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CP-VIOLATING GAUGE INTERACTIONS *

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ABSTRACT

Maximally CP-violating weak interactions mediated through intermediate bosons, similar to those suggested by Okubo, can be constructed through an extension of the gauge principle.

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1. Most models of weak interaction Lagrangians contain two pieces: one, CP-conserving; a second, CP-violating with a coupling 10^{-3} times smaller. An attractive exception is Okubo's model 1) *) which (following the ideas originally suggested by Nishijima 2)) consists of but one single purely CP-violating piece with a coupling constant $f \approx 10^{-3}$. The second-order (f^2) effects of this interaction give the normal CP-conserving weak decays, while the third order (f^3) gives rise to CP-violating decays like $K_L \to 2\pi$. Conceptually the appeal of the model lies in what Okubo calls its "maximality" (of CP-violation) and the economy of couplings introduced.

Now, in recent years, the idea of obtaining $\mathcal{L}_{\text{weak}}$ by exploiting "the gauge principle" has gained currency 3). The gauge principle automatically yields: a) vector-type interactions, b) universality, c) restrictions on the possible interaction types and, finally, d) renormalizability of the theory through use of Higgs-Kibble-t'Hooft mechanism. It appears something of a challenge to derive an Okubo-like Lagrangian using the gauge principle. In this note we present a first attempt, which secures points a) to c) above, but not d). To get the basic idea across we have couched the exposition in $O(3) \approx SU(2)$ language. Extensions to SU(3) or SU(4) or still higher groups are immediate.

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*) Okubo gives two models, based on SU(3) symmetry; one works with an octet of W's and the other with a (complex) triplet of W's. R.E. Marshak and his collaborators (Proceedings of the CERN Topical Conference on Weak Interactions, 1969 and Nucl. Phys. 11, 253 (1969)) have particularly emphasised the merits of the triplet W-model. Whereas the octet model presents no difficulties from a gauge point of view, the triplet model does not appear to be so amenable.

2. It is clear that no gauging of conventional C- (or CP-) conserving kinetic energy terms is likely to give rise to C- (or CP-) violating interactions.

Following a remark of Gregor Wentzel (Rochester Conference, 1956), our basic idea is to make use of surface terms in the prototype action functional. These terms have no effect on the dynamics before gauging, but provided that they define a fixed direction in the symmetry-space, we shall find that the minimal gauge ansatz gives rise to interactions which inevitably violate C (or CP).

As an example, consider

$$\mathcal{L}_{\text{surface}} = f\, \partial_\mu (\phi^k J^k_\mu) \tag{1}$$

where ϕ^k denotes a triplet of (normal C) scalar fields with charges ±1,0, and J^k_μ are the weak currents. To fix ideas one may take the Cabibbo currents,

$$J^\pm_\mu = I^\pm_\mu \cos\theta + V^\pm_\mu \sin\theta + \ell^\pm_\mu \quad . \tag{2}$$

J^3_μ is arbitrary;the only restriction we propose is the conventional one, that it does not contain <u>both</u> $|\Delta Y| \neq 0$ pieces <u>and</u> leptons.

Clearly if the C- (or CP-) behaviour of ϕ^k and J^k_μ is normal, $\mathcal{L}_{\text{surface}}$ is odd under C (or CP). To emphasise this further, we introduce with Okubo a <u>strongly</u> interacting gauge field W^k_μ in that O(3) space which treats ϕ^k as a triplet. All other fields (including J^k) are singlets in O(3). The gauge-principle gives

$$\mathcal{L} = -\frac{1}{4}(W^k_{\mu\nu})^2 + \frac{1}{2}(\nabla_\mu \phi^k)^2 + fg\, \varepsilon^{k\ell m} W^k_\mu \phi^\ell J^m_\mu \tag{3}$$

where

$$W^k_{\mu\nu} = \partial_\mu W^k_\nu - \partial_\nu W^k_\mu + g\, \varepsilon^{k\ell m} W^\ell_\mu W^m_\nu$$
$$\nabla_\mu \phi^k = \partial_\mu \phi^k + g\, \varepsilon^{k\ell m} W^\ell_\mu \phi^m$$

and $g \approx 1$.

Assuming that f in appropriate units (see below) is small, the last term in (3) which has arisen from gauging $\mathcal{L}_{\text{surface}}$ is the weak Lagrangian which, relative to the first two strong terms, violates CP.

To complete the Lagrangian, we add a mass term for W-mesons, $(M_0^2/2)(W_\mu^k)^2$ and mass-like terms for the ϕ-fields, which we choose of the form:

$$- \left[\frac{m_0^2}{8c^2} (\phi^k \phi^k - c^2)^2\right] - \left[\frac{m_1^2}{2} (\phi \times n)^2\right] \quad . \qquad (4)$$

Here $\underline{n}$ is the vector (0,0,1) which picks out the third axis in O(3). The particular form (4) is designed to favour the emergence of a non-vanishing expectation value, $\langle\phi^3\rangle = c$, while at the same time avoiding Goldstone particles through the explicit symmetry-breaking term within the second bracket in (4). Note that $\langle\phi^3\rangle$ can be expected to be a "strong magnitude" and large.[*]

The effective $\mathcal{L}_{\text{weak}}$ reads (on substituting $\phi^k = cn^k + \varphi^k$)

$$\mathcal{L}_{\text{weak}} = 2ifgc(J^+W^- - J^-W^+) + fg(\varepsilon^{k\ell m} W_\mu^k \varphi^\ell J_\mu^m) \quad . \qquad (5)$$

Both terms in (5) violate CP and give rise to similar effects. The first term in (5), and the simpler to calculate with, is of Okubo form. By arranging that c and the masses of W and φ mesons are large, while $fgc \approx 10^{-3}$, one may hope that at small momenta it is necessary only to work with the first term. Calculations with the second term are subject to the uncertainties of divergent closed loop calculations in a non-renormalizable theory. To facilitate such calculations, however, we set down the propagators for the ϕ and W fields. On account of $\langle\phi^3\rangle \neq 0$, these fields mix. One finds (with $M_1^2 = M_0^2 + g^2 c^2$):

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[*] Since W^k and ϕ^k have no other than mutual strong interactions, $\langle\phi^3\rangle$ does not affect "strongly" the rest of physics.

$$\langle T\, \varphi^k \varphi^\ell \rangle = \frac{i}{p^2 - m_0^2}\, n^k n^\ell + \frac{i}{p^2 - m_1^2}\, \frac{M_1^2}{M_0^2}\, (\delta^{k\ell} - n^k n^\ell)$$

$$\langle T\, \varphi^k W_\mu^\ell \rangle = - \frac{p_\mu}{p^2 - m_1^2}\, \frac{\sqrt{M_1^2 - M_0^2}}{M_0^2}\, \varepsilon^{k\ell j}\, n^j$$

$$\langle T\, W_\mu^k W_\nu^\ell \rangle = \frac{i}{p^2 - M_0^2} \left(-\eta_{\mu\nu} + \frac{p_\mu p_\nu}{M_0^2} \right) n^k n^\ell +$$

$$+ \frac{i}{p^2 - M_1^2} \left(-\eta_{\mu\lambda} + \frac{p_\mu p_\nu}{M_1^2} \right) \left(\eta_{\lambda\nu} + \frac{M_1^2 - M_0^2}{M_0^2}\, \frac{p_\lambda p_\nu}{p^2 - m_1^2} \right) (\delta^{k\ell} - n^k n^\ell) \, .$$

As in all situations where Cabibbo-like currents are used, it is necessary to assume octet dominance in order to achieve a dynamical suppression of $|\Delta I| = \frac{3}{2}$ and $\Delta S = 2$ effects. Alternatively, it is also possible to construct elaborate gauge models with neutral currents *) which will do this group-theoretically up to order f^3. We shall not discuss these models here. Our chief purpose in this note was to demonstrate the natural clash of C (or CP) which arises from gauging kinetic-energy versus surface terms.

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*) A direct SU(3) analogue of the first term in (5) would be $\sum_{i = 3,6,8} \lambda_i\, f^{ijk}\, W^j\, J^k$. One can show that Nishijima's theory of CP-violation possesses a Lagrangian equivalent to this in a quark model with quarks interacting strongly with vector and axial vector mesons. Nishijima claims to have shown (Coral Gables Conference 1968, p. 175, W.A. Benjamin Inc., New York) that a dynamical suppression of $\Delta S = 2$ effects actually does take place.

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