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# INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

PCAC ANOMALIES AND GRAVITATION

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PCAC ANOMALIES AND GRAVITATION \*

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### ABSTRACT

The Feynman rules of electron-graviton interaction are used to calculate the graviton self-energy and the  $\pi^0 \rightarrow 2g$  matrix element, the direct analogues of the photon self-energy and  $\pi^0 \rightarrow 2\gamma$ . The lowest-order perturbation result leads to the modified PCAC relation,

$$\partial_\alpha j_{\alpha 5} = 2mj_5 + e^2 \epsilon_{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta} / 16\pi^2 + \epsilon_{\kappa\lambda\mu\nu} R_{\kappa\lambda\rho\sigma} R_{\mu\nu\rho\sigma} / 768\pi^2$$

where the anomalous terms on the right are the lowest-order non-generally covariant contributions of infinite-mass regulator fermion loops. The important question of whether the gravitational anomaly should be generally covariant and (if higher-order calculations show that this is not the case) whether compensating leptons are an absolute must, is, unfortunately, left open to the order we have worked.

## I. INTRODUCTION

Anomalous Ward-Takahashi identities have had an important impact <sup>1)</sup> on at least two branches of elementary particle physics. In the first place, they have proved relevant to the determination of pseudoscalar meson lifetimes and, in the second place, they have placed strong restrictions on the acceptability of renormalizable models <sup>2)</sup> based on spontaneous symmetry breaking <sup>3)</sup> which unify weak and electromagnetic interactions <sup>4)</sup>. This paper generalizes the triangle loop anomaly to gravitation. We find that in lowest-order perturbation theory the PCAC relation is modified to

$$\partial_\alpha j_{\alpha 5} = 2mj_5 + e^2 \epsilon_{\alpha\beta\gamma\delta} F_{\alpha\beta} F_{\gamma\delta} / 16\pi^2 + \epsilon_{\kappa\lambda\mu\nu} R_{\kappa\lambda\rho\sigma} R_{\mu\nu\rho\sigma} / 768\pi^2 \quad (\text{A})$$

where the corrections on the right are associated with infinite-mass regulator loops.

Since the Riemann tensor  $R_{\alpha\beta\gamma\delta}$  plays the analogous role in gravity theory to the electromagnetic tensor  $F_{\alpha\beta}$ , it is interesting that the form of the gravitational anomaly for  $\pi^0 \rightarrow 2g$  parallels the form for the electromagnetic anomaly  $\pi^0 \rightarrow 2\gamma$ . An important problem, however, remains. Are there also anomalous terms in the  $\pi^0 \rightarrow 3g$ ,  $\pi^0 \rightarrow 4g$ , ... processes of a form such that they all sum together to make the expression  $\epsilon RR$  manifestly generally covariant? If this does not happen, would there be a disaster for gravity theory - e.g. emergence of a non-zero graviton self-mass or eventually a breakdown of the equivalence principle (i.e. a clash between the predictions of the theory and the Eötvös experiment)? If this happened, the existence of compensating leptons to remove the anomaly would become a necessity, not just for reasons of renormalizability, but for deeper physical reasons connected with the equivalence principle.

Unfortunately the work presented in this paper does not include  $\pi^0 \rightarrow 3g$ ,  $\pi^0 \rightarrow 4g$ , or higher processes, and we cannot make any definitive statement about the question raised above, though a few discussion remarks are presented in the last section. We do, however, consider another problem which concerns the general attitude about the interpretation of the anomalies.

While one cannot dispute the truth of this relation (A) - it is an algebraic identity once the  $j_{\alpha 5}$  and  $j_5$  elements are given <sup>5)</sup> - one can still question the interpretations that have been given to it. To take a famous example, the photon self-energy  $\Pi_{\alpha\beta}$  when calculated in lowest-order perturbation theory shows up a quadratic self-mass  $\Lambda^2 \eta_{\alpha\beta}$  which is just as anomalous and quite intolerable in view of the fact that a miraculous cancellation between bare and self mass is called for with little sign of gauge invariance at the level of the bare Lagrangian. Of course the satisfactory way out of the dilemma is well known: one must regularize the self-energy gauge invariantly <sup>6)</sup> from  $\Pi$  to the correct value  $\overline{\Pi}$  which then becomes divergenceless. But one must apply this procedure everywhere else for consistency. We wish to argue that the so-called anomalies under discussion possess the same status in basic theory as the self-mass of the photon, no more and no less. They represent shortcomings of the conventional calculations method and do not represent a breakdown of basic things like gauge invariance or W-T identities.

The plan of the paper is as follows. We illustrate the gauge-invariant method of regularization for vacuum charge and mass polarization in Sec.III after evolving the Feynman rules of calculation for the electron in Sec.II; we also show, so far as two-point functions are concerned, that such regularization is essential to establish the Borchers' equivalence between Lagrangians involving electron fields connected by a chiral transformation. In Sec.IV we carry the argument over the famous VVA triangle graph where the anomaly is connected with the regulation of the element <sup>1)</sup>  $\langle 2mj_5 \rangle$  and one is able to assert that the "correct" elements do satisfy PCAC in the form  $\langle \partial_\alpha \bar{j}_{\alpha 5} \rangle_7 = \langle 2mj_5 \rangle$ . This is similar to a point of view argued also by Hagen <sup>7)</sup>. Our contribution is to show that precisely the same viewpoint is necessary in order that the axiomatic Borchers' equivalence theorems <sup>8)</sup> for chirally related Lagrangians continue to hold for the theory in question. Lastly, we deal with the gravity analogue in Sec.V and once again find that the  $\tilde{R}\tilde{R}$  anomaly can be incorporated with  $j_5$  into the correct pseudoscalar element  $\overline{2mj_5}$  in accordance with Borchers' equivalence theorem.

There are two physical consequences when one adopts this point of view. First, one is no longer able to calculate the  $\pi^0$  lifetime from smooth PCAC <sup>9)</sup> because one is asserting that the perturbation result for the fermion loop is wrong and requires a regularization. Second, the problem of non-renormalizability remains, regardless of one's interpretation. For spontaneously broken gauge models of weak interactions one is led to consider compensation mechanisms <sup>10)</sup> to guarantee renormalizability. For gravity, with its intrinsic non-polynomiality, compensation may not be necessary as finiteness can be achieved in other ways <sup>11)</sup>. However, here the demand for general covariance may lead to the same conclusion, that new leptons are indeed needed, as discussed before.

## II. FEYNMAN RULES IN A SPINOR MODEL

Anomalous Ward-Takahashi identities <sup>12)</sup> are associated with fermion loops <sup>1)</sup>. As a necessary preliminary to their study we shall set up the Lagrangian framework and ensuing Feynman rules for a charged massive fermion field which therefore undergoes electromagnetic and gravitational interactions. Later we shall describe the modifications needed for dealing with neutrinos and further pseudoscalar-pseudovector interactions.

Begin with a world of photons, gravitons and electrons, where the Lagrangian is known to be <sup>11)</sup>

$$\begin{aligned} \mathcal{L}(A, g, \psi) | -g |^{\frac{1}{2}} = & -\frac{1}{4} g^{\kappa\lambda} g^{\mu\nu} F_{\kappa\mu} F_{\lambda\nu} + \frac{1}{8} f^{-2} g^{\mu\nu} ( \Gamma_{\mu\kappa}^{\lambda} \Gamma_{\nu\lambda}^{\kappa} - \Gamma_{\mu\nu}^{\lambda} \Gamma_{\kappa\lambda}^{\nu} ) \\ & + | -g |^{-w} L^{\mu\nu} \bar{\psi} \gamma_{\mu} ( \frac{1}{2} i \vec{\sigma}_{\mu} + e A_{\mu} ) \psi \\ & - | -g |^{-w} \bar{\psi} ( m + \frac{1}{4} i \epsilon^{k\ell mn} B_{k\ell m} \gamma_n \gamma_5 ) \psi \end{aligned} \quad (1)$$

w signifying the weight of the fermion field. As usual,

$$\begin{aligned} F_{\mu\nu} &= \partial_{\mu} A_{\nu} - \partial_{\nu} A_{\mu} \\ \Gamma_{\mu\nu}^{\kappa} &= g^{\kappa\lambda} \{ \mu\nu, \lambda \} = \frac{1}{2} g^{\kappa\lambda} ( \partial_{\nu} g_{\mu\lambda} + \partial_{\mu} g_{\nu\lambda} - \partial_{\lambda} g_{\mu\nu} ) \\ B_{\ell mn} &= L_{\ell}^{\lambda} B_{\lambda mn} = L_{\ell}^{\lambda} L_m^{\mu} ( L_{n\nu}^{\nu} \Gamma_{\lambda\mu}^{\nu} - \partial_{\lambda} L_{n\mu} ) \end{aligned} \quad (2)$$



and  $B$  and  $\Gamma$  are the spinorial and vectorial connections needed for covariant differentiation. The symmetric tetrad field  $L_m^\mu$  is defined by  $L_m^\mu L_n^\nu \eta^{mn} = g^{\mu\nu}$ ,  $L_{m\mu} = g_{\mu\nu} L_m^\nu$ ,  $L^{n\mu} = \eta^{mn} L_m^\mu$ , etc.,  $\eta$  being the Minkowski metric (latin indices) and  $g$  the space-time metric (greek indices).

Because we shall be setting up a perturbation expansion in powers of the fine-structure constant  $\alpha = e^2/4\pi$  and the Newtonian gravitational constant  $G = f^2/8\pi$ , it will be necessary to expand the metric about the local Minkowski value. The question of how to choose a suitable interpolating graviton field then arises. Now it is known that the awkward factors involving determinant of  $g$  can be absorbed for the pure gravitational Lagrangian when the Lagrangian is expressed in terms of the density<sup>13)</sup>

$$\tilde{g}^{\mu\nu} = g^{\mu\nu} |g|^{-1/2} = \eta^{mn} L_m^\mu L_n^\nu |L|^{-1} \quad (3)$$

Clearly then, if we attach weight  $w = -\frac{1}{4}$  to the fermion field, the same can be done for the kinetic part of the graviton-spinor Lagrangian which reads:

$$\bar{\psi} \left[ \tilde{L}^{n\mu} \gamma_n \left( \frac{1}{2} i \overleftrightarrow{\partial}_\mu + e A_\mu \right) - \frac{1}{4} \epsilon^{k\ell mn} B_{k\ell m} \gamma_n \gamma_5 - m \left[ -\tilde{L} \right]^{1/2} \right] \psi \quad (4)$$

where  $\tilde{g}^{\mu\nu} = \eta^{mn} \tilde{L}_m^\mu \tilde{L}_n^\nu$ ,  $\tilde{L}_m^\mu = L_m^\mu |L|^{-1/2}$ . In this form the non-poly-nomiality is carried by the photon-graviton interaction Lagrangian  $\frac{1}{4} \tilde{g}^{\kappa\lambda} \tilde{g}^{\mu\nu} F_{\kappa\mu} F_{\lambda\nu} |-\tilde{g}|^{-1/2}$  and the mass and spin terms. Define the physical graviton field  $h_n$  through the relation

$$\tilde{L}_n^\mu = \delta_n^\mu + f h_n^\mu \quad (5)$$

The substitution of (5) into (1) will yield an expansion of  $\mathcal{L}$  to any desired power of  $fh$  — in fact for our discussion of the simplest anomalies the order  $f^2 h^2$  will prove sufficient. In this linearized version, all summations, etc., have meaning relative to the Minkowski

