A NOTE ON THE $l$-DEPENDENCE OF THE $\alpha-\alpha$ INTERACTION *

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ABSTRACT

The present note, based on an analysis of low-energy $\alpha-\alpha$ scattering, emphasises that the $l$-dependence of the $\alpha-\alpha$ interaction is not essential and that the existence of a repulsive part over and above the Coulomb potential is not indispensable. Both these findings point towards the possible existence of a static $\alpha-\alpha$ interaction.

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Till recently, various investigations have been made into the energy dependence and the orbital angular momentum dependence of the $\alpha - \alpha$ interaction. Phenomenological studies reveal that the interaction between two $\alpha$-particles is dependent on their relative orbital angular momentum $\ell$ but is independent of their relative energy. Resonating group studies of the $\alpha - \alpha$ interaction built from the basic N-N forces show that the $\alpha - \alpha$ interaction is energy dependent as well as $\ell$-dependent. It was seen, however, that for a given $\ell$ value, the position of the node in the radial wave function is insensitive to energy. Thus the energy dependence of the $\alpha - \alpha$ interaction is not found to be very serious; it is the $\ell$-dependence that has always been considered to be one of the salient features of the $\alpha - \alpha$ interaction. One of the earlier attempts was made to investigate if even at low bombarding energies, for which polarization of $\alpha$-particles does not play a major role, two $\alpha$-particles would interact according to a velocity-independent potential (which would include independence of $\ell$). It was found that this was just not possible even up to laboratory energies $\approx 6$ MeV. Later investigations have confirmed more or less the same findings. The best that one has been able to do phenomenologically with $\ell$-dependence has been to find a common outer attractive part and ascribe the $\ell$-dependence to the repulsive part of the $\alpha - \alpha$ interaction.

The purpose of the present note is to emphasise that the $\ell$-dependence of the $\alpha - \alpha$ interaction is not a closed subject but is still open to question. There are phenomenological indications that a purely attractive $\ell$-independent $\alpha - \alpha$ potential can reproduce the scattering phase shifts quite satisfactorily. This finding, which contradicts all past investigations, was arrived at by the authors from a rather different context.

Recently Brown and Tang extended the resonating group calculations to energies above the reaction threshold of 17.4 MeV for the reaction $\alpha + \alpha \rightarrow ^7\text{Li} + p$, by introducing a local, phenomenological, imaginary potential into their formulation. Their purpose was to account approximately for the influence of reactions on elastic scattering. They found that a simple, surface-peaked imaginary potential, with a strength which varies smoothly with energy, reproduces quite adequately the imaginary parts of the experimentally determined $\alpha - \alpha$ phase shifts from 50 to 120 MeV (lab). However, although the addition of an imaginary potential to the resonating group formulation...
serves a definite purpose, the basic spirit of resonating group formalism seems to get lost somewhat through the introduction of phenomenology. It would seem, therefore, that if one has to use an imaginary potential for \( \alpha-\alpha \) scattering at high energies, then instead of using a combination of resonating group and phenomenological potentials one might as well consider a purely phenomenological optical model analysis of \( \alpha-\alpha \) scattering.

Optical model analyses have in the past been made by several authors\(^1\),\(^5\),\(^7\)) covering the energy range of 23-120 MeV. Again, the optical model potentials were found to be \( l \)-dependent. For both the real and imaginary parts of the \( \alpha-\alpha \) potential, the Woods-Saxon form was used. This form, although used extensively in optical model analysis, has not been used for \( \alpha-\alpha \) scattering below the reaction threshold. Since, from the phenomenological point of view, the \( l \)-dependence of the \( \alpha-\alpha \) interaction has been studied with only certain chosen shapes (in fact, in the past mainly square-well and Gaussian potentials have been used), the question remains whether this \( l \)-dependence is true for all assumed shapes and flexibilities of the \( \alpha-\alpha \) potentials. It is in this particular context that the suitability of the Woods-Saxon form is explored and commented upon here in relation to that of other forms used in the past.

The \( l = 0, 2, 4 \) phase shifts for energies \( E \leq 24 \) MeV only were considered and hence, for the nuclear part of the \( \alpha-\alpha \) potential, a completely real potential was used:

\[
V_{\alpha\alpha}^N(r) = -\frac{U}{1 + \exp[r-R/a]} \quad \text{with} \quad R = r_0 A^{1/3} \quad . \tag{1}
\]

For the Coulomb potential \( V_c \) we have used

\[
V_c(r) = \frac{Z_1 Z_2 e^2}{2 R_c} \left( 3 - \frac{r^2}{R_c^2} \right) , \quad r < R_c
\]

\[
= \frac{Z_1 Z_2 e^2}{r} , \quad \quad \quad \quad \quad r > R_c \quad . \tag{2}
\]

where \( R_c = r_0 A^{1/3} \). \( U, a, r_0 \) and \( r_c \) have their usual meaning and \( r_c \) is taken equal to \( r_0 \). The complete potential is

\[
V_{\alpha\alpha}(r) = V_{\alpha\alpha}^N(r) + V_c(r) \quad . \tag{3}
\]

Attempts were made to reproduce \( \delta_0, \delta_2 \) and \( \delta_4 \) up to 24 MeV (lab) using only one set of parameters. The expectation of obtaining the phase shifts
with an attractive part only arises because in previous investigations it had been found that the repulsive part needed for the higher partial waves was effectively masked by the large centrifugal barriers. In fact, it was found that the G-wave phase shifts can be reproduced even without any repulsive part. The phase shifts for lower \( l \) values, however, needed some repulsion. Since potential (3) used in the present work has a different shape and flexibility, it is worthwhile exploring if the repulsive part is really essential for the lower partial waves. Naturally, we started with an attractive part only and looked for a common set of parameters. In fact, it turned out that such a set does exist. The potential corresponding to this set \((U = 127.5 \text{ MeV}, r_0 = 1.15 \text{ fm}, a = 0.6 \text{ fm})\) is plotted in Fig.1. The phase shifts for this potential are shown in Fig.2 and are compared with the experimental ones of Refs.16-18. Remembering that the emphasis here is not on reproducing the phase shifts for one particular partial wave as well as possible but, rather, on having an overall fit to the S, D and G phase shifts, the agreement with experimental phase shifts is quite striking — more so because there is no repulsion in the potential! The absence of repulsion should be a surprising result not so much for the G and D waves but certainly for the S wave which has no centrifugal barrier. It seems as if the repulsion thought to be needed for the low-energy S-wave \( \alpha-\alpha \) interaction may perhaps be provided by only the \( \alpha-\alpha \) Coulomb interaction. This point, however, needs further attention. Thus the present analysis emphasises two features of the \( \alpha-\alpha \) interaction:

1) Phenomenologically, \( l \)-dependence of the \( \alpha-\alpha \) interaction is not essential.

2) The existence of a repulsive part in the two-body \( \alpha-\alpha \) interaction over and above the actual Coulomb potential is not indispensable.

Since we are considering low-energy \( \alpha-\alpha \) scattering, it seems plausible that the chances of interpenetration of the \( \alpha \)'s are rather small and hence the inside repulsion which usually results from antisymmetrization of the total wave function against overlap in presumably not expected to be important.

We thus remark that the previous observations which were made on the \( l \)-dependence of the \( \alpha-\alpha \) interaction appear now to be a result mainly of the restricted flexibility of the potential shapes assumed. One also notes that the Woods-Saxon form does not only describe the scattering of nucleons off nuclei but also provides an adequate description of scattering of composite particles.
It is of interest to see how the above features could emerge from resonating group studies of the $\alpha-\alpha$ interaction. In this connection it is worth noting that the attractive potential which reproduces the $S$, $D$ and $G$ phase shifts has central depth ($\approx 122$ MeV) and range ($\approx 5.5$ fm) which are compatible with those of the direct part of the $\alpha-\alpha$ interaction furnished by resonating group studies. However, a particular point of interest would be to examine in detail the $l$-dependence and repulsive character of the kernels for fairly realistic $N-N$ interactions. Some preliminary results seem to point towards a weak $l$-dependence and are in line with the present findings. Detailed calculations in this regard are in progress. We conclude by saying that the concept of a static $l$-independent $\alpha-\alpha$ interaction at low energies may not be untenable.

After this paper had been written, a paper by Neudatchin et al.\textsuperscript{19} came to our attention in which a local optical model analysis of $\alpha-\alpha$ scattering has been made using the Woods-Saxon form. The emphasis of the present note is, however, more on the properties of the $\alpha-\alpha$ interaction at low energies. Nevertheless, it is gratifying to note that the present investigation, which was made independently, does confirm the results of Neudatchin et al. and further questions the faith established so far in the non-static nature of the $\alpha-\alpha$ interaction.

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REFERENCES

1. For a systematic survey of the $\alpha$-$\alpha$ interaction see:
The nuclear and the Coulomb part of the $\alpha-\alpha$ potential and their sum are plotted as a function of $\alpha-\alpha$ separation.

Calculated phase shifts $\delta_0$, $\delta_2$, and $\delta_4$ for $\alpha-\alpha$ scattering have been plotted as a function of the laboratory energy. The experimental points are from Refs. 16-18.