International Atomic Energy Agency
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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

SPECTRUM-GENERATING ALGEBRAS AND CANONICAL REALIZATIONS †

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ABSTRACT

A general formalism for the embedding of operators into canonical realizations of Lie algebras is applied to Hamiltonians and to so(2,1). A fairly complete list of potentials with so(2,1) as spectrum-generating algebra is derived.

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July 1972

† To be submitted for publication.
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1. INTRODUCTION

The spectrum, the eigenfunctions and the matrix elements of a linear operator $A$ with domain $\mathcal{D}_A$ in a Hilbert space $\mathcal{H}$ can be determined by pure algebraic methods if there exists a Lie-algebra $G$ with a representation $D(G)$ on $\mathcal{D}_A$ such that $A$ can be identified with a linear combination of the generators $D(g_i)$, $i = 1, \ldots, m$, of $D(G)$.

For applications to physically interesting operators, e.g. quantum-mechanical observables, it is reasonable to consider only those $D(G)$ which are canonical realizations $\sigma(G)$ of $G$. These are representations with generators $\sigma(g_i)$ given as functions of momentum $P_j$ and position $Q_j$ operators, which form together with the mass operator $C$, an integrable irreducible representation $\tilde{H}_n$ of the abstract Heisenberg algebra $H_n$

$$[P_j, Q_k] = \delta_{jk}, \quad [P_k, P_j] = [Q_k, Q_j] = 0, \quad j, k = 1, \ldots, n$$

and which are given up to unitary equivalence in the Hilbert space $L^2(\mathbb{R}^n, dx^n)$ by

$$P_j = -i\partial_j, \quad Q_j = x_j, \quad C = -im \quad .$$

Strictly speaking, $\sigma(G)$ is obtained through an isomorphism $\sigma$ mapping $G$ into a suitably defined function space $F_n$ over $2n$ partly non-commuting variables $P_j$, $Q_j$. $F_n$ is equipped with a Lie bracket through the operator product in $L^2$. To simplify its calculation we choose for $F_n$ a tensor product of two function spaces $W_n$, $V_n$ defined over the abelian subalgebras $P_j$ and $Q_j$, $j = 1, \ldots, n$, respectively. To avoid functions like $(P_j)^{-1}$ we use for $W_n$ a polynomial space. This construction yields for $n = 1$ ($P = -id/dx$)

$$F^0_1 = \left\{ f(P, x) \mid f \in F_1^x, \ r \ \text{integer} \right\}$$

$\sigma(G)$ is a representation of $G$ in $L^2$; its integrability depends on $\sigma$. 

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with

\[ F_1^r = \{ f(P,x) \mid f = \sum_{i=1}^{r} v_i(x) P^i, v_i \in V_1 \} . \]

If a linear operator \( A \) can be written as

\[ A = \sum_{i=1}^{m} \alpha_i \sigma(g_i) \in F_n, \alpha_i \text{ real or complex}, \]

then \( \sigma(G) \) is called a canonical \( F_n \) embedding of \( A \) in \( G \), and it yields a method for a discussion of differential operators \(^2\) and for an algebraic formulation of non-relativistic and relativistic quantum systems \(^3\)-\(^7\). In this context it is interesting to calculate the set \( \{ G, F_n \} \) of all Lie algebras \( G \) with canonical \( F_n \) realization and the set of operators \( \{ A \} \) with a canonical \( F_n \) embedding. To carry out this programme in its full generality does not seem feasible; some restrictions are necessary.

We assume that \( G \) is given, and we take \( G = so(2,1) \) with commutation relations in its standard basis

\[(I) \quad [g_1, g_2] = -ig_3, \quad (II) \quad [g_2, g_3] = ig_1, \quad (III) \quad [g_3, g_1] = ig_2.\]

Furthermore, we discuss only \( F_n^G \) embeddings of the following class of Hamiltonians:

\[ H_1^2 = \{ H \mid H = aP^2 + v_1(x)P + v_2(x) \mid a \neq 0, \text{ real}; v_1(x) \in V_1 \} , \]

i.e., we calculate those Hamiltonians in \( \{ A \} \)

\[ \{ A \} = \{ A \mid A = \sum_{i=1}^{3} v_i \sigma(g_i) \mid \sigma(g_i) \in \sigma(so(2,1)) \subseteq F_1 \}

which can also be found in \( H_1^2 \). A canonical embedding of a Hamiltonian is referred to as spectrum-generating algebra.
Our main result is Theorem 1. To prove it we split \( H_1^2 \) into equivalence classes \( Y(H_1^2) \) each of which contains a representative element of the form \( \hat{H} = \frac{1}{3} \hat{P}^2 + V(x) \) (Sec.2.1). Then we find (Sec.2.2) all \( \hat{H} \in \{ \hat{A} \} \), i.e. all potentials \( V(x) \), which can be embedded into \( so(2,1) \) up to equivalence. Our solution is complete, except the "light-cone" case

\[
H = \sum_{i=1}^{3} \sigma_1 \sigma(G_1), \sigma(G_1) \in \sigma(so(2,1)) \text{ with } \sigma_1^2 + \sigma_2^2 = \sigma_3^2
\]

which is discussed in Appx.A.2.

2. A SPECIAL EMBEDDING PROBLEM

2.1 Equivalence classes of realizations

To simplify the calculations we introduce equivalence classes of canonical realizations of \( G \) in \( F_n \). Consider for a given \( F_n \) and \( \sigma(G) \subset F_n \) the set

\[
N_n(\sigma(G)) = \{ f | f, f^{-1}, f^{-1} \sigma(G)f \in F_n, \forall g \in G \}.
\]

Then with \( \sigma(G) \), also \( f^{-1} \sigma(G)f = \sigma'(G) \) is a \( F_n \) realization of \( G \). If \( N_n \) is closed under multiplication, \( \sigma \) and \( \sigma' \) are called equivalent; and the set \( \{ \sigma \} \) of canonical \( F_n \) realizations of \( G \) decomposes into equivalence classes

\[
Y(\sigma(G)) = \{ \sigma'(G)| \sigma'(G) = f^{-1} \sigma(G)f, f \in N_n(\sigma(G)) \}.
\]

As we have chosen \( F_1^0 \) for \( F_1 \), the set \( N_1 \) is contained in \( V_1 \) and is independent of \( \sigma(G) \). The mapping \( \tau(f) = v^{-1}fv, v \in N_1 \), sends \( H_1^2 \) into itself and \( H_1^2 \) can be split into equivalence classes in respect to \( N_1 \).

Hence it is necessary to determine canonical \( F_1^0 \) embeddings in \( so(2,1) \) only for one representative element \( \hat{H} \) in each equivalence class \( Y(H_1^2) \) of \( H_1^2 \).
Choose \( v = 2v_1 \), put for technical reasons \( a = -1/\hbar \), and we find as representative Hamiltonian

\[
\hat{H} = -\frac{\hbar^2}{4} P^2 + V(x).
\]

2.2. Potentials with \( \text{so}(2,1) \) as spectrum-generating algebra

The problem is now to calculate the potentials \( V(x) \) for which an embedding exists. The result is presented in Lemmas 1, 2 and 3 and is summarized in Theorem 1. The proof of Lemma 1 is given in the appendix, Lemma 2 is obvious and Lemma 3 can be verified using the same methods as in Ref. 4.

Lemma 1:

Let \( g_1, g_2, g_3 \) be a standard basis of \( \text{so}(2,1) \). Then every isomorphism \( \sigma \) mapping \( \text{so}(2,1) \) into \( F_1 \) with \( \sigma(g_1) = \hat{H} \) or \( \sigma(g_2) = \hat{H} \) or \( \sigma(g_3) = \hat{H} \) is an isomorphism into the subspace \( H_1^2 \) of \( F_1^0 \).

Lemma 2:

Let \( g_1', g_2', g_3' \) be a standard basis of \( \text{so}(2,1) \) and let \( M \) be an operator with

\[
\sum_{i=1}^{3} w_i \sigma(g_{1}) = M, \quad w_i \text{ real, } w_1^2 + w_2^2 \neq w_3^2.
\]

Then there is a transformation into another standard basis such that \( \sigma(g_1) = k \cdot M \) or \( \sigma(g_3) = k \cdot M \), \( k \) real.
Lemma 3:

Any isomorphism $\sigma$ mapping $\mathfrak{so}(2,1)$ into $\mathbb{H}$ with $\sigma(g_3) = \hat{H}$ (case I) or $\sigma(g_1) = \hat{H}$ (case II) is determined by three (complex) numbers $a_1$, $b_2$, $\lambda$ (case I) or $\alpha_3$, $b_2$, $\lambda$ (case II), respectively. The generators $\sigma(g_i)$ are:

1. For case I: $\epsilon_1 = 1$, $\epsilon_3 = -1$; $\nu = 1$, $\mu = 3$ for case I; $\nu = 3$, $\mu = 1$ for case II.

$$
\begin{align*}
\sigma(g_1) &= a^v (d/dx)^2 + 2i\epsilon_3 a^x - \frac{3}{2}\epsilon_3 a^x - 2a^v c - \epsilon_5 \gamma_v, \\
\sigma(g_2) &= a^v (d/dx)^2 + 2(\epsilon_3 a^x - 2a^v c - \epsilon_5 \gamma_v), \\
\sigma(g_3) &= -\frac{1}{4} (d/dx)^2 + c.
\end{align*}
$$

The constants are related by

$$
\begin{align*}
a_1^2 + \epsilon_3 a_1^2 &= \frac{1}{16}, \\
a_2 b_2 + \epsilon_3 a_2 b_2 &= 0, \\
\gamma_v &= \frac{1}{2} a_2 + 4a_2 (b_2^2 + \epsilon_3 b_2^2), \\
\gamma_2 &= \frac{1}{2} a_2^2 - \epsilon_3 a_2 (b_2^2 + \epsilon_3 b_2^2).
\end{align*}
$$

The generators $\sigma(g_i)$ are symmetric on a dense set $D_0$ in $L^2$ for real $a_2$, $a_3$, and imaginary $b_2$, $b_3$.

Theorem 1:

Let $g_1$, $g_2$, $g_3$ be a standard basis of $\mathfrak{so}(2,1)$. Let $\sigma$ be an isomorphism of $\mathfrak{so}(2,1)$ into $\mathbb{F}_1$ and let $w_1$, $w_2$, $w_3$ be real numbers such that

$$
w_1^2 + w_2^2 \neq w_3^2 \quad \text{and} \quad \sum_{i=1}^3 w_i \sigma(g_i') = \hat{H} \epsilon_1^2
$$

holds. Then
1. \( \sigma \) is an isomorphism of so(2,1) into \( H^2 \subset F^1 \).

2. All Hamiltonians equivalent to

\[ \hat{H} = a(d/dx)^2 + b(x-d)^2 + c(x-d)^{-2}, \quad a \neq 0; \quad a, b, c, d \text{ real} \]

have a canonical \( F^\infty \) embedding in so(2,1).

We remark that the representations \( \sigma(\text{so}(2,1)) \) obtained in Lemma 3 are integrable if the generators are symmetric on \( \mathcal{D}_0 \) and if \( \lambda > 0 \) holds. This can be shown through a discussion of its Nelson operator \( \hat{N} \):

\[ \Delta = \sum_{i=1}^{3} \sigma(g_i)^2 = 2\sigma(g_3)^2 - C_0 \]

which is essentially self-adjoint on \( \mathcal{D}_0 \). The Casimir operator of \( \sigma(\text{so}(2,1)) \) is denoted by \( C_0 \) and \( C_0 = \lambda - 3/16 \) holds.

For the light-cone case \( \omega_1^2 + \omega_2^2 = \omega_3^2 \) the theorem does not apply and other types of operators or potentials appear (see Appx. A.2), e.g. the potential of a constant force. A complete classification of all potentials in this case is not known.

The physically most interesting problem of embedding Hamiltonians in \( F^3_3 \) can be treated along the same lines, but unless restrictive conditions are imposed on the Hamiltonians \( 3 \) the necessary computational work becomes difficult. The same holds for the construction of operator embeddings via canonical realizations of higher-dimensional Lie algebras.

**ACKNOWLEDGEMENTS**

It is a pleasure to thank Dr. J. Hennig for interesting discussions. We are grateful to Prof. Abdus Salam, the International Atomic Energy Agency and UNESCO for hospitality at the International Centre for Theoretical Physics, Trieste. One of us (BP) wants to thank the Vermittlungsstelle für Deutsche Wissenschaftler im Ausland for partial financial support.

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APPENDIX

A.1 Proof of Lemma 1

We consider the case $\sigma(g_3) = \hat{H}$; the calculation for the other cases is analogous. The generators $\sigma(g_i)$ are of the form

$$\sigma(g_i) = \sum_{j=0}^{r} a_{ij}^r(x) \frac{d}{dx}^{r-j}, \quad i = 1, 2$$

$$\sigma(g_3) = -\frac{1}{4} \left( \frac{d}{dx} \right)^2 + V(x).$$

The commutation relations are given in (I), (II), (III).

We start with $r = 3$ and assume $a_{10}^3(x) \neq 0, \quad i = 1, 2$. Apply both sides of (I) on a function $f(x)$ taken from $C^0(R)$ and compare the coefficients for $f^{(j)} = (d/dx)^j f(x), \quad j = 5, \ldots, 0$. The resulting equations are denoted by $(m)$, with $m = 6 - j$. Similar equations follow from (II) and (III) denoted by $(m')$ and $(m'')$, respectively; $m', \quad m'' = 1, \ldots, 5$. All these equations can easily be computed and will not be given here. They yield a system of first-order differential equations for $a_{ij}^r(x)$ (constants of integration $a_{ij}^r$).

Eqs. $(1')-(3'), (4'$); $(1'')-(3''), (4''$) yield (the upper index 3 is omitted) $a_{10}, \ldots, a_{12}^3, a_{13}^3, a_{20}, \ldots, a_{22}^3, a_{23}^3$ respectively. It is easy to check that (1) is automatically fulfilled. Eqs. $(2)-(4)$ give relations between the $\alpha_{ij}^r$, e.g. $\alpha_{10}^1 = \pm i \alpha_{20}^1$ is derived from (2). But (5) and (5'') are both differential equations for $V(x)$ having different solutions. So the commutation relations cannot be valid except for $a_{10}^r = a_{20}^r = 0$ and our assertion is proven for $r = 3$. The case $r = 4$ can be handled in the same way and with the corresponding result.

Suppose now $r > 5$ and $a_{10}^r(x) \neq 0, \quad i = 1, 2$. Commutator (II) leads to a recursion formula for $a_{1j}^r (B_j^r$ are binomial coefficients; in $a_{1j}^r$ the index $r$ is omitted; $a_{1j}^r = 0$ for $j < 0$):

$$a_{1j}^{(1)} = 0$$

$$a_{1j}^{(1)} = -2a_{2(j-1)}^{(1)} - \frac{1}{2} a_{1(j-1)}^{(2)} - 2 \sum_{k=0}^{j-2} B_k^{r} a_{1k}^r V^{(j-k-1)}.$$
Commutator (III) implies
\[
\begin{align*}
a_{20}^{(1)} &= 0 , \\
a_{2j}^{(1)} &= 2ia_{1(j-1)} - \frac{1}{2}a_{2(j-1)} - 2 \sum_{k=0}^{j-2} \beta_k a_{2k} v^{(j-k-1)} .
\end{align*}
\]

Commutator (I) gives 2r equations. They imply
\[
\alpha_{1j} = +ia_{2j} \quad \text{or} \quad \alpha_{1j} = -ia_{2j} , \quad j = 0,1,\ldots,r-1
\]
which can be proven by induction. From the general form of \( a_{1j}^{(1)} \) and \( a_{2j}^{(1)} \), one derives
\[
\begin{align*}
a_{1j}^{(1)} &= +ia_{2j}^{(1)} \quad \text{or} \quad a_{1j}^{(1)} = -ia_{2j}^{(1)} , \quad j = 0,1,\ldots,r-1,r
\end{align*}
\]
Thus we have
\[
a_{1j} = \pm ia_{2j} , \quad j = 0,1,\ldots,r-1
\]
and with complex \( \delta \)
\[
a_{1r} = \pm ia_{2r} + \delta
\]
and (I) cannot be valid except for \( a_{10} = a_{20} = 0 \). This completes the proof.

A.2 The case \( v_1^2 + v_2^2 = v_3^2 \)

We consider only the case \( r = 4 \). After a \( \text{so}(2,1) \) basis transformation such that \( \hat{H} = k^{-1} (\sigma(g_1) + \sigma(g_3)) \) (this transformation exists because of \( v_1^2 + v_2^2 = v_3^2 \)) the generators are
\[
\begin{align*}
\sigma(g_1) &= \sum_{j=1}^{4} a_{1j}(x) (d/dx)^j , \quad i = 1,2,3 \\
\sigma(g_1) + \sigma(g_3) &= -k \cdot (d/dx)^2 + V(x) \quad (*)
\end{align*}
\]
The commutation relations yield
\[
\begin{align*}
a_{1h} &= a_{1h} \\
a_{13} &= a_{13} \\
a_{12} &= \frac{2}{k} a_{1h} v + \frac{i}{2k} a_{23} x + a_{12}
\end{align*}
\]
\[ a_{11} = \frac{2}{k} \alpha_{14} V^{(1)} + \frac{3}{2k} \alpha_{13} V + \frac{i}{2k} \alpha_{22} x + \alpha_{11} \]

\[ a_{10}^{(1)} = \frac{1}{k} (\alpha_{14} V^{(3)} + \frac{3}{4} \alpha_{13} V^{(2)} + \alpha_{12} V^{(1)} + \frac{2}{k} \alpha_{14} VV^{(1)} + \]

\[ + \frac{i}{2k} \alpha_{23} x V^{(1)} + \frac{3i}{4k} \alpha_{23} V + \frac{1}{4} x + \frac{i}{2} \alpha_{21}) \]

\[ a_{24} = 0 \]

\[ a_{23} = a_{23} \]

\[ a_{22} = a_{22} \]

\[ a_{21} = \frac{3}{2k} \alpha_{23} V - \frac{1}{2} x + \alpha_{21} \]

\[ a_{20} = \frac{3}{4k} \alpha_{23} V^{(1)} + \frac{1}{k} \alpha_{22} V + \alpha_{20} \]

\[ a_{3j} \] can be obtained from the \[ a_{1j} \] with (*).

The \[ a_{ij} \] fulfill the relations

\[ \alpha_{14} = -(2k)^{-1} \alpha_{23} \]

\[ \alpha_{13} = -k^{-1} \alpha_{23} \alpha_{22} \]

\[ \alpha_{12} = \frac{k}{2} - (2k)^{-1} \alpha_{22} \]

\[ \alpha_{11} = i (2k)^{-1} \alpha_{23} \]

\[ \alpha_{10} = i (4k)^{-1} \alpha_{22} \]

\[ \alpha_{21} = 0 \]

\[ \alpha_{20} = -\frac{i}{4} \]

\[ \alpha_{22} \] and \[ \alpha_{23} \] are arbitrary constants.

\[ V \] is a solution of the non-linear differential equation

\[ \frac{1}{4} \alpha_{23} V^{(3)} + \frac{3}{2k} \alpha_{23} VV^{(1)} - \frac{i}{2} x V^{(1)} = i \]

\[ V \]

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with solutions, e.g.,

\[ V(x) = -2k x^{-2} \]

or

\[ V(x) = i k (\alpha_{23})^{-1} x \]

Hence in this case \( r = 4 \) potentials appear which have not been obtained from Theorem 1. The type of differential equation for \( V \) depends on \( r \). One can check that for \( r = 5,6,7 \) the equation has exactly the same form as above; for \( r = 8,9,10,11 \) it reads

\[
\alpha_{25} \left[ \frac{1}{16} V^{(5)} + \frac{5}{8k} V^{(3)} + \frac{5}{4k} V^{(1)}V^{(2)} + \frac{15}{8k^2} V^{2}V^{(1)} \right] + \\
\alpha_{23} \left[ \frac{1}{4} V^{(3)} + \frac{3}{2k} V^{(1)} \right] + \left[ -\frac{i}{2} x + \alpha_{21} \right] V^{(1)} = i V
\]
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