IC/71/145

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INTERNATIONAL ATOMIC ENERGY AGENCY

UNITED NATIONS EDUCATIONAL, SCIENTIFIC AND CULTURAL ORGANIZATION

1971 MIRAMARE-TRIESTE

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MIRAMARE - TRIESTE
November 1971

* To be submitted for publication.

ABSTRACT

It is known that the spontaneous violation of a gauge symmetry of the second kind results in the appearance not of Goldstone bosons but, rather, of massive gauge particles. The path-integral quantization of such theories is discussed here in general terms. The primary consideration is that quantities of physical significance, such as matrix elements of the scattering operator or the energy momentum tensor, should be independent of the gauge in which the quantization rules are formulated. In particular, if it is possible to find one gauge in which the theory is unitary and another in which it is renormalizable then the gauge-independent quantities must enjoy both these qualities. These ideas are applied to a simple model with massive Yang-Mills fields and to a model which unifies the weak and electromagnetic interactions of electron-type leptons. Both these models appear to be unitary and renormalizable. The lepton theory is a relatively economical one. It involves five independent parameters: the electron charge and mass, the mass of the charged intermediate vector boson, and the masses of a neutral scalar and a neutral vector boson.

I. INTRODUCTION

One of the major achievements of the last two decades in particle physics has been the discovery of the internal (including chiral) symmetries of hadrons and leptons. Most symmetries are only approximate however and, no matter how much one may disguise it, the concept of broken symmetry is unaesthetic. Fortunately, the aesthetic balance is to some extent restored by the notion that symmetry breaking may be spontaneous [1]. In this view, the universal Lagrangian is supposed to be fully symmetric while the apparent lack of symmetry in the solutions of the equations of motion is ascribed to degeneracy in the physical ground state or, equivalently, to the existence of certain fields in the theory whose ground state expectation values do not vanish when calculated self-consistently.

It is well known that the straightforward implementation of this appealing idea leads to the appearance of massless, scalar, Goldstone excitations [2]. Such unwanted particles will not arise, however, if the underlying lagrangian symmetry is a gauge symmetry of the second kind. It has been shown by Higgs [3] and Kibble [4] that the spontaneous symmetry violation is manifested not in the appearance of massless particles but rather in the acquiring of finite mass by the gauge fields.

Massive vector particles are clearly more acceptable than massless scalars in any model which claims to be realistic. Another reason for favouring the models with gauge symmetries of the second kind is that, in many cases, they can be shown to be renormalizable in the conventional sense [5]. This is in spite of the presence of massive vector particles. Perhaps, in view of the existence of finite, non-polynomial lagrangian theories, renormalizability in the conventional sense is not a criterion which should be decisive in selecting or rejecting Lagrangians. However, conventionally renormalizable Lagrangians still score on one point. The perturbation expansion in each order of the (major) coupling constant is Froissart-bounded. This does not happen with non-polynomial Lagrangians although one may possibly achieve it for physical S-matrix elements after a summation over the major coupling constant.

In this paper, following the recent stimulating work of t'Hooft [6] we examine this problem of renormalizability in the case of a gauge theory of lepton interactions [7,8]. The underlying symmetry U(2) is associated with four gauge fields. Three of these (with charges +1, -1 and 0) acquire mass through the spontaneous breakdown of U(2) to U(1). They are supposed to mediate the weak interactions. The remaining (massless) field is associated with the unbroken symmetry U(1) and is identified with the electromagnetic field. The spontaneous symmetry breakdown serves also to give mass to the charged leptons [9]. This model contains, in addition to the gauge fields and the lepton fields, a doublet of scalar fields whose interactions are set in a form which favours the emergence of a symmetry-breaking solution. The doublet comprises a charged field and a complex neutral field, and the symmetry breaking finds expression in the non-vanishing vacuum expectation value of the neutral component. It turns out that only one massive scalar particle is associated with the doublet of fields. The other components, the Goldstone bosons, are gauge effects which can be transformed away.

The decisive new factor which permits us to discuss renormalizability in this model is the existence of a clear and unambiguous technique for quantizing gauge theories. Now, the view has been often expressed in the past that gauge theories should be renormalizable and that the weak interactions in particular should be governed by a gauge principle. However, attempts to realize this idea always faltered over the problem of giving mass to the gauge particles - a clearly essential feature of any realistic theory. It was conjectured [5] that mass generation by the Higgs-Kibble mechanism would not interfere with renormalizability but the quantization programme could not be formulated with precision. The missing element has now become available in the work of Faddeev and Popov [10]. It is now possible to quantize in a wide variety of gauges and, what is more, to state rules for transforming Green's functions from one gauge to another. In effect, we have an equivalence theorem.

It will turn out, in the examples to be discussed, that different gauges present various advantages in that some property may be apparent in one gauge but not in another. In particular the (formal) unitarity of the physical S-matrix is evident in what we shall call the canonical gauge while its renormalizability is made clear in another, the Landau gauge. If we fully accept the gauge independence of the S-matrix, however, it must be that appearances are deceptive. The S-matrix of this theory must be both unitary <u>and</u> renormalizable.

The paper is planned as follows. In Sec.II we review the Faddeev-Popov quantization technique and present a formal derivation - by means of Feynman path-integrals - of the basic equivalence theorem for gauge theories. Here we discuss briefly the distinction between gauge symmetries of the first and second kind in order to indicate that the spontaneous breakdown of the former does not interfere with the operation of the basic theorem. An illustration of the method is given in Sec.III where a massive Yang-Mills theory is treated. This example will serve, we hope, to make very clear the distinction between first- and second-kind symmetries since, in spite of the spontaneous breakdown which generates mass, there remains a conserved isospin in the theory. It is shown also in Sec.III that the Faddeev-Popov technique coincides with the traditional method of canonical quantization in a suitably chosen gauge. One curious feature of the model is the appearance of a non-polynomial term in the Hamiltonian. The proposed gauge theory of lepton interactions is discussed in some detail in Sec.IV (while the detailed expression for the Lagrangian is given in Appendix I). It will be emphasised that a correct choice of independent parameters must be made in order that finite categories of graphs should be gauge independent. Sec.V is devoted to concluding remarks and speculations concerning the lepton model and possibilities for making it less arbitrary by means of self-consistent techniques for computing some of the coupling parameters. Appendix II is devoted to the derivation of Ward-Takahashi identities.

II. THE FADDEEV-POPOV TECHNIQUE

A general way to quantize theories with a gauge symmetry has emerged only in recent years with the work of Faddeev and Popov[10]. The old methods - for example the Gupta-Bleuler method - are valid for the abelian symmetry of electrodynamics provided a linear gauge condition is adopted. They fail, however, to cope with non-linear gauge conditions (such as Nambu's, $A_\mu^2 =$ constant) or with any of the non-abelian gauge symmetries. In this section we shall recapitulate the new method and discuss some of its more important features including particularly the insight which it gives into questions of gauge dependence in quantized amplitudes.

To pose the general problem, suppose we are asked to quantize a system of fields ϕ whose classical equations of motion are governed by the action functional

$$\mathcal{S}_{\mathcal{L}}(\phi) = \int d_4x \; \mathcal{L}(\phi) \; . \tag{2.1}$$

If the classical Euler-Lagrange equations are not underdetermined, i.e., if a unique solution corresponds to the initial data, then the general solution for the quantized amplitudes can be represented by the Feynman path-integral

$$\langle T \; F(\phi) \rangle = \int (d\phi) \; F(\phi) \; \exp\left[\frac{i}{\hbar} \mathcal{S}_{\mathcal{L}}(\phi)\right] , \tag{2.2}$$

where the functional $F(\phi)$ may, for example, take the form of a simple product,

$$F(\phi) = \phi(x_1) \; \phi(x_2) \cdots \phi(x_n) \tag{2.3}$$

in which case (2.2) gives the Green's functions. The S-matrix, on the other hand, is obtained by substituting

$$F(\phi) = \; : \exp \int dx \; dy \; \phi_{in}(x) \; Z^{-\frac{1}{2}} \; D^{-1}(x-y) \; \phi(y) \; : \tag{2.4}$$

where $D(x-y)$ denotes the renormalized free propagator.

The general solution (2.2) is not appropriate for theories with a gauge symmetry of the second kind. Suppose we have a (pseudo) group of transformations

$$\Omega(x) : \phi \to \phi^{\Omega} \tag{2.5}$$

which leave the action, $\mathcal{S}_{\mathcal{L}}$, invariant. In such cases the Euler-Lagrange equations are underdetermined. In order to obtain a unique classical solution from given initial data it is necessary to impose a gauge condition, i.e. a supplementary set of equations (whose number coincides with the number of group parameters Ω)

$$h(\phi) = 0 \ . \tag{2.6}$$

Such conditions can always be obtained, along with the original (and deficient) Euler-Lagrange equations, by varying a suitably modified action functional. It has not been usual to follow this course in classical work because it complicates the equations a good deal and offers no compensating advantage. However, for the quantized case it is essential. This is because the quantization programme demands - for its internal consistency - that the complete set of equations of motion be derivable from an action principle. This is true both of the standard canonical quantization procedure and of the Feynman path-integral procedure. Fortunately, a general method for determining the supplementary terms in the action has been given by Faddeev and Popov.

The procedure is as follows. To the original, invariant part of the action add a supplementary, non-invariant piece $\mathcal{S}_h$ to define the total action

$$\mathcal{S} = \mathcal{S}_{\mathcal{L}} + \mathcal{S}_h \ . \tag{2.7}$$

The supplementary action, which serves to fix the gauge, must satisfy two conditions:

a) it must break all the second-kind gauge symmetries;

b) it must obey the normalization condition

$$1 = \int (d\Omega) \ \exp\left[\frac{i}{\hbar} \ \mathcal{S}_h(\phi^\Omega)\right] \quad , \tag{2.8}$$

which we shall call the Faddeev-Popov (F-P) constraint. The path-integral (2.8) is supposed to extend over those elements $\Omega(x)$ of the pseudogroup which vanish asymptotically. The measure $(d\Omega) = \prod_x d\Omega(x)$ is supposed to be a group invariant. Finally, the F-P constraint must be satisfied identically in ϕ .

To construct a supplementary action which satisfies the requirements a) and b) is straightforward. If we write $\mathcal{S}_h$ as the sum of two pieces,

$$\mathcal{S}_h(\phi) = \mathcal{A}_h(\phi) + W_h(\phi) \quad , \tag{2.9}$$

where $\mathcal{A}_h$ is freely chosen and non-invariant - so as to meet the requirement a) - and $W_h(\phi)$ is invariant, then we can satisfy the requirement b) by re-writing it in the form

$$\exp[-\frac{i}{\hbar} W_h(\phi)] = \int (d\Omega) \exp\left[\frac{i}{\hbar} \mathcal{A}_h(\phi^\Omega)\right] \tag{2.10}$$

which now serves as the definition of W_h . The group invariance of this integral assures the invariance of W_h .

A pseudodynamical interpretation can be given to W_h by observing that the path-integral (2.10) represents a "transition amplitude", viz. the Ω-vacuum transition amplitude. That is, if we regard $\Omega(x)$ as a quantized field whose classical equations of motion are governed by the "action" $\mathcal{A}_h(\phi^\Omega)$, which includes the effects of perturbing external fields ϕ , then the right-hand side of (2.10) is precisely the Feynman representation of the vacuum amplitude. If we represent this amplitude by graphs with virtual Ω-lines then the functional $(-i/\hbar)W_h(\phi)$ is represented by the subset of connected graphs. It is sometimes convenient, therefore, to think of $\Omega(x)$ as a "fictitious" field whose Green's functions represent the propagation of fictitious particles. For the computation of physical ϕ-amplitudes, however, we need to include only the Ω-vacuum amplitudes. In performing such computations it is necessary to take account of one rather subtle point. That is, since the connected part of the Ω-vacuum amplitude represents the functional $(-i/\hbar)W_h(\phi)$, whereas what one needs for the total action is $(+i/\hbar)W_h(\phi)$, one must multiply the amplitude corresponding to any given graph - containing both Ω- and ϕ-lines - by $(-)^N$, where N denotes the number of Ω-connected pieces in it.

The significance of the requirement a) is clear: it serves to render the Euler-Lagrange equations well determined. Requirement b) is less transparent. Its true significance is revealed only when we try to transform a quantized amplitude (a Green's function, say) from one gauge to another. The transformation rule between two gauges - characterized by the supplementary actions $\mathcal{S}_{h_1}$ and $\mathcal{S}_{h_2}$, respectively - is obtained as follows. Consider the path-integral

$$\int (d\phi)(d\Omega)\, F(\phi) \exp\left[\frac{i}{\hbar}\left(\mathcal{S}_{\mathcal{L}}(\phi) + \mathcal{S}_{h_1}(\phi) + \mathcal{S}_{h_2}(\phi^\Omega)\right)\right] \quad . \tag{2.11}$$

If we perform the integral over $\Omega(x)$ keeping $\phi(x)$ fixed, then, according to the F-P constraint (2.8), we are left with the expression

$$\int(d\phi)\ F(\phi)\ \exp\left[\frac{i}{\hbar}\left(\mathcal{S}_{\mathcal{L}}(\phi)+\mathcal{S}_{h_1}(\phi)\right)\right] = \langle T\ F(\phi)\rangle_{h_1} \tag{2.12}$$

which is just the quantized amplitude in the first gauge. Alternatively, we could have integrated first over $\phi(x)$ keeping $\Omega(x)$ fixed. In this case we can first make a change of variable in the ϕ-integration, viz. $\phi \to \phi' = \phi^{\Omega}$. If the measure $(d\phi)$ is gauge invariant - which we shall always require - then, since $\mathcal{S}_{\mathcal{L}}(\phi)$ is also invariant, the integral (2.11) takes the form

$$\int (d\Omega)(d\phi)\ F(\phi^{\Omega})\ \exp\left[\frac{i}{\hbar}\left(\mathcal{S}_{\mathcal{L}}(\phi) + \mathcal{S}_{h_2}(\phi) + \mathcal{S}_{h_1}(\phi^{\Omega})\right)\right]$$

$$= \int (d\Omega)\ \left\langle T\ F(\phi^{\Omega})\ \exp\left[\frac{i}{\hbar}\ \mathcal{S}_{h_1}(\phi^{\Omega})\right]\right\rangle_{h_2} \tag{2.13}$$

(after removing the primes and replacing Ω^{-1} with Ω). On comparing the expressions (2.12) and (2.13) we find the basic transformation rule,

$$\langle T\ F(\phi)\rangle_{h_1} = \int (d\Omega)\ \left\langle T\ F(\phi^{\Omega})\ \exp\left[\frac{i}{\hbar}\ \mathcal{S}_{h_1}(\phi^{\Omega})\right]\right\rangle_{h_2} \tag{2.14}$$

and, similarly, its converse

$$\langle T\ F(\phi)\rangle_{h_2} = \int (d\Omega)\ \left\langle T\ F(\phi^{\Omega})\ \exp\left[\frac{i}{\hbar}\ \mathcal{S}_{h_2}(\phi^{\Omega})\right]\right\rangle_{h_1} \ . \tag{2.14'}$$

Thus, one sees that in general the gauge dependence of Green's functions is quite complicated. However, a great simplification occurs - and this is the real justification of the F-P constraint - when the rules (2.14) and (2.14') are applied to functionals $F(\phi)$ which are gauge invariant in the classical sense. The classical invariance

$$F(\phi^{\Omega}) = F(\phi) \tag{2.15}$$

implies the quantum gauge <u>independence</u>,

$$\langle T\ F(\phi)\rangle_{h_1} = \langle T\ F(\phi)\rangle_{h_2} \tag{2.16}$$

as can be seen by taking the Ω-integration in (2.14) inside the T-bracket and using the F-P constraint. Since the functional (2.14) which defines the physical S-matrix is invariant in the sense (2.15), it follows that the S-matrix is gauge independent. Thus we have the fundamental equivalence theorem of gauge theories:

Those quantized amplitudes which go over in the classical correspondence limit to gauge-invariant functionals are themselves independent of the gauge in which quantization is carried out.

Of course we have not "proved" this theorem since the arguments given above are only formal. There remains the important task of showing that these manipulations with ill-defined (because ultraviolet divergent) path-integrals can be justified through regularization and the incorporation of counterterms. Such justification will no doubt prove very arduous. However, we believe that the great formal simplicity of the arguments used here must signify their fundamental correctness.

The discovery that the "gauge-compensating term" W_h must be included in the action was made by Feynman[11]. His approach differs from that of Faddeev and Popov in being based on the requirement that the S-matrix should be unitary while the latter authors require that it should be gauge independent. It is possible to show, however, that the method of Faddeev and Popov also defines a unitary S-matrix and so is, at least to that extent, equivalent to Feynman's. To demonstrate this one need only find a gauge in which the F-P quantization prescription reduces to the standard canonical quantization. An example of this will be discussed in the next section where, in the so-called "canonical gauge",the compensating term is precisely cancelled when the path-integral is put into canonical form. That is, in the canonical gauge one can write

$$\langle T\,F(\phi)\rangle_{can} = \int (d\phi)\; F(\phi)\; \exp\left[\frac{i}{\hbar}\left\{\mathcal{S}_{\mathcal{L}} + \mathcal{A}_{can} + W_{can}\right\}\right]$$
$$= \int (d\phi\; d\pi)\; F(\phi)\; \exp\left[\frac{i}{\hbar}\int dx\;\left\{\pi\dot{\phi} - H\right\}\right] \tag{2.17}$$

where $H = H(\phi,\pi)$ denotes the classical hamiltonian density: it does not depend on $\hbar$ and contains no remnant of the fictitious particle structure. Now, since the S-matrix is gauge independent when computed by the F-P method, it must coincide with the canonical S-matrix and therefore be, at least formally, unitary.

The canonical action functional is distinguished by its lack of $\hbar$-dependence. In general the total action contains non-classical terms, in W_h, which reflect the fictitious particle structure. An expansion in powers of Planck's constant,

$$W_h(\phi) = \sum_{L=0}^{\infty} \hbar^L \, W_h^L(\phi) \tag{2.18}$$

singles out the parts W_h^L which are represented by fictitious particle vacuum graphs with L loops. The gauges to be used in the following sections are rather special in that only one-loop graphs contribute, i.e. $W_h \sim \hbar$. For these gauges W_h can be given in closed form: it is related to a functional determinant. This simplicity is a feature of Landau-type gauges where the classical gauge condition $h(\phi) = 0$ is incorporated in the action by means of Lagrange multipliers. Explicitly,

$$\begin{aligned} \exp\left[\frac{i}{\hbar} \mathcal{A}_h(\phi)\right] &= \int (dC) \exp\left[\frac{i}{\hbar} \int dx \; Ch(\phi)\right] \\ &= \delta(h(\phi)) \quad . \end{aligned}$$

In this way the quantized fields are constrained to satisfy the condition $h(\phi) = 0$. In Fermi-type gauges, where

$$\mathcal{A}_h(\phi) = \int dx \; \tfrac{1}{2}\big(h(\phi)\big)^2 \quad ,$$

it is possible to show that all values $L \geq 1$ are present in (2.18). One can also invent gauges where all values $L \geq 0$ contribute so that even the classical part of the action is modified. Finally, one can sometimes find a gauge in which W_h vanishes identically[12].

In deriving the transformation rule (2.14) an essential requirement was the invariance of the path-integral over ϕ with respect to those gauge transformations $\Omega : \phi \to \phi^{\Omega}$ which enter the normalization condition (2.8). Not only the measure $(d\phi)$ but also the domain itself must be invariant. Now, if a symmetry is spontaneously broken one expects this to be reflected in a lack of domain invariance. This is certainly the case with non-linear realizations of chiral symmetry where both the action functional and the measure are invariant. One can see that the spontaneous violation of chiral symmetry is associated with a conflict between the chiral transformation law,

$\delta\pi^k = f^{k\ell}(\pi)\varepsilon^\ell$, and the asymptotic condition $\pi^k \to 0$, which characterizes the path-integral domain for a vacuum amplitude. Since $f^{k\ell}(0) \neq 0$, there can be no asymptotic symmetry [13].

In the following sections we shall be treating models in which the gauge symmetry is not asymptotic. In these models there will appear complex scalar fields which are not gauge invariant, $\phi^\Omega \neq \phi$, yet which do not vanish asymptotically but rather approach non-vanishing constant values (their vacuum expectation values). In such models the domain is not asymptotically invariant and the transformations $\phi \to \phi^\Omega$ with constant Ω are certainly not symmetries of the theory. The gauge symmetries of the second kind are symmetries only insofar as they approach the identity asymptotically - and so do not affect the asymptotic states. They do not include the first-kind gauge transformations as a subgroup.

It is fortunate that the integral (2.18), which defines the gauge-compensating functional W_h as the connected part of the fictitious particle vacuum amplitude, involves only such fields Ω as vanish asymptotically. (More precisely, the independent fields which parametrize the group matrix Ω vanish asymptotically so that Ω itself approaches the identity matrix.) Under these transformations the domain of the path-integral over ϕ is indeed invariant in spite of the non-vanishing asymptotic limits which characterize it. The gauge symmetry of the second kind remains a symmetry with dynamical consequences but it must be logically distinguished from the symmetry of the first kind - which governs the classification of physical states - with which it may or may not be associated.

III. A MASSIVE YANG-MILLS THEORY

The quantization technique of Faddeev and Popov was applied by them originally to the example of a pure, massless Yang-Mills field. We give now its application to a simple case where the operation of the Higgs-Kibble mechanism gives mass to the Yang-Mills field[14]. This will serve to illustrate the main features of the method in a simpler context than the lepton theory to which Sec.IV is devoted.

Consider the system of a Yang-Mills field in interaction with a doublet of scalar fields which is characterized by the Lagrangian

$$\mathcal{L} = -\frac{1}{4}(F^k_{\mu\nu})^2 + \nabla_\mu \bar{K}^a \nabla_\mu K_a - \frac{g^2}{8}\left(\frac{m}{M}\right)^2\left(\bar{K}^a K_a - \frac{2M^2}{g^2}\right)^2 \tag{3.1}$$

where

$$F^k_{\mu\nu} = \partial_\mu A^k_\nu - \partial_\nu A^k_\mu - g\,\varepsilon^{k\ell m} A^\ell_\mu A^m_\nu$$

$$\nabla_\mu K_a = \partial_\mu K_a + ig\, A^k_\mu (\tau^k/2)_a{}^b K_b$$

$$\nabla_\mu \bar{K}^a = \partial_\mu \bar{K}^a - ig\, A^k_\mu \bar{K}^b (\tau^k/2)_b{}^a \quad . \tag{3.2}$$

The indices $a, b, \ldots$ take the values $1, 2$ (and $\bar{K}^a = K_a{}^\dagger$) while $k, \ell, \ldots$ take the values $1, 2, 3$. The Lagrangian (3.1) is invariant under the Yang-Mills pseudogroup whose infinitesimal transformations take the form

$$\delta A^k_\mu = \varepsilon^{k\ell m} A^\ell_\mu \Omega^m - \frac{1}{g}\partial_\mu \Omega^k \quad ,$$

$$\delta K_a = i\Omega^k (\tau^k/2)_a{}^b K_b \quad ,$$

$$\delta \bar{K}^a = -i\Omega^k \bar{K}^b (\tau^k/2)_b{}^a \quad . \tag{3.3}$$

The form of the last term in (3.1) favours a solution in which the gauge symmetry appears to be broken. That is, in the tree approximation we expect

$$\langle K_1 \rangle = \frac{\sqrt{2}M}{g}$$

$$\langle K_2 \rangle = 0 \quad . \tag{3.4}$$

Hence, if the transformations (3.3) are extended to include <u>asymptotic</u> transformations, Ω = constant, then this symmetry is clearly violated by (3.4).

We must therefore regard these transformations as having no asymptotic significance, i.e., as having no effect on physical states. We must require $\Omega(x)$ to vanish asymptotically. However, this does not mean that there is no asymptotic SU(2) symmetry. In fact there is. To see this, introduce a new set of real variables σ and B^k to represent the scalar fields,

$$K_1 = \frac{\sqrt{2}M}{g} + \frac{1}{\sqrt{2}} (\sigma + i\, B^3)$$
$$K_2 = \frac{1}{\sqrt{2}} (i\, B^1 - B^2) \quad . \tag{3.5}$$

In terms of these variables the Lagrangian (3.1) takes the rather complicated form:

$$\begin{aligned}\mathcal{L} = &- \frac{1}{4} (F^k_{\mu\nu})^2 + \frac{M^2}{2} (A^k_\mu)^2 + M\, A^k_\mu\, \partial_\mu B^k + \frac{1}{2} (\partial_\mu B^k)^2 \\ &+ \frac{1}{2} (\partial_\mu \sigma)^2 - \frac{m^2}{2} \sigma^2 + \frac{g}{2} A^k_\mu \left(\sigma \partial_\mu\, B^k - B^k \partial_\mu \sigma - (B \times \partial_\mu B)^k\right) \\ &+ \frac{Mg}{2} \sigma (A^k_\mu)^2 + \frac{g^2}{8} (\sigma^2 + B^2)(A^k_\mu)^2 - \frac{gm^2}{4M} \sigma(\sigma^2 + B^2) \\ &- \frac{g^2 m^2}{32M^2} (\sigma^2 + B^2)^2 \quad .\end{aligned} \tag{3.6}$$

This Lagrangian is invariant under the transformations

$$\begin{aligned}\delta A^k_\mu &= \epsilon^{k\ell m}\, A^\ell_\mu\, \Omega^m - \frac{1}{g} \partial_\mu \Omega^k \\ \delta\sigma &= - \frac{1}{2} B^k\, \Omega \\ \delta B^k &= \frac{M}{g} \Omega^k + \frac{1}{2} \sigma \Omega^k + \frac{1}{2} \epsilon^{k\ell m}\, B^\ell\, \Omega^m\end{aligned} \tag{3.7}$$

which are obtained from (3.3). However, (3.6) is clearly invariant also under the SU(2) transformations

$$\begin{aligned}\delta A^k_\mu &= \epsilon^{k\ell m}\, A^\ell_\mu\, \omega^m \\ \delta\sigma &= 0 \\ \delta B^k &= \epsilon^{k\ell m}\, B^\ell\, \omega^m\end{aligned} \tag{3.8}$$

with constant ω . These transformations belong to a gauge group of the first kind which we shall call the "true" I-spin group. It is quite distinct from the Yang-Mills pseudogroup. It is the true I-spin group which yields conserved quantities and according to which the asymptotic states must be classified.

The vacuum condition (3.4) does no violence to the true symmetry according to which

$$\delta K_1 = \frac{i}{2}(\omega_1 - i\omega_2) K_2 + \frac{i}{2}(\omega_1 + i\omega_2) \bar{K}^2 \quad ,$$
$$\delta K_2 = \frac{i}{2}(\omega_1 + i\omega_2)(K_1 - \bar{K}^1) - i\omega_3 K_2 \quad . \tag{3.9}$$

Corresponding to the two symmetries outlined here, there exist two distinct currents. Only one of these is conserved and so yields the usual Ward-Takahashi identities. The other current is partially conserved and yields a distinct family of Ward-Takahashi identities which reflect the underlying gauge symmetry of the second kind - the one which has no influence on the classification of physical states. These currents and their respective families of identities will be discussed in Appendix II.

Consider now the problem of quantizing (3.6). The first step in the quantization procedure is the choice of gauge. Here we shall consider two covariant gauges which between them illustrate the main properties of the model. These gauges are:

a) The canonical gauge

The supplementary terms in the action take the form

$$\exp\left[\frac{i}{\hbar}\ \mathcal{S}_{can}\right] = \int (dC)\ \exp\left[\frac{i}{\hbar}\int dx\ C^k B^k + \frac{i}{\hbar} W_{can}\right]$$
$$= \delta(B)\ \exp\left[\frac{i}{\hbar} W_{can}\right] \ . \tag{3.10}$$

This supplementary action incorporates the classical gauge condition,

$$B^k = 0 \quad .$$

The field $C^k(x)$ in (3.10) plays the role of Lagrange multiplier and results in the appearance of the δ-functional $\delta(B)$ in the path-integrals. The compensating functional W_{can} is very easy to evaluate in this case. According to the prescription (2.10),

$$\exp\left[-\frac{i}{\hbar} W_{can}\right] = \int (d\Omega)\ \delta(B^{\Omega}) \quad . \tag{3.11}$$

Since we shall need the values of W_{can} only in the subspace $B = 0$ we can evaluate (3.11) by including only infinitesimal transformations Ω in the integration, i.e. for $B = 0$,

$$\begin{aligned}\exp\left[-\frac{i}{\hbar} W_{can}\right] &= \int (d\Omega)\ \delta\left(\left(\frac{M}{g} + \frac{\sigma}{2}\right)\Omega^k\right)\\ &= \left|\mathrm{Det}\left(\frac{M}{g} + \frac{\sigma}{2}\right)\delta(x - x')\delta^{k\ell}\right|^{-1}\\ &= \exp\left[-\ 3\delta(0)\int dx\ \ell n\left(\frac{M}{g} + \frac{\sigma}{2}\right)\right] \times \text{constant} .\end{aligned}$$

If we choose the constant to make W_{can} vanish when $\sigma = 0$ then the result is

$$W_{can} = -\ 3i\hbar\delta(0)\int dx\ \ell n\ \left(1 + \frac{g}{2M}\sigma\right) \cdot \tag{3.12}$$

This structure can be understood as due to the "propagation" of a fictitious particle round simple closed loops with a propagator proportional to the Dirac delta function. At each vertex on the loop one σ-line is attached. The fact that only single-loop graphs contribute to W_{can} is reflected in the factor $\hbar$ which stands in front of the integral (3.12).

We have called this gauge canonical because it lends itself to a canonical quantization of the model. The action density (3.6) can be replaced by the canonical form,

$$\begin{aligned}\mathcal{L}_1 &= E^k_a\,\partial_0\,A^k_a + \pi\,\partial_0\,\sigma\ -\\ &-\left\{\frac{1}{2}(E^k_a)^2 + \frac{1}{4}\left[\partial_a A^k_b - \partial_b A^k_a - g(A_a \times A_b)^k\right]^2 - \frac{M^2}{2}(A^k_\mu)^2 + E^k_a\,\partial_a\,A^k_0\right.\\ &\quad - M\,A^k_\mu\,\partial_\mu\,B^k + \frac{1}{2}\pi^2 + \frac{1}{2}(\partial_a\sigma)^2 + \frac{m^2}{2}\sigma^2\\ &\quad - \frac{g}{2}A^k_\mu\left[\sigma\,\partial_\mu\,B^k - B^k\,\partial_\mu\sigma - (B \times \partial_\mu\,B)^k\right] - \frac{Mg}{2}\sigma(A^k_\mu)^2\ -\end{aligned}$$

$$- \frac{g^2}{8} (\sigma^2 + B^2)(A_\mu^k)^2 + \frac{gm^2}{4M} \sigma(\sigma^2 + B^2) + \frac{g^2 m^2}{32M^2} (\sigma^2 + B^2)^2 \Big\} \tag{3.13}$$

where A_0^k and B^k must be eliminated since they have no canonical conjugates. Such eliminations are performed, in the language of path-integrals, by integration over the undesirable variables. The Green's functions are represented in this gauge by the path-integral

$$\langle T\ F(A,\sigma) \rangle_{can} = \int (dA_\mu\ dE_a\ d\sigma\ d\pi\ dB)\ F(A,\sigma)\ \delta(B) \cdot \exp\left[\frac{i}{\hbar} \int dx \left[\mathcal{L}_1 - 3i\hbar\delta(0)\ \ln\left(1 + \frac{g}{2M}\sigma\right)\right]\right] \tag{3.14}$$

since, if the canonical momenta E and π are eliminated by performing the necessary (gaussian) integrations, the resulting integral over A_μ, σ and B takes the form prescribed in Sec.II. On the other hand, to obtain the purely canonical representation we should integrate over A_0 and B. The integral over B is trivial due to the presence of $\delta(B)$. The integral over A_0 is gaussian and proceeds as follows:

$$\int (dA_0) \exp\left[\frac{i}{\hbar} \int dx \left[\frac{1}{2} (M + \frac{g}{2}\sigma)^2 A_0^2 + A_0\ \partial_a\ E_a\right]\right]$$

$$= \left|\mathrm{Det}(M + \frac{g}{2}\sigma)\delta(x - x')\delta^{k\ell}\right|^{-1} \exp\left[\frac{i}{\hbar} \int dx \left[- \frac{1}{2} (M + \frac{g}{2}\sigma)^{-2} (\partial_a E_a)^2\right]\right]$$

$$= \exp\left[- 3\delta(0) \int dx\ \ln\left(1 + \frac{g}{2M}\sigma\right) - \frac{i}{\hbar} \int dx\ \frac{1}{2} \frac{(\partial_a\ E_a)^2}{(M + \frac{g}{2}\sigma)^2}\right] \tag{3.15}$$

where we have shown only that part of $\mathcal{L}_1$ which depends on A_0. It is very gratifying to find that the fictitious particle contribution to (3.14) is precisely cancelled by the result of the A_0-integration. We are left with the canonical path-integral,

$$\langle T\ F(A,\sigma) \rangle_{can} = \int (dA_a\ dE_a\ d\sigma\ d\pi)\ F(A,\sigma) \exp\left[\frac{i}{\hbar} \int dx \left[E_a\ \dot{A}_a + \pi\dot{\sigma} - H\right]\right] \tag{3.16}$$

where the hamiltonian density is given by

$$H = \frac{1}{2}(E^k_a)^2 + \frac{1}{4}(\partial_a A^k_b - \partial_b A^k_a - g\epsilon^{k\ell m} A^\ell_a A^m_b)^2 + \frac{M^2}{2}(A^k_a)^2 \left[1 + \frac{g}{2M}\sigma\right]^2$$

$$+ \frac{1}{2}\pi^2 + \frac{1}{2}(\partial_a\sigma)^2 + \frac{m^2}{2}\sigma^2\left[1 + \frac{g}{4M}\sigma\right]^2 + \frac{1}{2M^2}\frac{(\partial_a E_a)^2}{\left[1 + \frac{g}{2M}\sigma\right]^2} \quad . \tag{3.17}$$

It thus appears that the system described by the Lagrangian (3.6) contains two fundamental states, a vector isotriplet with mass M and a scalar isosinglet with mass m . These particles have positive metric and the theory should therefore yield a unitary S-matrix, in the perturbation sense at least.

This formulation of the theory is not renormalizable, however, since the vector propagator takes the form

$$\langle T\, A^k_\mu(x)\, A^\ell_\nu(0)\rangle = \hbar \int \frac{dk}{(2\pi)^4} \frac{i}{k^2 - M^2}\left[-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{M^2}\right] e^{-ikx} \tag{3.18}$$

in zeroth order. That the most singular parts of the Feynman integrals should cancel from physical S-matrix elements appears very unlikely. To see that this does in fact happen we must use another gauge.

b) The Landau gauge

The Landau supplementary action takes the form

$$\exp\left[\frac{i}{\hbar}\mathcal{S}_{Lan}\right] = \int (dC)\, \exp\left[\frac{i}{\hbar}\int dx\, C^k\, \partial_\mu A^k_\mu + \frac{i}{\hbar} W_{Lan}\right]$$
$$= \delta(\partial_\mu A_\mu)\, \exp\left[\frac{i}{\hbar} W_{Lan}\right] \tag{3.19}$$

which incorporates the gauge condition

$$\partial_\mu A^k_\mu = 0 \quad .$$

The compensating functional W_{Lan} is evaluated in the same way as before (on the subspace $\partial_\mu A_\mu = 0$),

$$\exp\left[-\frac{i}{\hbar} W_{Lan}\right] = \int (d\Omega)\; \delta(\partial_\mu A^\Omega_\mu)$$

$$= \int (d\Omega)\ \delta(\partial_\mu \nabla_\mu\ \Omega + \cdots)$$

$$= \left| \mathrm{Det}\ \partial_\mu \nabla_\mu^{k\ell}\ \delta(x - x') \right|^{-1} \times \text{constant}$$

since we need to keep only infinitesimal Ω . With a suitable choice of constant this gives

$$W_{Lan} = -i\hbar\ \mathrm{Tr}\ \ell n \left(\frac{1}{\partial^2}\ \partial_\mu \nabla_\mu \right) \quad . \tag{3.20}$$

The structure of this functional is perhaps made clearer by the integral representation,

$$\exp\left[-\frac{i}{\hbar} W_{Lan}(A) \right] = \int (d\Omega\ dC)\ \exp\left[\frac{i}{\hbar} \int dx\ \partial_\mu C^k\ (\partial_\mu \Omega^k - g\varepsilon^{k\ell m}\ A_\mu^\ell \Omega^m) \right] \tag{3.21}$$

which can be interpreted graphically. The exponent in the integrand of (3.21) defines the fictitious particle action. This action yields for the "fields" C and Ω the chronological pairings

$$\begin{aligned}
\langle T\ \Omega^k(x)\ \Omega^\ell(0) \rangle &= 0 \\
\langle T\ \Omega^k(x)\ C^\ell(0) \rangle &= \frac{\hbar}{i} \frac{1}{\partial^2}\ \delta(x)\delta^{k\ell} = \hbar\ D(x)\delta^{k\ell} \\
\langle T\ C^k(x)\ C^\ell(0) \rangle &= 0
\end{aligned} \tag{3.22}$$

where $D(x)$ denotes the usual zero-mass causal function. The only allowed vertices join one C-, one Ω- and one A-line. It can be seen that there is only one connected graph of order g^n . It consists of a single directed loop of the massless fictitious particle with one A-line attached at each vertex. It can also be seen that the higher powers of Ω which were discarded from ∂A^Ω cannot contribute because of the peculiar structure of the pairings (3.22).

The chronological pairings of the fields A_μ , B and σ are given by

$$\langle T\ A_\mu^k\ A_\nu^\ell \rangle = \frac{i\hbar}{k^2 - M^2} \left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2} \right)$$

$$\langle T\ B^k\ B^\ell \rangle = \frac{i\hbar}{k^2}$$

$$\langle T\,\sigma\,\sigma\rangle = \frac{i\hbar}{k^2 - m^2} \quad . \tag{3.23}$$

These propagators, taken with the vertices determined by (3.6), give a renormalizable perturbation series, as can be shown by the usual power-counting method. The theory appears, however, to be non-unitary. This is due to the massless scalar ghost which appears in the vector propagator. The off-shell S-matrix elements and gauge-dependent quantities in general do indeed contain these ghosts and the B-particle (Goldstone boson) as well in this gauge. But since neither of these features appeared in the canonical gauge it must be that they are pure gauge effects. The ghosts and Goldstone bosons must cancel from gauge-independent quantities such as physical S-matrix elements. Conversely, the canonical gauge problem of unrenormalizability with its implication of the absence of Froissart-boundedness must likewise be a gauge effect since it does not appear in the Landau gauge.[15] The physical S-matrix must be both Froissart-bounded and free of ghosts and Goldstone particles. In order to have these properties in a perturbation expansion it is of course necessary always to include gauge-independent sets of graphs, i.e. all graphs of fixed order g^n .

IV. A LEPTON MODEL

One of the main purposes of this paper is to present in some detail a gauge model which unifies the weak and electromagnetic interactions of electron-type leptons [9]. The underlying symmetry according to which the fields in this model are classified - but which is spontaneously violated - is the gauge group U(2). The left-handed electron and neutrino are treated as a doublet while the right-handed electron remains a singlet. In addition to the four gauge fields - triplet and singlet - the model contains a doublet of scalar fields, one component of which will have a non-vanishing expectation value in the physical vacuum.

To fix the notation consider a multiplet of fields ψ which belong to a representation of U(2). Under an infinitesimal transformation of the group we have

$$\delta\psi(x) = i\Omega^k(x)\, I^k\psi(x) + i\Omega^0(x)\, I^0\psi(x) \tag{4.1}$$

where the infinitesimal parameters $\Omega^k(x)$ and $\Omega^0(x)$ are real. The matrices I^k and I^0 represent the algebra of U(2),

$$[I^k, I^\ell] = i\, \varepsilon^{k\ell m}\, I^m \quad ,$$

$$[I^k, I^0] = 0 \quad .$$

The covariant derivative of $\psi(x)$ is given by

$$\nabla_\mu\psi = \partial_\mu\psi + i\, g\, X_\mu^k\, I^k\, \psi + i\, \frac{g_1}{2}\, X_\mu^0\, I^0\, \psi \tag{4.2}$$

where the gauge fields X_μ^k and X_μ^0 transform according to

$$\delta X_\mu^k = \varepsilon^{k\ell m}\, X_\mu^\ell\, \Omega^m - \frac{1}{g}\, \partial_\mu \Omega^k$$

$$\delta X_\mu^0 = -\frac{2}{g_1}\, \partial_\mu \Omega^0 \quad . \tag{4.3}$$

One combination of the gauge fields is to be identified with the electromagnetic field and a corresponding combination of the coupling constants, g and g_1, with the electric charge, e . These combinations are determined by the basic identification of the charge operator Q . We adopt the definition

$$Q = I^3 + \frac{I^0}{2} \quad . \tag{4.4}$$

An examination of (4.2) shows immediately that the combination

$$\frac{1}{g} X_\mu^3 + \frac{1}{g_1} X_\mu^0 = \left(\frac{1}{g^2} + \frac{1}{g^2}\right)^{\frac{1}{2}} A_\mu \tag{4.5}$$

couples to the charge operator. We therefore identify A_μ with the electromagnetic field. The orthogonal combination

$$\frac{1}{g_1} X_\mu^3 - \frac{1}{g} X_\mu^0 = \left(\frac{1}{g^2} + \frac{1}{g_1^2}\right)^{\frac{1}{2}} U_\mu \tag{4.6}$$

defines the other neutral gauge field. The remaining gauge fields are taken in the combinations

$$\frac{1}{\sqrt{2}} (X_\mu^1 - iX_\mu^2) = W_\mu^+ \quad \text{and} \quad \frac{1}{\sqrt{2}} (X_\mu^1 + iX_\mu^2) = W_\mu^- \tag{4.7}$$

which carry positive and negative charge, respectively. In terms of the new gauge fields, formula (4.2) for the covariant derivative reads

$$\begin{aligned} \nabla_\mu\psi = \partial_\mu\psi &+ i \frac{1}{\sqrt{2}} (W_\mu^+ I^+ + W_\mu^- I^-)\psi \\ &+ i \frac{gg_1}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}} A_\mu \left(I^3 + \frac{I^0}{2}\right)\psi \\ &+ i \frac{1}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}} U_\mu \left(g^2 I^3 - g_1^2 \frac{I^0}{2}\right)\psi \end{aligned} \tag{4.8}$$

where $I^\pm = I^1 \pm iI^2$. The electric charge must therefore be given by

$$e = \frac{gg_1}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}} \quad . \tag{4.9}$$

The left-handed electron and neutrino fields, e_L and ν_L, comprise the doublet

$$\ell_a = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \tag{4.10}$$

while the right-handed electron e_R is a singlet. In order that these particles should be correctly charged, we must assign the following values to I^0 :

$$I^0 \ell_a = -\ell_a \quad \text{and} \quad I^0 e_R = -2\, e_R \quad . \tag{4.11}$$

The doublet of scalar fields $T_a = (T^+, T^0)$ is given charges $+1$ and 0 by the assignment $I^0 = 1$. These assignments are summarized in Table I.

The Lagrangian of the proposed model [8] for lepton interactions takes the gauge-invariant form,

$$\begin{aligned}\mathcal{L} = &-\tfrac{1}{4}(X^k_{\mu\nu})^2 - \tfrac{1}{4}(X^0_{\mu\nu})^2 + \nabla_\mu \bar{T}^a \nabla_\mu T_a - \kappa(\bar{e}_R \ell_a \bar{T}^a + \bar{\ell}^a e_R T_a) \\ &+ \bar{\ell}^a\, i\, \gamma_\mu \nabla_\mu\, \ell_a + \bar{e}_R\, i\, \gamma_\mu \nabla_\mu\, e_R - \lambda^2 \left(\bar{T}^a T_a - \frac{\rho^2}{2}\right)^2\end{aligned} \tag{4.12}$$

where $\bar{T}^a = T_a^{\dagger}$, $\bar{\ell}^a = \ell_a^{\dagger}\gamma_0$, $\bar{e}_R = e_R^{\dagger}\gamma_0$ and the field strengths are defined by

$$\begin{aligned} X^k_{\mu\nu} &= \partial_\mu X^k_\nu - \partial_\nu X^k_\mu - g\epsilon^{k\ell m} X^\ell_\mu X^m_\nu \\ X^0_{\mu\nu} &= \partial_\mu X^0_\nu - \partial_\nu X^0_\mu \quad . \end{aligned} \tag{4.13}$$

These field strengths can be expressed in terms of the physical components by means of the following formulae:

$$\begin{aligned} X^1_{\mu\nu} &= \frac{1}{\sqrt{2}}(W^+_{\mu\nu} + W^-_{\mu\nu}) \\ X^2_{\mu\nu} &= \frac{i}{\sqrt{2}}(W^+_{\mu\nu} - W^-_{\mu\nu}) \\ X^3_{\mu\nu} &= \frac{g_1 A_{\mu\nu} + g\, U_{\mu\nu}}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}} \\ X^0_{\mu\nu} &= \frac{g\, A_{\mu\nu} - g_1 U_{\mu\nu}}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}} \end{aligned} \tag{4.14}$$

where $W^\pm_{\mu\nu}$, $A_{\mu\nu}$ and $U_{\mu\nu}$ are given by

$$W^\pm_{\mu\nu} = \partial_\mu W^\pm_\nu - \partial_\nu W^\pm_\mu \pm \frac{igg_1}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}}(A_\mu W^\pm_\nu - A_\nu W^\pm_\mu) \pm \frac{ig^2}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}}(U_\mu W^\pm_\nu - U_\nu W^\pm_\mu)$$

$$A_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \frac{igg_1}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}} \; (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-)$$

$$U_{\mu\nu} = \partial_\mu U_\nu - \partial_\nu U_\mu + \frac{ig^2}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}} \; (W_\mu^+ W_\nu^- - W_\nu^+ W_\mu^-) \qquad , \tag{4.15}$$

In (4.12) we can substitute

$$-\frac{1}{4} (X_{\mu\nu}^k)^2 - \frac{1}{4} (X_{\mu\nu}^0)^2 = -\frac{1}{2} W_{\mu\nu}^+ W_{\mu\nu}^- - \frac{1}{4} (A_{\mu\nu})^2 - \frac{1}{4} (U_{\mu\nu})^2$$

to eliminate the old field strengths from the Lagrangian. The gauge fields appear elsewhere in the Lagrangian only through the various covariant derivatives which we list:

$$\nabla_\mu \nu_L = \left[\partial_\mu + \frac{i}{2} \left(g^2 + g_1^2\right)^{\frac{1}{2}} U_\mu\right] \nu_L + i \frac{g}{\sqrt{2}} W_\mu^+ e_L$$

$$\nabla_\mu e_L = \left[\partial_\mu - \frac{igg_1}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}} A_\mu - \frac{i}{2} \frac{g^2 - g_1^2}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}} U_\mu\right] e_L + i \frac{g}{\sqrt{2}} W_\mu^- \nu_L$$

$$\nabla_\mu e_R = \left[\partial_\mu - \frac{igg_1}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}} A_\mu + \frac{i}{2} \frac{2g_1^2}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}} U_\mu\right] e_R$$

$$\nabla_\mu T^+ = \left[\partial_\mu + \frac{igg_1}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}} A_\mu + \frac{i}{2} \frac{g^2 - g_1^2}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}} U_\mu\right] T^+ + i \frac{g}{\sqrt{2}} W_\mu^+ T^0$$

$$\nabla_\mu T^0 = \left[\partial_\mu - \frac{i}{2} \left(g^2 + g_1^2\right)^{\frac{1}{2}} U_\mu\right] T^0 + i \frac{g}{\sqrt{2}} W_\mu^- T^+ \qquad . \tag{4.16}$$

The last term in the Lagrangian (4.12) favours the emergence of a symmetry-violating solution. The neutral field T^0 will develop a non-vanishing expectation value in the physical vacuum. In the classical approximation,

$$\left\langle T^0 \right\rangle = \frac{\rho}{\sqrt{2}} \qquad . \tag{4.17}$$

In the same approximation various other fields acquire a mass by the same mechanism. To reveal these masses it is useful to replace the complex field

T^0 by the real components σ and χ ,

$$T^0(x) = \frac{1}{\sqrt{2}}\left[\rho + \sigma(x) + i\,\chi(x)\right] \cdot \tag{4.18}$$

After making all these substitutions one arrives at a rather complicated Lagrangian (Appendix I) which involves the fields $W_\mu^\pm$, A_μ , U_μ , $T^\pm$, σ , χ and the leptons ν_L , e_L and e_R . This Lagrangian is invariant under the group of U(2) gauge transformations:

$$\delta W_\mu^\pm = \pm i\, W_\mu^\pm\, \Omega^3 \mp i\, \frac{g_1 A_\mu + g U_\mu}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}}\,\Omega^\pm - \frac{1}{g}\,\partial_\mu \Omega^\pm$$

$$\delta A_\mu = \frac{i g_1}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}}\left(W_\mu^-\,\Omega^+ - W_\mu^+\,\Omega^-\right) - \frac{1}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}}\,\partial_\mu\left(\frac{g_1}{g}\,\Omega^3 + \frac{2g}{g_1}\,\Omega^0\right)$$

$$\delta U_\mu = \frac{i g}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}}\left(W_\mu^-\,\Omega^+ - W_\mu^+\,\Omega^-\right) - \frac{1}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}}\,\partial_\mu\left(\Omega^3 - 2\Omega^0\right)$$

$$\delta T^\pm = \pm\,\frac{i}{2}\left(\Omega^3 + 2\Omega^0\right)T^\pm \pm \frac{i}{2}\,\Omega^\pm(\sigma \pm i\chi) \pm \frac{i}{2}\,\Omega^\pm\rho$$

$$\delta\sigma = \frac{i}{2}\left(\Omega^- T^+ - \Omega^+ T^-\right) + \frac{1}{2}\left(\Omega^3 - 2\Omega^0\right)\chi$$

$$\delta\chi = \frac{1}{2}\left(\Omega^- T^+ + \Omega^+ T^-\right) - \frac{1}{2}\left(\Omega^3 - 2\Omega^0\right)(\rho + \sigma)$$

$$\delta\nu_L = \frac{i}{2}\left(\Omega^3 - 2\Omega^0\right)\nu_L + \frac{i}{\sqrt{2}}\,\Omega^+\, e_L$$

$$\delta e_L = -\,\frac{i}{2}\left(\Omega^3 + 2\Omega^0\right) e_L + \frac{i}{\sqrt{2}}\,\Omega^-\,\nu_L$$

$$\delta e_R = -\,2i\Omega^0\, e_R \quad , \tag{4.19}$$

where $\Omega^\pm = (1/\sqrt{2})(\Omega^1 \mp i\Omega^2)$.

We consider now the question of fixing the gauge in order to proceed with quantizing the theory. It is necessary to impose four gauge conditions and we shall deal with two alternatives:

a) <u>a "canonical" gauge</u>

$$\partial_\mu A_\mu = 0 \quad , \quad \chi = 0 \quad , \quad T^\pm = 0 \; , \tag{4.20}$$

b) <u>the Landau gauge</u>

$$\partial_\mu A_\mu = 0 \quad , \quad \partial_\mu U_\mu = 0 \quad , \quad \partial_\mu W^\pm_\mu = 0 \quad . \tag{4.21}$$

To treat a truly canonical gauge it would be necessary to impose a non-covariant condition on the electromagnetic field, e.g., $\partial_a A_a = 0$ or $A_0 = 0$. However, since our object in using the canonical gauge is only to show that ghosts do not appear in gauge-independent quantities and since everyone accepts this in electromagnetic theory, we shall forego that refinement.

The canonical gauge-compensating terms are given by the integral

$$\exp\left[-\frac{i}{\hbar} W_{can}\right] = \int (d\Omega)\delta(\partial_\mu A^\Omega_\mu)\delta(\chi^\Omega)\delta(T^{+\Omega})\delta(T^{-\Omega})$$

$$= \int (d\Omega)\delta\left(\partial^2\left\{\frac{g_1}{g}\Omega^3 + \frac{2g}{g_1}\Omega^0\right\} - ig_1\,\partial_\mu\left\{W^-_\mu\Omega^+ - W^+_\mu\Omega^-\right\}\right) \cdot$$

$$\cdot\ \delta\left((\rho + \sigma)(\Omega^3 - 2\Omega^0)\right)\delta\left((\rho + \sigma)\Omega^+\right)\delta\left((\rho + \sigma)\Omega^-\right)$$

on the subspace $\chi = T^+ = T^- = \partial_\mu A_\mu = 0$. The result is

$$W_{can} = -3i\hbar\delta(0)\int dx\ \ell n\left(1 + \frac{\sigma(x)}{\rho}\right) \quad . \tag{4.22}$$

This factor is exactly the same as the one met with in the Yang-Mills example of Sec.III and it will be cancelled in the same way when the path-integral is put into canonical form. Thus, to obtain the canonical form it will be necessary to integrate over the dependent variables W^+_0 , W^-_0 and U_0 which appear quadratically in the action density (4.12) due to the term

$$\nabla_\mu \bar{T}^a\, \nabla_\mu T_a = \cdots + \left\{\frac{g^2}{2} W^+_\mu W^-_\mu + \frac{g^2 + g_1^2}{4} U^2_\mu\right\} T_0^2$$

$$= \cdots + \left\{\frac{g^2}{2} W^+_0 W^-_0 + \frac{g^2 + g_1^2}{4} U^2_0\right\}\frac{1}{2}(\rho + \sigma)^2 \quad .$$

The integrals are therefore gaussian and can be evaluated explicitly as in Sec.III. The resulting expression for the hamiltonian density is very complicated. We shall not reproduce it here since it has no practical utility:

computations are always much simpler in the covariant lagrangian framework (where, of course, the supplementary term (4.22) must be included).

In the canonical gauge the particle spectrum is clear. Since the fields χ , T^+ and T^- have disappeared, we are left with the bosons corresponding to A , $W^\pm$, U and σ in addition to the leptons ν_L , e_L and e_R . The bare propagators are given by

$$\frac{1}{\hbar}\langle T\, A_\mu\, A_\nu\rangle = \frac{i}{k^2}\left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right) ,$$

$$\frac{1}{\hbar}\langle T\, W^+_\mu\, W^-_\nu\rangle = \frac{i}{k^2 - M_W^2}\left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{M_W^2}\right) ,$$

$$\frac{1}{\hbar}\langle T\, U_\mu\, U_\nu\rangle = \frac{i}{k^2 - M_U^2}\left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{M_U^2}\right)$$

$$\frac{1}{\hbar}\langle T\, \sigma\, \sigma\rangle = \frac{i}{k^2 - m_\sigma^2}$$

$$\frac{1}{\hbar}\langle T\, \nu_L\, \overline{\nu}_L\rangle = \frac{i}{\not p}\,\frac{1 - i\gamma_5}{2}$$

$$\frac{1}{\hbar}\langle T\, e\, \overline{e}\,\rangle = \frac{i}{\not p - m_e} \tag{4.23}$$

where the masses are expressed in terms of the parameters which appear in (4.12):

$$M_W = \rho\,\frac{g}{2}$$

$$M_U = \rho\,\frac{\left(g^2 + g_1^2\right)^{\frac{1}{2}}}{2}$$

$$m_\sigma = \sqrt{2}\rho\lambda$$

$$m_e = \frac{\rho}{\sqrt{2}}\,\kappa \quad . \tag{4.24}$$

The parameters g, g_1, λ, ρ and κ are unsuitable for computational purposes. This is because they appear through the masses (4.24) in the free Lagrangian. The amplitude corresponding to even a single graph will therefore contain arbitrarily high powers of these parameters - an effect which makes it difficult to organize a perturbation series. In particular, this effect obscures the underlying gauge symmetry and one might think that only infinitely large categories of graphs could be gauge independent. It is of crucial importance to be able to distinguish such gauge-independent categories since only for these will the unrenormalizabilities such as lack of Froissart-boundedness be avoided. Fortunately, it is possible to pick out finite sets which are gauge independent by a correct parametrization.

The true expansion parameter should appear only in the interaction terms (with positive powers) and not in the free Lagrangian. This can be arranged if we express our original parameters g, g_1, λ, ρ and κ in terms of the masses M_W, M_U, m_σ, m_e and the electric charge, e, given by (4.9). The true expansion parameter will then be the electric charge and the gauge-independent categories will then consist of sets of graphs of fixed order e^n. The correct parametrization of the Lagrangian (4.12) is therefore obtained by making the substitutions,

$$g = e \frac{M_U}{\left(M_U^2 - M_W^2\right)^{\frac{1}{2}}}$$

$$g_1 = e \frac{M_U}{M_W}$$

$$\kappa = \frac{e}{\sqrt{2}} \frac{m_e M_U}{M_W\left(M_U^2 - M_W^2\right)^{\frac{1}{2}}}$$

$$\lambda = \frac{e}{2\sqrt{2}} \frac{m_\sigma M_U}{M_W\left(M_U^2 - M_W^2\right)^{\frac{1}{2}}}$$

$$\rho = \frac{2}{e} \frac{M_W\left(M_U^2 - M_W^2\right)^{\frac{1}{2}}}{M_U} \tag{4.25}$$

which are obtained by solving (4.24) and (4.9). One can verify that the interaction Lagrangian consists of terms of order e and e^2 only (Appendix I).

There is, of course, a phenomenological constraint on these parameters in that the Fermi constant G_F is given in this model by

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{e^2}{8} \frac{M_U^2}{M_W^2(M_U^2 - M_W^2)} \quad . \tag{4.26}$$

Further constraints involving κ and λ will emerge when more data on high-energy lepton interactions become available.

Now consider the Landau gauge where the theory is expected to be renormalizable but ghost infected. The gauge conditions (4.21) lead to the compensating functional W_{Lan} defined by

$$\begin{aligned}
\exp\left[-\frac{i}{\hbar} W_{Lan}\right] &= \int (d\Omega)\delta(\partial_\mu A_\mu^\Omega)\delta(\partial_\mu U_\mu^\Omega)\delta(\partial_\mu W_\mu^{+\Omega})\delta(\partial_\mu W_\mu^{-\Omega}) \\
&= \int (d\Omega)\delta(\partial_\mu X_\mu^{k\Omega})\delta(\partial_\mu X_\mu^{0\Omega}) \\
&= \int (d\Omega)\delta(\partial_\mu \nabla_\mu \Omega^k)\delta(\partial^2\Omega^0) \\
&= \exp\left[-\frac{i}{\hbar} W_{Lan}(X^k)\right] \\
&= \exp\left[-\operatorname{Tr} \ell n \left(\frac{1}{\partial^2} \partial_\mu \nabla_\mu\right)\right]
\end{aligned} \tag{4.27}$$

where $W_{Lan}(X^k)$ denotes the same functional (3.20) which was discussed in Sec.III. Into this functional one must substitute for X^k the expressions

$$\begin{aligned}
X_\mu^1 &= \frac{1}{\sqrt{2}} (W_\mu^+ + W_\mu^-) \\
X_\mu^2 &= \frac{i}{\sqrt{2}} (W_\mu^+ - W_\mu^-) \\
X_\mu^3 &= \frac{g_1 A_\mu + g U_\mu}{\left(g^2 + g_1^2\right)^{\frac{1}{2}}} = \frac{M_U}{M_W} \left(\frac{M_U^2 - M_W^2}{M_U^2 + M_W^2}\right)^{\frac{1}{2}} A_\mu + \frac{M_U}{\left(M_U^2 + M_W^2\right)^{\frac{1}{2}}} U_\mu \quad .
\end{aligned} \tag{4.28}$$

The fictitious particle contributions in this gauge consist of simple disjoint loops to which the vectors attach.

The bare propagators in the Landau gauge are the same as those in the canonical gauge (4.23) except for the U and $W^\pm$ propagators which assume the transverse form

$$\frac{1}{\hbar}\langle T\, W_\mu^+\, W_\nu^-\rangle = \frac{i}{k^2 - M_W^2}\left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right)$$

$$\frac{1}{\hbar}\langle T\, U_\mu\, U_\nu\rangle = \frac{i}{k^2 - M_U^2}\left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right) \tag{4.29}$$

and which therefore contain scalar ghosts. These ghosts will be compensated in gauge-independent amplitudes by the Goldstone boson propagators,

$$\frac{1}{\hbar}\langle T\, T^+\, T^-\rangle = \frac{i}{k^2} \quad ,$$

$$\frac{1}{\hbar}\langle T\, \chi\, \chi\rangle = \frac{i}{k^2} \quad . \tag{4.30}$$

These cancellations of ghosts against Goldstone bosons will take place only in gauge-independent amplitudes such as physical S-matrix elements <u>provided</u> gauge-independent sets of graphs are computed. This means, in the parametrization (4.25), that all graphs of given order e^n must be taken together.

V. CONCLUSIONS

To what extent can the lepton model of Sec.IV be considered a realistic one? We wish to conclude by listing some general remarks and speculations concerning this question.

A) The theory as presented here is based on a supposed U(2) gauge symmetry of which the doublets (ν_μ,μ) and (ν_e,e) are independent representations. No vestige of any μ-e symmetry has been taken into account. It has in the past been proposed [16] to include such a symmetry in a more general scheme, based on U(3), where the four-component neutrino is grouped with the charged leptons, e^- and μ^+, to make up a triplet. In that scheme the left-handed component of the neutrino field is identified with ν_e while the right-handed component is identified with $\bar{\nu}_\mu$. In fact, the theory admits two independent lepton triplets:

$$\ell_L = \begin{pmatrix} \nu_L \\ e^-_L \\ \mu^+_L \end{pmatrix} \qquad \text{and} \qquad \ell_R = \begin{pmatrix} \mu^+_R \\ \nu_R \\ e^-_R \end{pmatrix}$$

and, correspondingly, two nonets of gauge fields, $Z_{\mu L}$ and $Z_{\mu R}$, to make up a $U(3) \times U(3)$ gauge-invariant system. The Lagrangian of Sec.IV is thus incorporated in the more general model of Ref.16.

Out of the two lepton triplets it is possible to construct six independent neutral currents (with zero lepton number) of which only two have been utilized in Sec.IV. This plague of neutral currents is a notorious feature of weak interaction symmetry schemes. Although they might function in the sphere of purely leptonic interactions without contradicting our meagre supply of information concerning such processes, they must be suppressed in the semi-leptonic and hadronic processes. A convincing way to do this has yet to be discovered. For this reason we have confined our considerations in this paper to those purely leptonic interactions which are governed by the smaller U(2) symmetry. Of the three neutral currents which can be made out of the electron-type leptons, only two are coupled to gauge fields. One of these gauge fields can be identified with the electromagnetic field A , while the other one, U , must be supposed to acquire a large mass through the spontaneous breakdown mechanism.

The muon-type leptons can be brought into the U(2) scheme by treating μ_L^+ as a singlet with $I^0 = 2$ and the pair (μ_R^+, ν_R) as a doublet with $I^0 = 1$. The lepton current to which U_μ couples is then given by

$$\frac{1}{4}\left(g^2 + g_1^2\right)^{\frac{1}{2}} \quad (\bar{e}i\gamma_\mu\gamma_5 e + \bar{\mu}i\gamma_\mu\gamma_5\mu - 2\bar{\nu}i\gamma_\mu\gamma_5\nu) + \frac{1}{4}\frac{g^2 - 3g_1^2}{\left(g^2 - g_1^2\right)^{\frac{1}{2}}}(\bar{e}\gamma_\mu e - \bar{\mu}\gamma_\mu\mu) \ .$$

It is amusing to note[16] that the special choice $g^2 = 3g_1^2$ (which implies $3M_U^2 = 4M_W^2$ according to (4.24)) gives rise to the pure axial vector coupling

$$(e/\sqrt{3})(\bar{e}i\gamma_\mu\gamma_5 e + \bar{\mu}i\gamma_\mu\gamma_5\mu - 2\bar{\nu}i\gamma_\mu\gamma_5\nu)$$

with the same sign of "axial charge"[17] for e^- and μ^+ . (This does not mean that U_μ conserves parity because its interactions with the charged bosons $W_\mu^\pm$ is pure vector.)

At the present time there is no compelling reason to prefer any particular value for the ratio g_1/g and so we have left it free in the discussion of Sec.IV. It may be noted, however, that if $g_1 \gg g$ then $M_U \gg M_W$ and the constraint (4.26) takes the form

$$\frac{G_F}{\sqrt{2}} = \frac{e^2}{8M_W^2}\left(1 + \frac{M_W^2}{M_U^2} + \cdots\right)$$

which indicates the lower bound on $M_W^2 \geq e^2/4\sqrt{2}\, G_F \sim (37 \text{ GeV})^2$. The choice favoured by Lee[18] is

$$g = e \quad , \quad g_1 = \infty \quad , \quad M_U = \infty \quad , \quad G_F/\sqrt{2} = e^2/8M_W^2 \quad .$$

B) It was emphasised in Sec.IV that the selection of graphs in any perturbation calculation must be controlled so as to assure both ghost cancellation and electromagnetic gauge independence. It was suggested that such control can best be exercised by working to a definite order in the electromagnetic coupling e while treating the bare masses $(M_W, M_U, m_\sigma$ and $m_e)$ as independent parameters. All the bare coupling constants $(g, g_1, \kappa, \lambda$ and $\rho)$ should be expressed in terms of these independent parameters. For the treatment of higher orders it may prove useful to look upon this as a self-consistency requirement of which the parametrizations (4.25) represent the zeroth-order solution.

C) An interesting limiting version of the theory is achieved by taking $m_\sigma \to \infty$. This removes the σ-particle from physics and causes the Lagrangian to take a non-polynomial form[19]. The action density (4.12) depends on m_σ (or, equivalently, on λ) through the term $-\lambda^2(\bar{T}^a T_a - \rho^2/2)^2$ and the effect of taking $m_\sigma \to \infty$ inside the path-integrals is given by

$$\lim_{m_\sigma \to \infty} \exp\left[-\frac{i}{\hbar}\int dx\ \lambda^2(\bar{T}^a T_a - \rho^2/2)^2\right] = \delta(\bar{T}^a T_a - \rho^2/2)$$

which embodies a constraint. In the parametrization of Sec.IV one can express the field σ in terms of the Goldstone fields,

$$\sigma = -\rho + \left(\rho^2 - \chi^2 - 2T^+T^-\right)^{\frac{1}{2}} .$$

Elimination of σ therefore results generally in a non-polynomial Lagrangian for the Goldstone fields. In the canonical gauge $\chi = T^+ = T^- = 0$, the Goldstone fields are suppressed along with σ and one is left with a massive Yang-Mills triplet interacting with leptons and the electromagnetic field. In the Landau gauge one obtains the same theory in the non-polynomial formulation given by Boulware[20].

For the model of Sec.III the constraint which results in the limit $m_\sigma \to \infty$ takes the form

$$\left(\frac{2M}{g} + \sigma\right)^2 + B^k B^k = \frac{4M^2}{g^2} .$$

An exponential parametrization of the constrained fields is given by

$$\frac{2M}{g} + \sigma + i\ B^k \tau^k = \frac{2M}{g}\ \exp\left[\frac{ig}{2M}\,\phi^k \tau^k\right] = S(\phi)$$

where the components $\phi^k(x)$ are independent variables. The Lagrangian (3.6) thereby becomes, in this limit,

$$\mathcal{L}_\infty = -\frac{1}{4}(F^k_{\mu\nu})^2 + \frac{M^2}{2}(A^k_\mu)^2 + M\ A^k_\mu\ L_{k\ell}(\phi)\ \partial_\mu\phi^\ell +$$

$$+ \frac{1}{2}\, g_{k\ell}(\phi)\ \partial_\mu\phi^k\ \partial_\mu\phi^\ell - i\hbar\delta(0)\ \ \ell n(\det g)^{\frac{1}{2}}$$

where the coefficients $L(\phi)$ and $g(\phi)$ are defined in terms of the SU(2) matrix $S(\phi)$ by

$$\frac{2M}{ig} S^{-1} \partial_\mu S = \tau^k L_{k\ell}(\phi) \partial_\mu \phi^\ell$$

$$g_{k\ell}(\phi) = \sum_m L_{mk}(\phi) L_{m\ell}(\phi) \quad .$$

Thus one arrives at a localizable non-polynomial form of the theory with the minor coupling constant $g/2M$. From this point of view we make contact with non-polynomial lagrangian theories previously considered by the authors[21]. Such theories possess an inbuilt cut-off given by the inverse of the minor coupling constant. This cut-off will in the present case regularize only some (ϕ-containing) processes. Other (A-containing) processes will retain the conventional infinities associated with renormalizable theories.

D) The model presented in this paper has the usual infinities of a renormalizable lagrangian theory. If one were to include gravitational couplings so as to make the Lagrangian generally covariant then all these infinities would be regularized[22]. A conjecture is that, owing to the gauge symmetry, only the logarithmic divergences would survive in the form $\ln G_{Newton}$, while the quadratic divergences, which might be expected to yield $1/G_{Newton}$, would be absent.

E) Can the form of the Lagrangian (4.12) be justified on general grounds? The various gauge field couplings are, of course, fixed by requiring the kinetic terms to be gauge invariant. The scalar doublet was introduced with a quartic self-coupling in order to catalyze the process of spontaneous symmetry breakdown which gives mass to the gauge fields. The coupling of this doublet to the leptons was needed in order that the charged leptons should acquire mass by the same mechanism. Having started the process, can one set up a programme for self-consistently computing the various couplings (g, g_1, κ, λ) and masses (M_W, M_U, m_σ) in terms of only the dimensionless electromagnetic parameter e and a dimensional parameter m_e (say)? That is, having set up equations for the physical values, can one set the bare values to zero? There would appear to be some scope for such an enterprise, particularly if all the infinities are regularized by means of a gravitational coupling. (On the other hand, one must first verify that the model is sensible at least to the extent that the presence of the massive intermediate bosons with couplings of electromagnetic strength does no violence to the very accurate predictions of lepton electrodynamics.)

F) In order to have a complete renormalizable theory of weak interactions it is essential that a spontaneously broken gauge theory of hadrons should also be considered together with the lepton theory of Sec.IV. It is not difficult to invent such models. Unhappily though, one must introduce a large collection of scalar fields (analogous to the doublet T_a) to secure that all symmetry breaking is spontaneous. (For conventional renormalizability there is no escaping the requirement that all symmetry breaking must be spontaneous. One cannot introduce explicit symmetry breaking without destroying the gauge symmetry that underlies the renormalizability.) In addition, the wealth of available data on weak processes involving hadrons puts many stringent conditions on any model which claims to be realistic.

G) The validity of the programme pursued in this paper depends fundamentally on the equivalence theorem of Sec.II. There are two aspects of this theorem which we have not gone into but which must be clarified before it can be applied with full confidence. First there is the regularization problem. The manipulations of Sec.II are applied to unregularized path-integrals and so are not strictly meaningful. They could be given a meaning if the action were replaced by a regularized functional containing appropriate counterterms. What remains to be proved is that the appropriate (i.e. gauge-invariant) regularization scheme can be invented[23)].

The second problem involves the presence of zero-mass particles, Goldstone bosons and ghosts, which could interfere in a serious way with the applicability of the equivalence theorem. Green's functions will not, because of infra-red effects, have a clearly defined pole structure. Since the pole structure in the external lines is an essential element in the defining of the S-matrix, there could be a difficulty here[24)].

It is encouraging that Lee's analysis[7)] shows that both these difficulties can be overcome in the abelian case. However, a new and disturbing feature arises in the non-abelian models. This is the fact that, in most gauges, the gauge field does not couple to the partially conserved current (as it does in the abelian case). The gauge field source generally contains an extra term whose form depends on the type of gauge used (see Appendix II). Because of this, one certainly cannot use a naive argument to prove, on the basis of the partial conservation law, that the Goldstone particles cancel the ghosts in physical amplitudes. If the equivalence theorem is true then such cancellations must follow a more subtle pattern. (It should be remarked that in none of the examples considered does the Yang-Mills field couple to a conserved quantity. Thus, in Sec.III, although there is a conserved current in the canonical gauge - where the question of ghosts and Goldstone particles does not arise - it differs from the vector field source.)

NOTE ADDED

We are indebted to Professor J. Prentki for showing us a recent paper by S. Weinberg which deals with the model of Sec.IV. Professors B. Zumino and W. Bardeen [25] have made the important point that, in a γ_5-containing theory, anomalous terms of the Adler type will inevitably appear. They have expressed the fear that the associated counterterms may render such theories unrenormalizable even in the Landau gauge. This difficulty would be in addition to the ghost question[23]. Clearly it is important to set up a detailed regularization scheme and to analyse carefully the renormalization programme.

APPENDIX I

THE LEPTON LAGRANGIAN

When the substitutions outlined in Sec.IV are made, the Lagrangian (4.12) assumes the form

$$\mathcal{L} = \mathcal{L}_0 + e\,\mathcal{L}_1 + e^2\,\mathcal{L}_2 \tag{AI.1}$$

where

$$\begin{aligned}
\mathcal{L}_0 = & -\frac{1}{2}(\partial_\mu W^+_\nu - \partial_\nu W^+_\mu)(\partial_\mu W^-_\nu - \partial_\nu W^-_\mu) + M_W^2 W^+_\mu W^-_\mu \\
& -\frac{1}{4}(\partial_\mu U_\nu - \partial_\nu U_\mu)^2 + \frac{M_U^2}{2} U_\mu^2 \\
& -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{1}{2}(\partial_\mu \sigma)^2 - \frac{m_\sigma^2}{2}\sigma^2 + \frac{1}{2}(\partial_\mu \chi)^2 \\
& + \partial_\mu T^+ \partial_\mu T^- + \overline{\psi}_e(i\not{\partial} - m_e)\psi_e + \frac{1}{2}\overline{\psi}_\nu\, i\not{\partial}(1 + i\gamma_5)\psi_\nu \\
& - iM_W(\partial_\mu T^+ W^-_\mu - \partial_\mu T^- W^+_\mu) - M_U U_\mu \partial_\mu \chi
\end{aligned} \tag{AI.2}$$

$$\begin{aligned}
\mathcal{L}_1 = & \frac{M_U M_W}{\left[M_U^2 - M_W^2\right]^{\frac{1}{2}}}\sigma W^+_\mu W^-_\mu + \frac{M_U^3}{2M_W\left[M_U^2 - M_W^2\right]^{\frac{1}{2}}}\sigma U_\mu^2 - \frac{M_U^2}{2\left[M_U^2 - M_W^2\right]^{\frac{1}{2}}} U_\mu (W^-_\mu T^+ + W^+_\mu T^-) \\
& + \frac{M_U}{2\left[M_U^2 - M_W^2\right]^{\frac{1}{2}}}\left[W^-_\mu\left\{(\chi + i\sigma)\overleftrightarrow{\partial}_\mu T^+\right\} + W^+_\mu\left\{(\chi - i\sigma)\overleftrightarrow{\partial}_\mu T^-\right\} + \frac{M_U}{M_W} U_\mu\left\{\sigma\overleftrightarrow{\partial}_\mu \chi\right\}\right] \\
& + \left[A_\mu + \frac{2M_W^2 - M_U^2}{2M_W\left[M_U^2 - M_W^2\right]^{\frac{1}{2}}} U_\mu\right]\left[M_W(W^-_\mu T^+ + W^+_\mu T^-) - i\,T^+\overleftrightarrow{\partial}_\mu T^-\right] + \\
& + \left[A_\mu + \frac{M_W}{\left(M_U^2 - M_W^2\right)^{\frac{1}{2}}} U_\mu\right]\left[i\,W^+_\nu\overleftrightarrow{\partial}_\mu W^-_\nu - i(\partial_\nu W^+_\mu W^-_\nu - W^+_\nu \partial_\nu W^-_\mu)\right] \\
& + i\,W^+_\mu W^-_\nu\left[\partial_\mu A_\nu - \partial_\nu A_\mu + \frac{M_W}{\left[M_U^2 - M_W^2\right]^{\frac{1}{2}}}(\partial_\mu U_\nu - \partial_\nu U_\mu)\right]
\end{aligned}$$

$$+ A_\mu \overline{\psi}_e \gamma_\mu \psi_e + U_\mu \left[\frac{M_U^2}{4M_W\left(M_U^2 - M_W^2\right)^{\frac{1}{2}}} \left\{ \overline{\psi}_e i\gamma_\mu \gamma_5 \psi_e - \overline{\psi}_\nu \gamma_\mu (1 + i\gamma_5)\psi_\nu \right\} - \right.$$

$$\left. - \frac{3M_U^2 - 4M_W^2}{4M_W\left(M_U^2 - M_W^2\right)^{\frac{1}{2}}} \overline{\psi}_e \gamma_\mu \psi_e \right]$$

$$- \frac{M_U}{2\sqrt{2}\left(M_U^2 - M_W^2\right)^{\frac{1}{2}}} \left[\overline{\psi}_e \gamma_\mu (1 + i\gamma_5)\psi_\nu W_\mu^- + \overline{\psi}_\nu \gamma_\mu (1 + i\gamma_5)\psi_e W_\mu^+ \right]$$

$$- \frac{m_e M_U}{2M_W\left(M_U^2 - M_W^2\right)^{\frac{1}{2}}} \left[\overline{\psi}_e (\sigma + \chi\gamma_5)\psi_e + \frac{1}{\sqrt{2}} \overline{\psi}_e (1 + i\gamma_5)\psi_\nu T^- + \frac{1}{\sqrt{2}} \overline{\psi}_\nu (1 - i\gamma_5)\psi_e T^+ \right]$$

$$- \frac{m_\sigma^2 M_U}{4M_W\left(M_U^2 - M_W^2\right)^{\frac{1}{2}}} \sigma(\sigma^2 + \chi^2 + 2T^+T^-) \tag{AI.3}$$

$$\mathcal{L}_2 = \frac{M_U^2}{4(M_U^2 - M_W^2)} W_\mu^+ W_\mu^- (\sigma^2 + \chi^2 + 2T^+T^-) + \frac{M_U^4}{8M_W^2(M_U^2 - M_W^2)} U_\mu^2 (\sigma^2 + \chi^2)$$

$$+ \left(A_\mu + \frac{M_W}{\left(M_U^2 - M_W^2\right)^{\frac{1}{2}}} U_\mu \right)^2 T^+T^- - \frac{M_U^2}{2(M_U^2 - M_W^2)} (W_\mu^+ W_\mu^- W_\nu^+ W_\nu^- - W_\mu^+ W_\mu^+ W_\nu^- W_\nu^-)$$

$$+ W_\mu^+ W_\nu^- \left[\eta_{\mu\nu} \left(A_\lambda + \frac{M_W}{\left(M_U^2 - M_W^2\right)^{\frac{1}{2}}} U_\lambda \right)^2 - \left(A_\mu + \frac{M_W}{\left(M_U^2 - M_W^2\right)^{\frac{1}{2}}} U_\mu \right) \left(A_\nu + \frac{M_W}{\left(M_U^2 - M_W^2\right)^{\frac{1}{2}}} U_\nu \right) \right]$$

$$- \frac{M_U}{2M_W} \left(U_\mu - \frac{M_W}{\left(M_U^2 - M_W^2\right)^{\frac{1}{2}}} A_\mu \right) \left[(\sigma - i\chi)T^+ W_\mu^- + (\sigma + i\chi)T^- W_\mu^+ \right] -$$

$$- \frac{m_\sigma^2 M_U^2}{32M_W^2(M_U^2 - M_W^2)} (\sigma^2 + \chi^2 + 2T^+T^-)^2 \quad . \tag{AI.4}$$

In these expressions the convention of summing over repeated indices is used in the forms

$$A_\mu^2 = A_\mu A_\mu = \eta_{\mu\nu} A_\mu A_\nu = A_0 A_0 - A_1 A_1 - A_2 A_2 - A_3 A_3 \quad .$$

The Dirac matrices satisfy the anticommutation relations $\{\gamma_\mu,\gamma_\nu\} = 2\eta_{\mu\nu}$, so that γ_0 is hermitian while γ_1, γ_2, γ_3 and $\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$ are anti-hermitian. The covariants $\bar{\psi}\gamma_\mu\psi$, $\bar{\psi}\gamma_5\psi$ and $\bar{\psi}i\gamma_\mu\gamma_5\psi$ are real. In (AI.3) we use the abbreviation $f\overleftrightarrow{\partial}_\mu g = (\partial_\mu f)g - f(\partial_\mu g)$.

The bare propagators are determined by (AI.2) together with a gauge condition. In the Landau gauge they take the form:

$$\langle T\, W^+_\mu(x)\, W^-_\nu(0)\rangle = \int \frac{dk}{(2\pi)^4} \frac{i\hbar}{k^2 - M_W^2} \left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right) e^{-ikx}$$

$$\langle T\, U_\mu(x)\, U_\nu(0)\rangle = \int \frac{dk}{(2\pi)^4} \frac{i\hbar}{k^2 - M_U^2} \left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right) e^{-ikx}$$

$$\langle T\, A_\mu(x)\, A_\nu(0)\rangle = \int \frac{dk}{(2\pi)^4} \frac{i\hbar}{k^2} \left(-\eta_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\right) e^{-ikx}$$

$$\langle T\, \sigma(x)\, \sigma(0)\rangle = \int \frac{dk}{(2\pi)^4} \frac{i\hbar}{k^2 - m_\sigma^2}\, e^{-ikx}$$

$$\langle T\, T^+(x)\, T^-(0)\rangle = \int \frac{dk}{(2\pi)^4} \frac{i\hbar}{k^2}\, e^{-ikx}$$

$$\langle T\, \chi(x)\, \chi(0)\rangle = \int \frac{dk}{(2\pi)^4} \frac{i\hbar}{k^2}\, e^{-ikx}$$

$$\langle T\, \psi_e(x)\, \bar{\psi}_e(0)\rangle = \int \frac{dk}{(2\pi)^4} \frac{i\hbar}{\not{k} - m_e}\, e^{-ikx}$$

$$\langle T\, \psi_\nu(x)\, \bar{\psi}_\nu(0)\rangle = \frac{1 + i\gamma_5}{2} \int \frac{dk}{(2\pi)^4} \frac{i\hbar}{\not{k}}\, e^{-ikx} \quad . \tag{AI.5}$$

The contributions of the supplementary action W_{Lan} are obtained by adjoining to the Lagrangian (AI.1) the effective term

$$\mathcal{L}_{sup} = \partial_\mu C^k\, \nabla_\mu B^k$$

$$= \partial_\mu C^0\, \partial_\mu B^0 + \partial_\mu C^+\, \partial_\mu B^- + \partial_\mu C^-\, \partial_\mu B^+ -$$

$$- \frac{ieM_U}{\left(M_U^2 - M_W^2\right)^{\frac{1}{2}}} \left[W_\mu^-(B^+\partial_\mu C^0 - B^0\partial_\mu C^+) - W_\mu^+(B^-\partial_\mu C^0 - B^0\partial_\mu C^-)\right]$$

$$- ie \left(A_\mu + \frac{M_W}{\left(M_U^2 - M_W^2\right)^{\frac{1}{2}}} U_\mu\right)(B^-\partial_\mu C^+ - B^+\partial_\mu C^-) \quad . \tag{AI.6}$$

The bilinear terms in this expression define the bare propagators,

$$\langle T\ B^0(x)\ C^0(0)\rangle = \langle T\ B^+(x)\ C^-(0)\rangle = \int \frac{dk}{(2\pi)^4} \frac{i\pi}{k^2} e^{-ikx} \quad . \tag{AI.7}$$

The other combinations, $\langle T\ B\ B\rangle$ and $\langle T\ C\ C\rangle$, vanish. It must be remembered that each fictitious particle loop carries the factor -1.

APPENDIX II

WARD-TAKAHASHI IDENTITIES

It is a general rule that the presence of a continuous symmetry in the action functional is manifested by a set of identities among the Green's functions of the quantized theory. If the symmetry is a gauge group of the first kind (constant parameters) then these identities relate the n+1-point function involving the conserved current j_μ and n fields to a combination of n-point functions involving only the fields. If the symmetry is a gauge group of the second kind (spacetime-dependent parameters) then the current acts as the source of the gauge field so giving rise to identities between n+1- and n-point functions involving only fields. This latter statement must, in some cases, be qualified as will be seen in the following.

Ward-Takahashi identities of the first kind can be derived very easily in the path-integral formalism. To illustrate the procedure consider a system of fields ϕ whose Lagrangian is invariant with respect to the transformations

$$\delta\phi = \omega^k \, t^k \, \phi \tag{AII.1}$$

where the t^k are traceless generator matrices which characterize the representation and the parameters ω^k are infinitesimal constants. Then the Noether current j_μ^k satisfies the classical identity

$$\partial_\mu \, j_\mu^k = - \frac{\delta \mathcal{S}}{\delta\phi} \, t^k \, \phi \tag{AII.2}$$

where $\mathcal{S}$ denotes the classical action functional. This identity leads immediately to the Ward-Takahashi identities in the quantized theory

$$\begin{aligned}
\partial_\mu \left\langle T \, j_\mu^k(x) \, F(\phi) \right\rangle &= \int (d\phi) \, \partial_\mu j_\mu^k(x) \, F(\phi) \, e^{\frac{i}{\hbar}\mathcal{S}} \\
&= \int (d\phi) \, F(\phi) \left(- \frac{\hbar}{i} \frac{\delta}{\delta\phi(x)} \, e^{\frac{i}{\hbar}\mathcal{S}} \right) t^k \phi(x) \\
&= \frac{\hbar}{i} \int (d\phi) \, \frac{\delta F}{\delta\phi(x)} \, t^k \phi(x) \, e^{\frac{i}{\hbar}\mathcal{S}} \\
&= \frac{\hbar}{i} \left\langle T \, \frac{\delta F}{\delta\phi(x)} \, t^k \phi(x) \right\rangle
\end{aligned} \tag{AII.3}$$

after an integration by parts. Here $F(\phi)$ denotes any functional of the fields. If, for example, $F(\phi)$ is a simple product (2.3) then (AII.3) coincides with the usual Ward-Takahashi identities for Green's functions. Alternatively, inserting the functional (2.4), which satisfies $\delta F/\delta\phi = 0$, one deduces from (AII.3) that matrix elements of $\partial_\mu j^k_\mu$ vanish between physical states.

The same technique can be used to derive identities in theories where the symmetry is violated asymptotically. For example, if a chiral symmetry is realized non-linearly then the linear homogeneous form $t^k\phi$ is replaced by the more general expression $t^k(\phi)$. If $t^k(0)$ fails to vanish then the asymptotic symmetry is violated even if the action functional and the measure are invariant. Although the Noether current is conserved it will contain a term which is linear in ϕ. One can show that this term is a linear combination of massless (Goldstone) fields. The presence of such a term makes it impossible to integrate the Noether density over a spacelike surface to obtain a conserved charge. (Since the symmetry is violated asymptotically there can be no conserved charge.) It has therefore become customary in such cases to subtract the offending linear term from the Noether current so as to define a new current which is only "partially" conserved but which is at least integrable. Define the new current

$$J^k_\mu = j^k_\mu - \text{linear terms} \tag{AII.4}$$

and, similarly, the source currents J for the fields

$$J = \frac{\delta \mathcal{S}}{\delta\phi} - \text{linear terms} \quad . \tag{AII.5}$$

The classical identity (AII.2) - with $t^k\phi$ replaced by $t^k(\phi)$ - now takes the form

$$\partial_\mu J^k_\mu + J\, t^k(0) = - \frac{\delta\mathcal{S}}{\delta\phi}\left[t^k(\phi) - t^k(0)\right] \tag{AII.6}$$

which contains no linear terms. The corresponding Ward-Takahashi identities are given by

$$\begin{aligned}\partial_\mu \langle T\, J^k_\mu(x)\, F(\phi)\rangle &+ \langle T\, J(x)\, F(\phi)\rangle\; t^k(0) \\ &= \frac{\hbar}{i}\left\langle T\, \frac{\delta F}{\delta\phi(x)}\left[t^k(\phi) - t^k(0)\right]\right\rangle\end{aligned} \tag{AII.7}$$

which results from a partial integration in the path-integral representation[26].
Insertion of, for example, the functional (2.4) into (AII.7) leads to the well-known result that the matrix elements of the Goldstone boson source, $J\, t^k(0)$, coincide with those of the operator, $-\partial_\mu J_\mu^k$.

The derivation of Ward-Takahashi identities given here is, of course, only formal. A correct derivation would have to employ a regularized action functional with counterterms and would bring out the anomalous terms which we have ignored.

For application to theories with gauge symmetries of the second kind a modification is needed. Since the total action discussed in Sec.II is not local one cannot apply the Noether theorem directly. Moreover, since the action is gauge dependent so also are the currents. In the canonical gauges, for example, they disappear altogether. The action functional (2.7),(2.9),

$$\mathcal{S} = \mathcal{S}_{\mathcal{L}} + \mathcal{A}_h + W_h \tag{AII.8}$$

generally contains, in addition to the invariant local piece $\mathcal{S}_{\mathcal{L}}$, the non-invariant, gauge-determining piece $\mathcal{A}_h$ and the invariant, non-local, gauge-compensating piece W_h . The gauge-determining piece must be non-invariant under transformations of the second kind - requirement a) of Sec.II - but it could all the same be invariant under first-kind transformations. If it is not then there is nothing more to be done: in such a gauge there will be no current which is even partially conserved. (This situation is exemplified by the canonical gauge.) Let us assume that $\mathcal{A}_h$ is not only invariant under first-kind transformations but also local so that it may be incorporated with $\mathcal{S}_{\mathcal{L}}$ to make up the lagrangian part of the action,

$$\mathcal{S}_{\mathfrak{L}} = \mathcal{S}_{\mathcal{L}} + \mathcal{A}_h \tag{AII.9}$$

where $\delta\mathfrak{L}$ vanishes for gauge transformations of the first kind. The corresponding Noether identity takes the form

$$\begin{aligned}\partial_\mu\, {}^N J_\mu^k &= -\frac{\delta \mathcal{S}_{\mathfrak{L}}}{\delta\phi} t^k(\phi) \\ &= -\frac{\delta \mathcal{S}}{\delta\phi} t^k(\phi) + \frac{\delta W_h}{\delta\phi}\; t^k(\phi) \quad .\end{aligned} \tag{AII.10}$$

Since W_h is invariant, however,

$$\delta W_h = \omega^k \int dx \frac{\delta W_h}{\delta\phi} t^k(\phi) = 0 \quad ,$$

it must always be possible to express the last term in (AII.10) as a total divergence,

$$\frac{\delta W_h}{\delta\phi} t^k(\phi) = - \partial_\mu w_\mu^k \quad . \tag{AII.11}$$

It is therefore natural to define the total current

$$j_\mu^k = {}^N j_\mu^k + w_\mu^k \quad , \tag{AII.12}$$

which satisfies the same identity as those discussed previously,

$$\partial_\mu j_\mu^k = - \frac{\delta \mathcal{S}}{\delta\phi} t^k(\phi) \quad . \tag{AII.13}$$

Thus, in theories with a gauge symmetry of the second kind, it is necessary to include along with the Noether current a non-local term (the fictitious particle contribution) in order to obtain a conserved current. Of course, if $t^k(0) \neq 0$ it will be necessary to separate the Goldstone term and deal with a partially conserved current as before.

There remains the problem, mentioned above, of relating the conserved current j_μ to the source of the gauge field. To do this it is necessary to elaborate the notation by distinguishing the gauge field A_μ from the other fields ϕ' . The gauge field has the characteristic transformation behaviour,

$$\delta A_\mu^k = - \frac{1}{g} \partial_\mu \Omega^k + (A_\mu \times \Omega)^k \quad . \tag{AII.14}$$

An infinitesimal transformation of the action gives

$$\begin{aligned} \delta \mathcal{S} &= \int dx \left[\frac{\delta \mathcal{S}}{\delta A_\mu^k} \delta A_\mu^k + \frac{\delta \mathcal{S}}{\delta \phi'} \delta\phi' \right] \\ &= \int dx \left[- \frac{1}{g} \frac{\delta \mathcal{S}}{\delta A_\mu^k} \partial_\mu \Omega^k + \frac{\delta \mathcal{S}}{\delta\phi} \Omega^k t^k(\phi) \right] \\ &= \int dx \, \Omega^k \, \partial_\mu \left[\frac{1}{g} \frac{\delta \mathcal{S}}{\delta A_\mu^k} - j_\mu^k \right] \end{aligned} \tag{AII.15}$$

after integrating by parts and using the identity (AII.13). On the other hand, $\delta\mathcal{S} = \delta\mathcal{A}_h$ since both $\mathcal{S}_{\mathcal{L}}$ and W_h are invariant. But $\mathcal{A}_h$ has been assumed to be invariant with respect to transformations of the first kind ($\partial_\mu \Omega^k = 0$) so that one can write

$$\delta \mathcal{A}_h = - \int dx \; \partial_\mu \Omega^k \; a^k_\mu \quad . \qquad \text{(AII.16)}$$

Since $\Omega^k(x)$ is arbitrary, the equality of (AII.15) and (AII.16) implies the identity

$$\frac{1}{g} \partial_\mu \frac{\delta \mathcal{S}}{\delta A^k_\mu} = \partial_\mu \; (j^k_\mu + a^k_\mu) \quad . \qquad \text{(AII.17)}$$

The gauge field source ${}^A J_\mu$ is defined (cf.(AII.5)) by

$${}^A J^k_\mu = \frac{\delta \mathcal{S}}{\delta A^k_\mu} - \text{linear terms} \qquad \text{(AII.18)}$$

and we are led to make the identification

$$\frac{1}{g} \, {}^A J^k_\mu = j^k_\mu + a^k_\mu - \text{linear terms} \quad . \qquad \text{(AII.19)}$$

Our derivation has fixed only the longitudinal part of this identity. We can regard (AII.19) as a definition insofar as the transverse part is concerned since the transverse part of the Noether current can be modified at will by adding divergence terms to the lagrangian density. It therefore turns out that the gauge particle source coincides with the conserved current j_μ (or the partially conserved J_μ defined by (AII.4)) apart from the non-linear terms (if any) in a_μ . It is possible to find gauges in which the latter term is absent (i.e. linear a_μ) but in general it must be taken into account.

This completes the general discussion of Ward-Takahashi identities. The formulae derived here can now be applied to the cases discussed in the body of the paper. Consider first the massive Yang-Mills theory of Sec.III.

The Landau gauge action takes the form

$$\mathcal{S} = \int dx \left(\mathcal{L} + C^k \partial_\mu A^k_\mu \right) + W_{Lan}(A) \qquad \text{(AII.20)}$$

where $\mathcal{L}$ is given by (3.6) and W_{Lan} by (3.20). The field C^k is a Lagrange multiplier to which we assign the transformation rule

$$\delta C = C \times \Omega \quad . \tag{AII.21}$$

The other fields transform according to the rules (3.7). (Only δB^k fails to vanish asymptotically for constant Ω .) The source currents for A and B are given by

$$
\begin{aligned}
{}^A J_\mu &= - g\, A_\nu \times F_{\nu\mu} - g\, \partial_\nu (A_\nu \times A_\mu) + \frac{g}{2} (\sigma \partial_\mu B - B \partial_\mu \sigma - B \times \partial_\mu B) \\
&\quad + M g \sigma\, A_\mu + \frac{g^2}{4} (\sigma^2 + B^2)\, A_\mu + \frac{\delta W_{Lan}}{\delta A_\mu}
\end{aligned}
$$

$$
{}^B J = - g\, \partial_\mu (\sigma A_\mu + B \times A_\mu) + \frac{g^2}{4}\, B\, A_\mu^2 - \frac{g m^2}{2M} B\sigma - \frac{g^2 m^2}{8M^2} B(\sigma^2 + B^2) \quad . \tag{AII.22}
$$

The contribution a_μ due to the gauge-determining Lagrange multiplier term is given by

$$a_\mu = - \frac{i}{g} \nabla_\mu C \quad . \tag{AII.23}$$

The partially conserved current which corresponds to the Yang-Mills symmetry is given by substituting from (AII.19) into (AII.4). The result is

$$
\begin{aligned}
J_\mu &= - A_\nu \times (F_{\nu\mu} + \eta_{\nu\mu} C) - \partial_\nu (A_\nu \times A_\mu) + \frac{1}{g} \frac{\delta W_{Lan}}{\delta A_\mu} \\
&\quad + \frac{1}{2} (\sigma \partial_\mu B - B \partial_\mu \sigma - B \times \partial_\mu B) + M\sigma\, A_\mu + \frac{g}{4} (\sigma^2 + B^2)\, A_\mu
\end{aligned}
\tag{AII.24}
$$

where $\eta_{\nu\mu}$ denotes the minkowskian metric tensor. This current satisfies the identity

$$\partial_\mu J_\mu^k + \frac{M}{g}\, {}^B J^k = - \epsilon^{k\ell m} \frac{\delta \mathcal{L}}{\delta A_\mu^\ell} A_\mu^m - \frac{1}{2} \frac{\delta \mathcal{L}}{\delta B^k} \sigma - \frac{1}{2} \epsilon^{k\ell m} \frac{\delta \mathcal{L}}{\delta B^\ell} B^m + \frac{1}{2} \frac{\delta \mathcal{L}}{\delta \sigma} B^k \tag{AII.25}$$

from which the Ward-Takahashi identities follow:

$$
\begin{aligned}
&\partial_\mu \left\langle T\, J_\mu^k\, F \right\rangle + \frac{M}{g} \left\langle T^B\, J^k\, F \right\rangle = \\
&= \frac{\hbar}{i} \left\langle T \left\{ \epsilon^{k\ell m} \frac{\delta F}{\delta A_\mu^\ell} A_\mu^m + \frac{1}{2} \frac{\delta F}{\delta B^k} \sigma + \frac{1}{2} \epsilon^{k\ell m} \frac{\delta F}{\delta B^\ell} B^m - \frac{1}{2} \frac{\delta F}{\delta \sigma} B^k \right\} \right\rangle
\end{aligned}
\tag{AII.26}
$$

where F denotes any functional of the fields A_μ, B and σ .

A distinct family of Ward-Takahashi identities corresponds to the conservation of the "true" isospin which is associated with the asymptotic symmetry (3.8). The current is given by

$$I_\mu = - A_\nu \times (F_{\nu\mu} + \eta_{\nu\mu}\, C) - \partial_\nu (A_\nu \times A_\mu) + \frac{1}{g} \frac{\delta W_{\mathrm{Lan}}}{\delta A_\mu} - B \times \partial_\mu B \tag{AII.27}$$

and the Ward-Takahashi identities take the usual form,

$$\partial_\mu \left\langle T\, I^k_\mu\, F \right\rangle = \frac{\hbar}{i} \left\langle T\, \varepsilon^{k\ell m} \left\{ \frac{\delta F}{\delta A^\ell_\mu} A^m_\mu + \frac{\delta F}{\delta B^\ell} B^m \right\} \right\rangle \quad . \tag{AII.28}$$

The same procedure could be applied to the model of Sec.IV whose action is given in detail in Appendix I. The resulting identities would, of course, be very complicated.

FOOTNOTES AND REFERENCES

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8) This model was proposed by S. Weinberg, Ref.5. It is included in a more general scheme for combining weak and electromagnetic interactions based on U(3) symmetry, by Abdus Salam and J.C. Ward, Phys. Letters 13, 168 (1964).

9) This paper is confined to a treatment of electron-type leptons. Some remarks concerning the inclusion of muon-type leptons are made in Sec.V.

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14) This theory was treated by t'Hooft, Ref.6, but in a different gauge from the ones considered here.

15) A canonical gauge computation of the one-loop corrections to the σ–σ elastic scattering amplitude in the abelian version of this theory is in progress. We have so far verified that the quartic asymptotic behaviour which one might have expected to find is in fact not present in the mass-shell amplitude.

16) Abdus Salam and J.C. Ward, Ref.8.

17) It appears that U_μ couples to a lepton magnetic monopole in this version.

18) T.D. Lee, "A speculative determination of the intermediate boson mass", Columbia preprint NYO-1932(2)-190 (1971). Note that $g_{\text{Lee}} = g/2\sqrt{2}$.

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21) See for example, Abdus Salam, "Computation of renormalization constants", in Proceedings of Coral Gables Conference (1971).

22) C.J. Isham, Abdus Salam and J. Strathdee, Phys. Rev. D3, 1805 (1971) and ICTP, Trieste, preprint IC/71/14 (to appear in Phys. Rev.).

23) A dangerous possibility would be the appearance of Adler's anomalies in the partial conservation laws (see Appendix II). Such an effect would cause one to doubt the gauge independence of physical S-matrix elements and, particularly, the cancellation of ghosts against Goldstone particles.

24) However, a preliminary analysis indicates that the branch point singularities in the external lines are very mild. Neither the masses nor the wave function renormalization constants appear to be infra-red divergent.

25) B. Zumino and W. Bardeen, private communication.

26) Some care must be exercised in this step since theories where $t^k(\phi)$ is not linear also have a path-integral measure which is not of the simple product type $\prod_x d\phi(x)$.

TABLE I

	I	I^3	I^0	Q	$2g^2I^3 - g_1^2I^0$
ν_L	$\frac{1}{2}$	$\frac{1}{2}$	-1	0	$g^2 + g_1^2$
e_L	$\frac{1}{2}$	$-\frac{1}{2}$	-1	-1	$-g^2 + g_1^2$
e_R	0	0	-2	-1	$2g_1^2$
T^+	$\frac{1}{2}$	$\frac{1}{2}$	1	1	$g^2 - g_1^2$
T^0	$\frac{1}{2}$	$-\frac{1}{2}$	1	0	$-g^2 - g_1^2$
W^+	1	1	0	1	$2g^2$
X^3	1	0	0	0	0
W^-	1	-1	0	-1	$-2g^2$
X^0	0	0	0	0	0

U(2) quantum number assignments. The neutral gauge fields, A_μ and U_μ, are mixtures of $I = 1$ and $I = 0$ fields, X^3 and X^0.

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