exp \left(-\frac{\kappa^2}{x^2}\right)$$

The important point about the factor $exp \left(-\kappa^2/x^2\right)$ is that if we approach
x^2 from the appropriate (time-like) direction and fill in for the other
directions by an appropriate analytic continuation, the factor $exp \left(-\kappa^2/x^2\right)$
tends to zero fantastically smoothly.

For the electron self-mass we now obtain

$$\frac{\delta m}{m} \approx e_o^2 \int \left(\frac{1}{x^2}\right)^2 e^{-\kappa^2/x^2} d^4x \quad .$$

Graphically we are summing the series of graphs

This summation over one, two, three, ..., gravitons, has thus given us a smoothing function $\exp(-\kappa^2/x^2)$ for the old infinity. The integral can be easily evaluated and is given in standard texts. We obtain.

$$\frac{\delta m}{m} \approx e_o^2 \left[\log(\kappa^2 m_e^2) + \kappa^2 \log(\kappa^2 m_e^2) + \cdots \right] \quad .$$

Writing, $R = 2G_N m_e = 2K^2 m_e$

when (R is the Schwarzschild radius of the electron) we can re-express

$$\frac{\delta m}{m} = e_o^2 \log(R\, m_e)$$

Note that when $R \to 0$ (i.e. when quantum gravity is ignored and $G_N \to 0$), we recover the old logarithmic infinity for δm. If gravity is not ignored, the theory has an inbuilt <u>cut-off length at the Schwarzschild radius of the electron</u>. It would seem that photons of wavelength smaller than the Schwarzschild radius are simply unable to affect the electron. Their emission and reabsorption by the electron – which is what was causing the infinity at the zero end of the scale $x = 0$ in $\int \frac{d^4x}{x^4}$ — is inhibited by the peculiar characteristics of space-time inside the Schwarzschild radius R.

And now comes an amazing numerical circumstance. Introducing all the appropriate factors of 2π, our result is

$$\frac{\delta m}{m} = \frac{3}{4\pi} \frac{e^2}{4\pi} \log \frac{1}{G_N m_e^2} + \text{terms in higher orders} \quad .$$

The very fact that $G_N m_e^2$ is such a tiny number means that $\log \frac{1}{G_N m_e^2}$ is not small. In fact, $\alpha \log \frac{1}{G_N m_e^2} \simeq \frac{105}{137}$ is a magnitude of the order of unity, so that $\frac{\delta m}{m} \simeq \frac{2}{11}$. The lowest-order approximation for $\frac{\delta m}{m}$ gives us a sensible result. The gravity correction to the electron's self-mass is not tiny. And

since in the next approximation (with two photons and trillions of gravitons exchanged) we may expect a contribution $\simeq \left| \alpha \log G_N m_e^2 \right|^2$ and so on for the third, fourth and other higher-order approximations in α - it is fully conceivable that the sum of the series for $\delta m/m$ is even nearer unity than the first approximation indicates. That is to say, gravity-modified quantum electrodynamics is likely - in a most natural manner, to fulfil Lorentz's conjecture - that all mass of the electron is due to its (gravity-modified) electromagnetic self-interactions (m = δm) and that the bare mass of the electron is indeed zero (m_0 = 0).

We can compute likewise the self-charge $\frac{\delta e}{e}$. This is also finite in a gravity-containing Dirac-Maxwell-Einstein theory. In this work it was important to check the electromagnetic gauge invariance of the theory. This has been done (see Isham, Strathdee & Salam, Physical Review 1971). One can also show that the civilizing effect of gravity in solving the Lorentz problem is a peculiarity of tensor-gravity (spin-2$^+$ gravitons) and not scalar-gravity of Brans-Dicke variety. Since the Schwarzschild singularity of the space-time manifold is also a peculiarity of Einstein's tensor theory, this supports the physical argument I gave - of the inter-relation of the electron's Schwarzschild radius with infinity suppression.

To conclude this part of my talk, quantum electrodynamics in its simple form is infinite because there was some missing physics - the physics of quantum gravity. Once that missing physics is supplied, the infinities are regularized in a natural manner. The amazing numerical circumstance is that the numerical value for δm, in the lowest order of the calculation accords sensibly with the conjecture of Lorentz that all inertia of the electron is due to its self-interactions.

One may in fact have started with Lorentz's conjecture in reverse, i.e., we could compute $\left| \alpha \log G_N m_e^2 \right|$ from the eigenvalue equation m_0 = 0, i.e.,

$$\frac{\delta m}{m} \left(\alpha \log G_N m_e^2 \right) = 1 .$$

One root of this equation (if the lowest order calculation is any guide) is

$$\left| \alpha \log G_N m_e^2 \right| \approx \frac{105}{137}$$

Thus, given $\alpha \simeq \frac{1}{137}$, we would know why the Newtonian constant $G_N m_e^2$ is so small and of the order 10^{-44} and vice versa.

IV   GAUGE MESONS AND SPONTANEOUS BREAKING OF SYMMETRIES

I turn now to the second major development in the conventional polynomial theories, forgetting about the gravity modifications for the present. In 1954 Yang and Mills (and independently my pupil R. Shaw for his Ph.D. thesis

at Cambridge) invented gauge meson theories corresponding to the internal istopic-spin symmetry SU(2). The pattern for these theories was provided by electrodynamics - the theory of the gauge particle (the photon) corresponding to the charge-conserving internal symmetry group U(1). Yang and Mills showed that for an exact gauge symmetry, these gauge particles must be massless, and their Yukawa couplings are renormalizable.

Now in nature, gauge particles do exist; the $(\ \rho,\ K^{*},\ \omega,\ \phi\ )$ nonet exists corresponding to the U(3) symmetry and also appears to possess Yukawa couplings with nucleons characteristic of gauge particles. However, the particles are massive and also the U(3) symmetry, to which the particles are supposed to correspond, is not exact. Either of the two circumstances - massiveness of the gauge mesons, or the broken character of the symmetry, is enough to make the theory non-renormalizable in general.

I said, "non-renormalizable in general". This qualification "in general" is extremely important. For arbitrary values of gauge meson masses and arbitrary values of the symmetry-breaking parameters the theory is indeed non-renormalizable. Could renormalizability, however, be restored, for special values of masses and a special form of symmetry-breaking?

The answer, devloped since 1963, through the work of Higgs, Kibble, Guralnik and Hagen, and finally clearly established during 1971 by G. 't Hooft, appears to be YES. I shall state it in the form of a theorem.

Theorem.

A Yang-Mills theory of gauge mesons interacting among themselves and with spin-½ fermions is renormalizable provided:

(1) The symmetry to which the mesons correspond is broken through Heisenberg's spontaneous symmetry-breaking mechanism.

(2) Provided this symmetry breaking is introduced through a multiplet of spin-zero mesons, with non-zero expectation values (ϕ).

(3) In this event, the masses of gauge particles as well as the Fermi particles are not arbitrary but are prescribed in terms of (ϕ).

(4) If the Fermi particles are coupled through axial-vector gauges in addition to the vector-gauge particles, an axial charge doubling of fermions is necessary in order to ensure renormalizability. To understand this theorem, let us introduce the concept of spontaneous symmetry breaking first.

Turn to the Dirac-Maxwell theory we considered in the first part of the lecture:

$$\mathcal{L} \rightarrow \bar{\psi}\,\gamma\partial\,\psi\ + e_0\,\bar{\psi}\,\gamma_\mu\psi\,A_\mu\ + m_0\,\bar{\psi}\,\psi$$

If $m_o = 0$ (zero bare mass), the theory possesses an additional symmetry - the so-called "neutrino" symmetry (γ_5 - invariance), specifically the transformation $\psi \rightarrow \left(exp\, i\gamma_5\, \theta\right)\psi$ leaves the Lagrangian form invariant. The bare mass term breaks this symmetry.

Question?

Can we start with <u>zero</u> bare mass $m_o = 0$ but compute self-consistently from within the theory a <u>non-zero</u> physical mass $m \neq 0$ $(m = m_o + \delta m)$? The answer is yes. In the first part of my talk I did precisely that; I set up an eigenvalue equation for $\dfrac{\delta m_e}{m_e}$:

$$\frac{\delta m_e}{m_e} = \frac{m_e - m_o}{m_e} = \alpha \, \log G_N \, m_e^2 \quad .$$

The form of this equation is always such (from Dyson's theorem) that we can set in it $m_o = 0$.

For the special values of m_e, α and G related through $\alpha \log G_N m_e^2 = 1$, we can solve and obtain a non-zero <u>physical mass</u> m_e.

Does the γ_5 - symmetry survive in the theory? In the Lagrangian, yes, since $m_o = 0$ - but not for the physical states, since the Hilbert space of physical particles describes electrons with <u>mass</u>. The γ_5 - symmetry has been "<u>spontaneously</u> and <u>self-consistently broken</u>" within the theory.

Now consider this as a Yang-Mills theory corresponding to the symmetry group U(1), generated by the γ_5-transformations $\psi \rightarrow e^{i\gamma_5\theta}\psi$. The theory will now be supplemented with an (axial-vector) gauge meson Z with the interaction $i e_o \, \bar{\psi} \gamma_\mu \gamma_5 \psi Z_\mu$

Now following Anderson, Higgs in 1963 proved a very important theorem about Z . This states that Z <u>must</u> possess a mass equal to em (where m is a solution of the eigenvalue equation $1 = \dfrac{e^2}{4\pi} \, \log G_N m_e^2$). Further, the theory <u>will</u> describe in its spectrum a spin-zero bound-state particle with an associated field ϕ, with a definite non-zero expectation value $\langle\phi\rangle$. And the coupling (f) of this field to electrons and its expectation value $\langle\phi\rangle$ <u>will</u> also be related to m thus:

$$f \, \langle\phi\rangle = m \, .$$

To restate all the ramifications of this closely knit theorem, let me summarise it again:

To summarise

Given a U(1) γ_5 - symmetry, one can set up a Yang-Mills gauge theory. If we introduce a spontaneous symmetry breaking, the theory <u>will</u> describe a spin-zero (bound) state with a non-zero expectation value and a definite

coupling to the Fermi particles. Also the mass of the gauge particle will not be arbitrary but a definite multiple of the non-zero expectation value. Finally this tightness of relations between the masses and the couplings of the spin-zero, spin-$\frac{1}{2}$ and spin-1 particles has the consequence that the complete theory is renormalizable.

In converse, the demand of renormalizability of a gauge theory can be met only if there is a definite set of scalar particles in the theory with non-zero expectation values, and with definite relations between the masses of the gauge bosons and these non-zero expectation values.

Table 2

(a) situation I

Exact γ_5 - symmetry $\rightarrow$ U(1) group

$m_e = 0$

$m_z = 0$ Theory renormalizable but not physical

(b) situation II

Spontaneously broken γ_5 - symmetry;

A. $m_e \neq 0 \rightarrow 1 = e^2/4\pi \log G_N m_e^2$
    $m_z \neq 0 \rightarrow m_z = e m_e$

B. Theory must contain a scalar particle ϕ with $f <\phi> = m_e$

C. There must exist an additional* heavy lepton to secure
    Renormalizability with opposite γ_5 - coupling to Z

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* I have no time to go into this rather technical point, associated with the so-called Schwinger - Bell - Jackiw - Adler anomaly. It appears that the demand of renormalizability cannot be met whenever axial-vector gauges are present unless there is a symmetry between sets of fermions possessing equal and opposite γ_5 - couplings to the axial-gauge particles.

The masses of these extra fermions (they need not be leptons) are not restricted by the theory, so far as we know.

V. A RENORMALIZABLE UNIFIED THEORY OF WEAK AND ELECTROMAGNETIC INTERACTIONS

One rather attractive instance of the application of ideas I have mentioned is that of a unified theory of weak and electromagnetic interactions. That both weak and electromagnetic interactions may in fact constitute different aspects of one symmetry scheme is a conjecture made long ago - to my knowledge, first, - by Julian Schwinger (1957). The conjecture was revived by Glashow (1959, 1961) and Salam & Ward (1959, 1964), who used a U(1)xSU(2) gauge group to consider four Yang-Mills-type gauge mesons three of which would mediate weak interactions and one would be the photon. Recently Weinberg (1967) and Salam (Nobel Symposium,Gothenburg, 1968) applied the ideas of spontaneous symmetry breaking to give the three weak gauge bosons their masses. Weinberg and Salam conjectured that the resulting theory may be renormalizable. The conjecture has now been proved by 't Hooft.

Specifically, the model works with the U(1) x SU(2) gauge symmetry, with four gauge particles, $W^{\pm}$, Z^{o} (mediating weak interactions) and the photon A^{o}. One implication of the symmetry scheme is that the weak coupling constant must equal the electromagnetic. The observed smallness of the effective weak constant G_F is then attributed to the large mass of $W^{\pm}$ particles; specifically we must have $e^2 = \dfrac{G_F}{m_w{}^2}$

Now in accordance with the Higgs-Kibble theorem, the emergence of these masses from a spontaneous symmetry breaking mechanism carries with it the following implications:

(1) There must exist in the theory a scalar bound state, with an associated field ϕ, whose expectation value $<\phi>\neq 0$.

(2) The masses of W and Z particles are given by the relations like

$$m_w = e<\phi>$$
$$m_z = \sqrt{2}\, e<\phi>$$

while $m_{electron} = f<\phi>$.

Here f is the direct coupling of the ϕ - particle to the electron.

(3) In order that the theory be renormalizable there must exist an additional heavy electron and a heavy muon with equal and opposite axial couplings to $W^{\pm}$ and Z particles.

(4) If the theory is extended to hadrons, either the hadronic quarks should also be doubled in number (with equal and opposite axial charges) or the symmetry group of strong interaction physics should be extended from SU(3) to at least SU(4), if not SU(5) or SU(3)xSU(3). This would

of course have profound consequences for spectroscopy in hadronic physics.

Summarising then, the renormalizability demand appears to leads to new and exciting physics. It is exciting physics in two directions. Firstly, it would appear that there is but one scheme of coupling constants and masses. There is no arbitrariness in assigning these parameters; renormalizability works for special values of these parameters and not otherwise. Secondly, there seem important restrictions on the types of internal symmetry schemes the physical particles belong to.

VI.   FROM RENORMALIZABLE TO FINITE THEORIES

I started this lecture by considering the prototype of all renormalizable theory - quantum electrodynamics. By considering its natural non-polynomial modification, when quantum gravitational interactions of electrons and photons are taken into account, we were able to solve the Lorentz problem - i.e. render the few remaining infinities of the theory also finite. What are the prospects of gravity rendering the renormalizable weak and strong interaction theories I have been speaking about <u>also finite</u>? After all, gravity affects protons and neutrons just as much as it affects electrons and muons.

Now gravity does affect hadrons just as much as it affects leptons but there appear to be indications that, in formulation of gravity theory for hadrons, one cannot follow Einstein blindly.

To illustrate what I mean, consider the analogous case of electrodynamics. Up to 1961, we believed that the electromagnetic interaction of protons is identical in form to the electromagnetic interaction of electrons. One believed that one would write a Dirac equation

$$\left(\gamma(\partial + ieA) + m_p \right) \psi_p = 0$$

for a proton completely similar to the corresponding equation for the electron:

$$\left(\gamma(\partial + ieA) + m_e \right) \psi_e = 0 \quad .$$
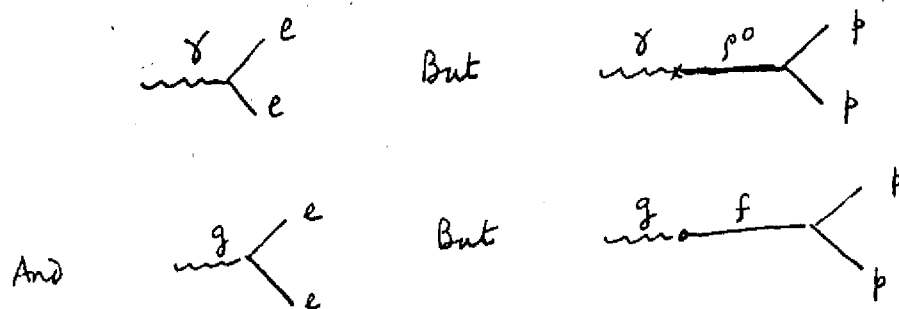
We now know this would be wrong.

Our present picture of the electromagnetic interaction of protons is a two-stage picture. Among the nonet of strongly interacting gauge particles (ρ, ω, ϕ, K^*),   there is one particle - ρ^0 - which has identical quantum numbers to the photon γ.

In quantum theory, whenever we have two particles with the same quantum numbers they <u>must</u> interconvert; there must be transitions between them.

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Our picture of the proton's electromagnetic interaction is therefore this. The proton emits and reabsorbs ρ^0; the ρ^0 then has a finite amplitude for conversion into a photon γ^0 - which in its turn can be emitted and absorbed by the electron or the muon. There are thus two distinct worlds - the world of the leptons, directly interacting with photons , and the world of the hadrons directly interacting with the gauge nonet $(\rho, \phi, \omega, K^*)$ among which is contained what may be called the heavy photon ρ^0. The two worlds communicate electromagnetically through an interconversion of ρ^0's to γ's. The picture I have outlined above is the one borne out by numerous experiments and is generally accepted.

Now the same situation appears to hold for gravity. Among the recently discovered mesons is the nonet of spin-2^+ which includes particles like f, f', A_2, etc.   The f^0 particle in particular possesses the special feature that all its quantum numbers are identical with those of the graviton. There is nothing on earth which can stop the interconversion of gravitons and f-mesons.

Isham, Strathdee and Salam have postulated the same equation as Einstein's for the f-particles. In this theory the f-meson _is_ the graviton of hadronic physics - just as the ρ^0 was its "photon". The f-meson possesses a universal coupling to matter, of the same non-polynomial variety as the graviton does. The gravitational force between hadrons and leptons or hadrons and hadrons proceeds through the mechanism of interconversion of f's to Einstein's gravitons, the latter in their turn coupling directly to leptons. Pictorially the respective interactions look like this:

Now what is the effect of this interposition of the f-particle in hadronic physics? You may recall that the effective cut-off (mass)^2 (from

the non-polynomiality) of gravity theory was proportional to the inverse of the gravitational coupling constant $\frac{1}{G_N}$. For f-gravity, the cut-off will come much earlier; it will come at $\frac{1}{G_f}$ where empirically $G_f\, m_N^2 \approx 1$.

This means that the cut-off (mass)^2 in strong interaction physics is around 1 BeV and not around 10^{19} BeV. This is precisely what is observed – the matrix elements in strong interaction physics do get damped around 1 BeV.

In summary then, the final finite theory of strong interactions may start as a renormalizable gauge theory with a spontaneous symmetry-breaking mechanism built into it to accord with the physically observed symmetry breaking. The final finiteness would be secured by considering the intrinsic non-polynomiality superimposed on the theory by the universal f-gravitational interaction. We must always start with a renormalizable theory before considering the effects of f-gravity, otherwise, according to some recent work of Lehmann, we are likely to run into ambiguities in the final numbers we will compute.

I have tried to convey to you some of the excitement of this field, its liveliness, its promise – particularly in leading us on to new and physically relevant physics. There are of course hosts of unsolved problems – not the least among them being, "does quantum gravity quench its own singularities?" And if so does this include also the singularities of space-time manifold about which we have heard such a lot at this conference? I should like to conclude with Oppenheimer:

"If what we have learned so far ··· is radical and unfamiliar and a lesson that we are not likely to forget, we think that the future will be only more radical and not less, only more strange and not more familiar, and that it will have its own new insights for the inquiring human spirit."