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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

FINITE FIELD THEORIES

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I. INTRODUCTION

Up to the end of the fifties, the chief problem in theoretical particle physics was a search for a finite (infinity-free) local, unitary and analytic field theory. As a result of the work done during the last 7-8 years this problem appears near to a solution. Entire function Lagrangians of the variety

$$\mathcal{L}_{int} = g \exp(\kappa\phi) \qquad (1)$$

and Lagrangians of the type

$$\mathcal{L}_{int} = g(\bar{\psi}\psi A) \exp(\kappa\phi) \qquad (2)$$

appear to give finite, local field theories where the so-called minor coupling constant κ (with dimensions of inverse mass) appears to play the role of an inbuilt cut-off. From this point of view the task of experiment and theory in weak, electromagnetic and strong interactions is not only to specify the major coupling constants g, but also to specify the inbuilt cut-offs κ^{-1} which may exhibit wide variations for different interactions. I wish briefly to review these developments in a non-rigorous fashion, emphasising in particular the following points:

1) Theories of types (1) and (2) are no strangers to physics. In strong interaction physics, the following theories are wholly or partially non-polynomial:

A. Chiral Lagrangians:

$$\mathcal{L} = \mathrm{Tr}\ \partial_\mu S \partial_\mu S^\dagger \ , \quad S = \exp(i\lambda\tau\cdot\pi) \ . \qquad (3)$$

The minor constant $\lambda \approx m_\pi^{-1}$.

B. Conformal Lagrangians containing dilatons.

C. Massive Yang-Mills Lagrangians for 1^- and 1^+ particles in the Boulware form.[1)]

D. Intermediate boson-mediated weak interactions where also the induced scalar and pseudoscalar interactions are intrinsically non-polynomial [2)] with the minor constant $\sqrt{G_F}$.

E. And finally the Einstein Lagrangian for gravity (intrinsically non-polynomial with the minor (and major) constant (both) equal to $\kappa \approx \sqrt{8\pi G_N} \approx 10^{-22}\ m_e^{-1}$) as well as <u>all gravity-modified matter Lagrangians</u>. If there is no inbuilt cut-off in any given theory lying lower than $1/\kappa \approx 10^{19}$ BeV, the <u>ultimate cut-off of gravity will always apply</u>.

2) The precise theorem which shows how the cut-off $1/\kappa$ acts is the following:

<u>Theorem</u>

Theories of type (1) and (2) possess an inbuilt cut-off $\approx 4\pi/\kappa$. Any matrix element computed to a given order g^n <u>and exactly to all orders in κ is finite</u>. In particular, if in (2) we take $\kappa \longrightarrow 0$ the "old" infinities of the renormalizable theory $g\bar{\psi}\gamma A\psi$ reappear. When $\kappa \neq 0$, these are regularized according to the following code:

The old $\log \infty$ is regularized to $\log \frac{4\pi}{\kappa}$;

the old linear infinities ∞^1 to $\frac{4\pi}{\kappa}$;

the old quadratic infinities ∞^2 to $\left(\frac{4\pi}{\kappa}\right)^2$,

and the old quintic infinity ∞^4 to $\left(\frac{4\pi}{\kappa}\right)^4$.

3) Finally, I wish to emphasise a feature of these finite theories which may be of great significance for strong interaction physics. This is the dependence of form factors on space-like momenta. In the very simplest of approximations (order g^2, all orders in κ) we obtain for large t ($t < 0$):

$$G(t) \approx g^2 \ \exp - |\kappa^2 t|^{\alpha} \ , \qquad (\alpha < \tfrac{1}{2})$$

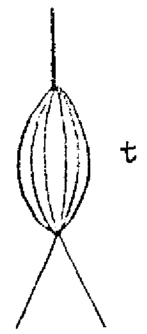


Fig.1

If strong interaction Lagrangians are chiral or conformal with a universal dilaton coupling (or both), such behaviour of G(t) may provide the readiest understanding of the mysterious empirical circumstance that transverse momenta $|p_T|$ of secondaries in inclusive reactions appear to be bounded.

II. EXPONENTIAL LAGRANGIAN THEORIES AS "GOOD" FIELD THEORIES

A detailed review of the technical aspects of non-polynomial theories was presented at the 1971 Miami Conference (IC/71/3) as also the credits to those who have worked in this field. I shall here simply summarize some of the recent major results.

1) The fundamental assumption which goes into the working out of non-polynomial theories is the euclidicity ansatz.

We always start perturbation theory computations in the Symanzik region of external momenta (this is the region where scalar products of all momenta are essentially space-like). There are no production thresholds and one can formally make a Wick-rotation to euclidean space-time. After the computations have been performed, one continues analytically to the physical subspace of momenta. It is a real miracle that the continued theory should exhibit unitarity and analyticity in the perturbation-theoretic sense. That this is indeed the case has recently been claimed by Taylor, Lehmann and Pohlmeyer.

2) All S-matrix computations are carried out to a given order in the major constant g but to all orders in the minor constant κ.

3) In computing S-matrix elements one needs to evaluate Fourier transforms of products of singular distributions like

$$\left(\frac{1}{x^2}\right)^{n_1} \otimes \left(\frac{1}{x^2}\right)^{n_2} \quad \text{or} \quad \left(\frac{1}{x^2}\right)^{n_1} \otimes (\log x^2)^{n_2} \quad . \tag{4}$$

This is when no derivatives are involved in the interactions. In derivative-containing theories, we also come across products like [3)]

$$\partial_\mu \left(\frac{1}{x^2}\right)^{n_1} \otimes \partial_\nu \left(\frac{1}{x^2}\right)^{n_2} \quad \text{or} \quad \partial^2 \left(\frac{1}{x^2}\right)^{n_1} \otimes \left(\frac{1}{x^2}\right)^{n_2} \quad . \tag{5}$$

The conventional field-theoretic infinities have their origin in the difficulties associated with defining these products and their Fourier transforms. Apparently the biggest disaster in the science of infinities has been the use of momentum space methods at too early a stage of the computations. Momentum space methods obscure the underlying distribution theory in x-space and tend to confuse the real physical origin of the infinities (polynomiality of the Lagrangians produced by taking the limit $\kappa \to 0$ in (2)) with (in this context spurious) mathematical ambiguities of ill-defined products of distributions like (5). Although no general theory of products of distributions exists at present, a consistent use of methods developed by Gel'fand and Shilov, together with appropriate analytic continuations, permits us to separate the two types of problems, as we shall see in the example below.

4) Consider the model electrodynamic interaction $\mathcal{L} = e\bar{\psi}\psi A \exp(\kappa\phi)$ where ψ is the electron field, A is the (scalar) photon and ϕ is a scalar field which I shall call "gravity", a nomenclature which I shall justify later.

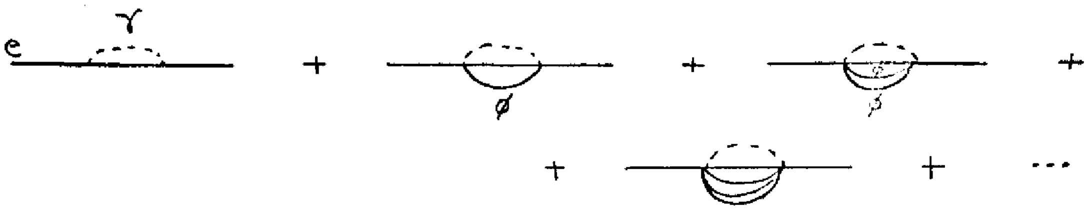
We wish to exhibit the realistic regularization of the otherwise infinite electron self-mass through gravity. Writing the most singular parts of the relevant propagators in the form (I have dropped some factors of 4π)

$$\langle \psi\bar{\psi} \rangle = -(i\,\gamma\cdot\partial + m)\,\frac{1}{x^2} + \text{less singular terms}$$

$$\langle AA \rangle = -\frac{1}{x^2}$$

$$\langle \phi\phi \rangle = -\frac{1}{x^2}$$

and noting that $\langle e^{\kappa\phi(x)}\, e^{\kappa\phi(0)} \rangle = \exp(-\kappa^2/x^2)$, the contribution to the electron self-mass from the sum of the chain of graphs



$$F(x) = \alpha \sum_{n=0} \frac{1}{n!} \left\{ (i\,\gamma\cdot\partial + m) \left[-\frac{1}{x^2}\right] \right\} \left[-\frac{\kappa^2}{x^2}\right]^n \left[-\frac{1}{x^2}\right] .$$

Using a Sommerfeld-Watson transform this equals

$$= \frac{\alpha}{2\pi i} \int dz\, \frac{(\kappa^2)^z\,(-\lambda)^z}{\Gamma(z+1)\,\sin\pi z} \left[-\frac{1}{x^2}\right]^{z+1} \left[i\,\gamma\cdot\partial + m\right] \left[-\frac{1}{x^2}\right] ,$$

where the contour lies round the positive real z-axis. This contour may be rotated to lie parallel to the imaginary axis to give

$$F(x) = \frac{\alpha}{2\pi i} \int_{\mathrm{Re}\,z<0} dz\, \frac{(\kappa^2)^z\,(-\lambda)^z}{\Gamma(z+1)\,\sin\pi z} \left(\frac{i\,\gamma\cdot\partial}{Z+2} + m\right) \left(-\frac{1}{x^2}\right)^{z+2} .$$

The advantage of doing this is that now we can use the unambiguous expression for the Fourier transform of the (classical) function $(-1/x^2)^z$ valid in the range $0 < \mathrm{Re}\, z < 2$, given by the expression $-1/(4\pi)^2\ (-p^2/(4\pi)^2)$ $(\Gamma(2-z))/(\Gamma(z))$ (see for example Gel'fand and Shilov, "Generalized Functions", Vol.I). Thus

$$\tilde{F}(p) = -\frac{\alpha}{(2\pi i)(4\pi)^2} \int_{\mathrm{Re}\, z\, 0} \frac{dz(\kappa^2)^z\,(-\lambda)^z}{(\sin\pi z)\Gamma(z+1)} \times$$

$$\times \left\{-\frac{\not{p}}{z+2} + m\right\} \left(-\frac{p^2}{(4\pi)^2}\right)^z \frac{\Gamma(-z)}{\Gamma(z+2)} \ .$$

$\tilde{F}(p)$ can be expressed as a sum of well-known Meijer G-functions. To see its behaviour as a power series in κ^2, we rotate the contour back to the real axis. The double pole in z-space at $z = 0$ gives regularized contributions to self-mass $\tilde{F}(p)|_{\not{p}-m=0}$ of the form $\alpha\, m \log(\kappa^2 m^2)$. The next term is $\alpha\,\kappa^2 m^2 \log(\kappa^2 m^2)$. The conventional Weisskopf logarithmic infinity of $\delta m/m$ is instantly recovered by taking the limit $\kappa \to 0$. For $\kappa \neq 0$, the old infinity is regularized in the manner stated in the Theorem in Sec.I ($\delta m/m \approx \alpha \log \kappa^2 m^2$).

I am sure you will agree that the infinity suppression mechanism of exponential theories is so transparent, so elegant and so easily exhibited that the calculation like the one above (with 4π's supplied) should form part of the first year courses in quantum field theory.

5) How unambiguous is the above procedure? To decide this, we must first distinguish between entire function non-polynomial Lagrangians like $\exp \kappa\phi$ which are localizable and micro-causal in the sense of Jaffe and rational non-polynomials like $1/(1+\kappa\phi)$ which are non-localizable and not causal. Lehmann and Pohlmeyer have considered possible ambiguities in local theories and have shown that all possible distribution-theoretic ambiguities in superpropagators like

$$\langle T{:}\exp\kappa\phi(x) - 1{:}\quad {:}\exp\kappa\phi(0) - 1{:}\rangle_0$$

can be removed from the theory by imposing on it one single ansatz which at the present stage of our understanding of these theories must be added to the list of assumptions made. What Lehmann and Pohlmeyer show is that in local theories one can define a "minimally singular" superpropagator. To this one may add the so-called ambiguity terms in a well-specified manner. However, the ambiguous terms are sharply distinguished from the minimal solution by their high-energy behaviour. The minimal superpropagator falls to zero along a specified direction in the complex energy-plane; the ambiguous terms never fall in any direction. In the example I worked out (to compute $\delta m/m$) I exhibited the Lehmann-Pohlmeyer minimal superpropagator. Why the minimally singular solutions of these non-polynomial theories should represent physics we do not know. We shall, however, for the present accept this ansatz, stressing once again that such an ansatz can be formulated for finite local non-polynomial theories only. It cannot be formulated for local polynomial theories nor, of course, for non-local theories. (In a recent Trieste preprint (IC/71/72), Bollini and Giambiagi have argued that for local theories the euclidicity hypothesis already guarantees freedom from ambiguities and automatically leads to the minimal solution of Lehmann-Pohlmeyer so that minimality ansatz is not a new ansatz but part of the euclidicity hypothesis.)

6) For local Lagrangians, one expects Borchers' theorem to apply. This should permit us to make such field transformations which preserve the locality of the Lagrangian. Under these, on-mass-shell S-matrix elements should remain unchanged.

7) According to a recently proved (Communications in Mathematical Physics, 1969) and extremely important theorem of Epstein, Glaser and Martin, the on-shell S-matrix elements in a local theory must be Froissart bounded. This means that, at any rate <u>after</u> summing the perturbation theory over the

major constant, Froissart boundedness must manifest itself. Since, in practice, such summations are difficult to carry out exactly, let us examine the mass-shell behaviour of individual or groups of graphs in non-polynomial theories.

As I stated earlier, the situation is beautiful for (space-like) momentum-transfer variables. A (form factor-like) graph, for example the one shown in Fig.2:

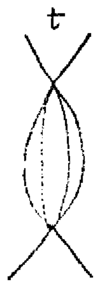
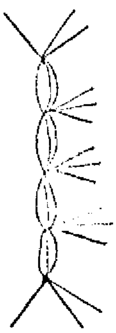


Fig.2

exhibits a dependence

$$F(t) \approx \exp\left(-|\kappa^2 t|^{(1/3)}\right) \quad \text{for large } t.$$

(This is the asymptotic expression for the relevant Meijer G-function.) This means that a multiperipheral graph of the type



(consisting of a string of bubble graphs - each behaving like $\exp(-|\kappa^2 t|^{(1/3)})$ will automatically give rise to boundedness among the transverse momenta of the secondaries emitted.

Consider now the situation for time-like energies. The Meijer's G-function referred to above now sums to

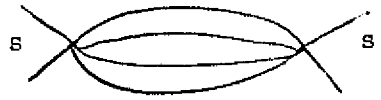


Fig.3

$$M(s) \approx i \exp|s\kappa^2|^{(1/3)} \quad .$$

The factor i signals that essentially the entire contribution comes from the many-body phase-space of the intermediate particles. Clearly this is unacceptable behaviour for a physical matrix element. It also contradicts the exact theorem of Martin, Glaser and Epstein. One needs a summation over the major constant; for example, a chain graph like

will sum to

$$\frac{1}{1 - M(s)}$$

and exhibit Froissart boundedness.

This summation, however, is naive. It will introduce its own unphysical singularities through the poles in the denominator. A superior procedure is to adopt the Redmond-Bogolubov method of summation of chain graphs. This method relies on noticing that the unitarity relation states

$$\text{Im } T^{-1}(s) = - \rho(s)$$

where $\rho(s)$ is the many-body phase-space term.

As stated before, for the superpropagator in Fig.3, the phase-space factor behaves like $\exp|\kappa^2 s|^{(1/3)}$. Thus

$$\text{Im } T^{-1}(s) \approx - \exp\left(|\kappa^2 s|^{(1/3)}\right)$$

or more exactly

$$\text{Im } T(s) = \text{Im } \frac{1}{1-M(s)} \approx \frac{\exp|\kappa^2 s|^{1/3}}{|1 + c \exp|\kappa^2 s|^{\frac{1}{3}}|^2} \approx \exp - |\kappa^2 s|^{1/3} \quad ,$$

which is consistent (though not identical) with the naive chain summation which gave

$$T \approx \frac{1}{1 - M} \quad .$$

One can now use a (no-subtraction) dispersion formula to write

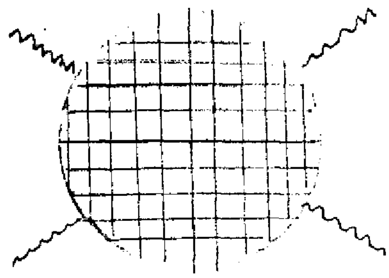
$$T(s) = \int \frac{ds'}{s' - s} \left\{ \text{Im } \frac{1}{1 - M(s')} \right\} \quad .$$

Applying this method to the 2-point propagator $\Delta(s)$ (and this applies also to theories with derivative couplings like $(\partial\varphi)^2 \, (e^{\kappa\phi} - 1)$) it is clear that the propagator will behave like

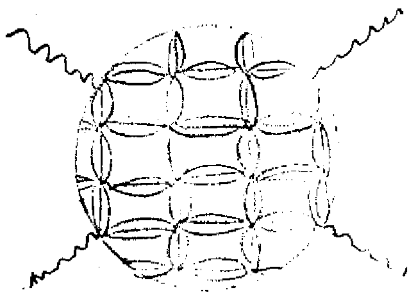
$$\Delta(s) = \int \frac{ds' \; e^{-|\kappa^2 s'|^{(1/3)}}}{s - s'} \, , \quad \text{i.e.} \quad \Delta(s) \sim \frac{1}{s} \, ,$$

i.e., the Lehmann-Källen theorem for propagators holds, as it should in a local theory. The subtraction constants of the theory, if any are needed, are all finite and the propagator on the light cone behaves like a free-field propagator $1/x^2$.[4] (As is well known, this is a key point when one considers whether a theory scales or not.)[5]

The summation of chain graphs for the exponential Lagrangian is particularly easy. The general case for other non-polynomial Lagrangians is no more difficult and has been worked out with Delbourgo and Strathdee. A different type of summation over fishnet diagrams has been mentioned by Sakita and Virasoro who, following Nielson, consider fishnets like



and show that the fishnet exhibits Veneziano-Regge behaviour, provided each line factor $1/p^2$ in the fishnet is replaced by a superpropagator factor $\approx \exp - (\kappa^2 p^2)$



I wish to stress that I consider the Bogolubov-Redmond or Sakita-Virasoro methods of summation of chain graphs as having only a suggestive and heuristic value and no more. The problem of implementing the Martin-Glaser-Epstein bound is the main unsolved problem of local non-polynomial theories and deserves further study.

TABLE I

Distinction between causal, local (e.g. exp $\kappa\phi$) and non-local theories (e.g. $1/(1+\kappa\phi)$). Local theories appear to possess all desirable features of good field theories.

Causal, local, exp $\kappa\phi$	Non-local $(1+\kappa\phi)^{-1}$
1) Superpropagators are entire functions of (κ^2/x^2) $\langle T \exp\kappa\phi(x) \exp\kappa\phi(0)\rangle = e^{-\kappa^2/x^2}$	Superpropagators are represented by divergent series $\langle T(1+\kappa\phi(x))^{-1}, (1+\kappa\phi(0))^{-1}\rangle = \sum n! \left(\frac{-\kappa^2}{x^2}\right)^n$
2) Theory analytic, unitary and positive definite	Unitary but positive-definiteness not clear
3) Borchers' theorem valid; changes of variables which preserve locality leave on-shell matrix elements unchanged	?
4) Martin-Glaser-Epstein theorem guarantees on-shell Froissart boundedness	?
5) Lehmann-Pohlmeyer ansatz guarantees freedom from ambiguities	?

cont.

TABLE I (continued)

Causal, local, exp $\kappa\phi$	Non-local $(1 + \kappa\phi)^{-1}$
6) Second order (or linear chain) form factor for large $t < 0$ $\lvert G(t) \rvert \approx g^2 \exp -\lvert \kappa^2 t \rvert^{1/3}$ (neglecting possible oscillations)	$G(t) \approx \dfrac{1}{(1 + \kappa^2 t)^3}$

8) Symanzik (private communication) and Fivel (Maryland preprint) have remarked that in an N-point graph any two points are joined through one and no more than one superpropagator (the total number of superpropagators for an N-point graph being equal to $N(N-1)/2$). This means that a typical graph in non-polynomial theories strongly resembles cluster graphs in statistical mechanics. Thus, in contrast to graphs in polynomial theories, their topological properties are much simpler. Exploiting this analogy, Fivel has made some exciting conjectures about the convergence of the entire perturbation series, defining correlation lengths for N-point graphs. I believe these analogies will prove of value in connection with giving a Lagrangian basis to the current models of hadrons as "Feynman liquids".

9) I have been discussing almost entirely theories with no derivatives in the coupling. Most of the physical theories of interest - chiral, Boulware-modified massive Yang-Mills and gravity theory - contain not one but two derivatives in the interaction. In addition, these theories pose the extra problem of arranging the calculation in such a manner that various types of gauge invariances are respected. We (Isham, Strathdee and Salam) have made a small beginning towards this - in particular towards arranging perturbation calculations in gravity-modified electrodynamics such that the electromagnetic invariance is manifest. In carrying through this programme [IC/71/13 (Phys. Letters 35B, 585 (1971)) & IC/71/14] we found we need to develop a calculus of derivatives (extending the Gel'fand-Shilov analytic continuation method) to products like

$$\partial_\mu \partial_\nu \frac{1}{x^2} \otimes \left(\frac{1}{x^2}\right)^z$$

(see also the related work of Patani and Lazarides IC/71/38 and Bollini and Giambiagi IC/71/72). This mathematical development, however, is still in its infancy. We have, for example, not yet shown that our computation of $\delta m/m$ in gravity-modified electrodynamics is gravitational gauge invariant (generally covariant) though we have exhibited its electromagnetic gauge invariance.

TABLE II

	Lagrangian	Inbuilt cut-off
1) Gravity-modified lepton electrodynamics	$\dfrac{L^{\mu a}\,\overline{\psi}\,\gamma_a(\partial_\mu - ie\,A_\mu)\psi + m\,\overline{\psi}\psi}{\det L}$ $L^{\mu a} = \exp(\kappa\phi^{ab}\,\gamma_{ab})$ where γ_{ab} are 4×4 symmetric matrices and $\phi^{ab} = \phi^{ba}$ are the 10 fields describing gravitons	at $\frac{4\pi}{\kappa}$ where $\kappa \approx \sqrt{16\pi G_N}$ $\approx 10^{-22}\,m_e^{-1}$ $\frac{\delta m}{m} = 3/4\pi \;\log \kappa^2 m^2 + O(\alpha, \kappa^2\alpha)$
2) Intermediate boson mediated weak (leptonic) interaction	A U(2) Yang-Mills theory of three weak fields W^+, W^-, X^0 together with the electromagnetic field A^0 proposed by Abdus Salam and J.C. Ward (Phys. Letters 13, 168 (1964)), the Nobel Symposium, Gothenburg (1968) and, independently, by S. Weinberg (Phys. Rev. Letters 19, 1264 (1967)) and considered by T.D. Lee (1971). ($M_{X^0} > M_{W^\pm} \gtrsim 37.8$ BeV) has been shown to be renormalizable by G. t'Hooft (Amsterdam Conf.) provided the theory contains one additional scalar field	Use either t'Hooft's formulation or our original version. Use the Boulware formulation to recast the theory. The pure spin-1 interactions are renormalizable but logarithmic self-mass and self-charge infinities still survive. The induced scalar and pseudoscalar interactions are finite, however, with cut-off at $4\pi/(\sqrt{G_F})$.
3) Strong chiral interactions	(Gürsey) exponential form (manifestly local) $\mathcal{L} = \mathrm{Tr}\,\partial_\mu S\,\partial_\mu S^\dagger$ where $S = \exp(i\lambda\tau\cdot\pi)$ Also chiral pions interacting with baryons, etc.	Cut-off $4\pi/\lambda \approx 4\pi m_\pi$; $\lvert p_T \rvert$ bounded

TABLE II (continued)

4) Strong conformal dilaton interactions	Dilatons couple universally to trace of strong stress tensor	Cut-off $\frac{4\pi}{\kappa_\sigma}$; $\lvert p_T \rvert$ bounded
5) Yang-Mills theories of 1^- and 1^+ particles	Use Boulware form, writing $W_\mu = A_\mu + (i/g) S \, \partial_\mu \, S^\dagger$ $(S = \exp i(g/m) \underline{\tau} \cdot \underline{B})$	A-field interactions renormalizable (with δm, δg still logarithmically infinite). B-field interactions are finite with cut-offs at $\frac{4\pi m}{g}$

III. INBUILT CUT-OFFS IN SOME CURRENTLY ACCEPTED FIELD THEORIES

Table II summarizes what I said earlier about the non-polynomialities in some of the Lagrangians currently in use (or which should be in use) to describe electromagnetic weak and strong phenomena.

What I wish you to notice is the patch-work pattern of cut-offs; some (in hadron physics) at relatively low energies like $4\pi m_\pi$, $4\pi m_\rho/g$; others (in lepton physics) at high energies like $G_F^{-1/2}$, and then the ultimate cut-off $G_N^{-1/2} \approx 10^{19}$ BeV. Let us discuss the theories one by one.

A. Gravity-modified lepton electromagnetism

In Table II, I have written in the conventional vierbein gravity Lagrangian. The connection between the vierbein $L^{\mu a}$ and Einstein's metric tensor $g^{\mu\nu}$ is given by the relation $L^{\mu a} L^{\nu}_{a} = g^{\mu\nu}$. I have used a manifestly local (exponential) parametrization to connect the vierbein and the physical graviton field ϕ^{ab} (the analogy for the chiral case is with the fields S and $\underline{\pi}$ which are connected by the relation $S = \exp(\lambda i \underline{\tau} \cdot \underline{\pi})$ with $\mathcal{L}$ given by $\mathrm{Tr}\, \partial_\mu S\, \partial_\mu S^+$). The exponential parametrization of $L^{\mu a}$ in the form $[\exp(\kappa\phi)]^{\mu a}$ automatically guarantees the important physical requirement on $\det L^{\mu a}$ - that it must not change sign, and that the signature of the metric tensor +--- does not, e.g., change to ++-- when the field evolves in time.

In Trieste we have carried through the calculation of electron self-mass using vierbein gravity (IC/70/131 (Phys. Rev. D3, 1805 (1971)) and IC/71/14). We needed during the course of this calculation superpropagators like

$$\left\langle \frac{L^{\mu a}(x)}{\det L(x)} \quad \frac{L^{\nu b}(0)}{\det L(0)} \right\rangle$$

with all their (tensor) indices. Such superpropagators have recently been worked out for exponential gravity by J. Ashmore and R. Delbourgo (Imperial College preprint). The numerical result of working with full vierbein gravity differs from the model calculation in the earlier part of this lecture by terms of order α (the term of order $\alpha \log \kappa^2 m^2$ is the same). Assuming that Maxwell's theory holds up to 10^{19} BeV - till gravity modifies it - we thus obtain the result

$$\frac{\delta m_e}{m_e} = \frac{3\alpha}{4\pi} \log (\kappa^2 m^2) \quad .$$

(Note $R = 2\kappa^2 m^2$ is the electron's Schwarzschild radius (in units of $1/m$).) Substituting $\kappa^2 \approx 10^{-44} m_e^{-2}$ we note that $\log \kappa^2 m^2 \approx 105 \approx \alpha^{-1}$ the gravity modification of the self-mass is large with $\delta m/m \approx 1/5$ in the lowest order of our calculation. Remembering that the effective parameter in gravity-modified electrodynamics is $(\alpha \log \kappa^2 m^2) \approx 105/137$, it is fully conceivable that, with higher orders included, one may after all realize the Lorentz conjecture that $\delta m/m = 1$, that indeed all self-mass of the electron is gravity-modified electromagnetic (or weak) in nature. [7] One may be tempted then to reverse the argument and, setting $\delta m/m = 1$, compute self-consistently in a field-theory bootstrap the value of $\alpha \log (G_N m^2)$ - possibly obtaining thereby the correct numerical result relating these basic constants of nature $\alpha \log G_N m^2$ (≈ 1).

But perhaps one should not be carried away by this relation $\alpha \log G_N m^2 \approx 1$. Perhaps one should systematically investigate what the possible local non-polynomial modifications to electrodynamics could be. Viewed this way, one can see that electromagnetic gauge invariance places strong restrictions on all possible modification of Maxwell's theory. Gauge invariance demands that any modification of the total electromagnetic Lagrangian should preserve the combination $(\partial_\mu - iA_\mu)$ intact. If it is a local scalar modification, then it must clearly have the form

$$\mathcal{L}_T = f(\kappa\phi) \quad \bar{\psi}\gamma_\mu(\partial_\mu - iA_\mu)\psi$$

where $f(\kappa\phi)$ is an entire function. However, we can now make the Borchers' transformation

$$\psi \rightarrow \psi\sqrt{f(\kappa\phi)}$$

and thereby remove the dependence of $\mathcal{L}_T$ on ϕ. This shows that a scalar modification would produce no regularization of infinities for (massless) electrons.

What is the next simplest modification? Clearly the simplest idea is to try a tensor modification like

$$L^{\mu\alpha}\,\bar{\psi}\,\gamma_\alpha(\partial_\mu - iA_\mu)\psi \quad .$$

We have preserved in this way the sacred gauge-invariant combination $(\partial_\mu - iA_\mu)$.

The emergence of the tensor modification from the requirement of electromagnetic gauge invariance together with the seductiveness of the relation $\alpha \log G_N m^2$ (≈ 1) make one feel one is on the right track in assuming it is gravity alone that modifies Maxwell's equations. But in the energy scale this is such a bold extrapolation that perhaps one should feel content at this stage simply to suggest to those performing accurate experiments in low-frequency lepton electrodynamics to express their cut-offs (where they expect electrodynamics to break down) not in the non-local form $1/(1 + t\kappa^2)$ (as Pipkin did during this conference) but in the local form through Meijer G-functions, i.e. through form factors of the type $\approx \exp -|t\kappa^2|^\alpha$ $(\alpha < 1/2)$.

B. Weak interactions

Consider leptonic $\mathcal{L}_{weak}$. As Table II shows, the Ward-Salam-Weinberg theory still contains a few logarithmic infinities. A gravity modification of this theory should regularize these remaining logarithmic infinities. This is the problem we are currently working on. We are very much hoping that some of the finite numbers we obtain refer to measurable magnitudes and not just to self-masses and self-charges.

C. Hadron physics

For hadrons physics two things are clear:

1) Empirically all hadronic matrix elements, whether in strong, weak or electromagnetic interactions, appear to possess cut-offs at energies of the order of a few BeV. Thus Einstein's ultimate cut-off is supplanted by a cut-off at a lower energy.

2) Even though the chiral and conformal cut-offs at around a BeV or so will regularize some of the infinities, we still need something as universal as gravity to make all matrix elements finite. To provide this universal regularizing non-polynomiality we (Isham, Strathdee and Salam) and, independently, Wess and Zumino - and more recently Arnowitt, Nath and Friedmann - have considered the possibility of taking over the Einstein equation into hadron physics and using it to describe a universal coupling of a 2^+ strongly interacting F-meson with all hadronic matter. (We replace the newtonian constant G_N by $G_F \approx (\mathrm{BeV})^{-1}$; $G_F^{-1/2}$ serves as the universal cut-off for hadron physics.) The interconversion of Einstein's g-field into the F-field, is described by a covariant mixing term, the entire theory being constructed in analogy with the $\rho - \gamma$ mixing theory. Since this two-tensor theory of gravity is described in detail in my Miami lecture, I shall not speak any more about it.

To conclude, I should like to recall the famous mathematician E. Hille's introduction to the first edition of his book on Semi-Groups, which goes something like this: [8] "Friends assure me that not everything I see around me is a Semi-Group. I simply do not believe them." I could say the same about non-polynomial Lagrangians. I believe in a few years all Lagrangian theories which will be considered will be of the local non-polynomial variety and through the interplay of the minor coupling constants, and especially of the dependence of self-masses and self-charges on logarithms of these constants (a feature of non-polynomial theories), we shall achieve an understanding of the outrageous magnitudes in particle physics (ranging from $G_{N\pi}^2/4\pi \approx 15$ to $G_N \approx 10^{-44}\ m_e^{-2}$ and of relations like $\alpha \log G_N m^2 \approx 1$.

FOOTNOTES

1) The Boulware split exhibits the Yang-Mills triplet W_μ, for example, in the form $W_\mu = A_\mu + (i/g) S\, \partial_\mu S^\dagger$ where $W_\mu = \underline{W}_\mu \cdot \underline{\tau}$ and $S = \exp(ig\tau \cdot B)$. Thus the conventional Lagrangian

$$\mathcal{L} = \mathrm{Tr}\,(\tilde{W}_{\mu\nu}\,\tilde{W}_{\mu\nu} + m^2\, W_\mu^2)$$

where

$$(\tilde{W}_{\mu\nu} = \partial_\mu W_\nu - \partial_\nu W_\mu + ig\,[W_\mu, W_\nu]$$

can be written in the (partially) non-polynomial form

$$\mathcal{L} = \mathrm{Tr}\;[\tilde{A}_{\mu\nu}\,\tilde{A}_{\mu\nu} + m^2\, A_\mu^2]$$

$$+\; m^2(2A_\mu\, S\, \partial_\mu S^\dagger + \partial_\mu S\, \partial_\mu S^\dagger) \quad .$$

The pure A part of the Lagrangian is polynomial and renormalizable. The "induced" scalar part containing the B-field exhibited within curly brackets is non-polynomial and finite with the minor constant g/m .

---

2) If weak intermediate bosons are of Yang-Mills variety, footnote (1) applies. A simpler case is that of a neutral intermediate boson W^0 with

$$\mathcal{L}_{int} = G_F m_W^2\, \bar{\psi}\, \gamma_\mu (1 + \gamma_5)\psi\, W_\mu \quad .$$

Make the conventional Stückelberg split of W_μ ($W_\mu = A_\mu + (1/m_W)\partial_\mu B$). One can rewrite

$$\mathcal{L}_{int} = \sqrt{G_F m_W^2}\,\bar{\psi}\,\gamma_\mu(1 + \gamma_5)\psi\, A_\mu + m_\psi\, \bar{\psi}[\exp(i\sqrt{G_F}\,\gamma_5\, B) - 1]\psi \quad .$$

---

3) Note that for $n_1 = 1$, the last product presents us with the problem of defining products like

$$\delta(x) \otimes \left(\frac{1}{x^2}\right)^{n_2} .$$

---

4) If we consider a linear chain of superpropagators with k initial and ℓ final particles

it is possible to write down an integral equation for the complete matrix element. Quite generally,

$$F_{k\ell}(s) = F^0_{k\ell} + \sum_{n=1}^{\infty} F^0_{kn} \frac{D_n(s)}{\Gamma(n+1)} F_{n\ell}(s)$$

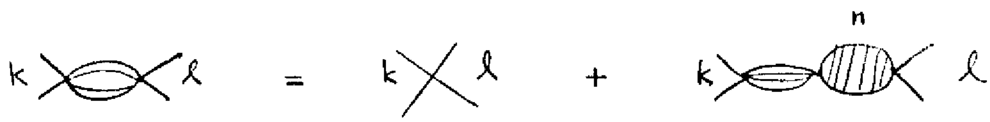
where

$$D_n(s) = \frac{1}{4\pi^2} \frac{\Gamma(2-n)}{(n)} \left(\frac{-s}{16\pi^2}\right)^{n-2}$$

and $F^0_{k\ell}$ is the contact (Born) term for the scattering which equals

$$F^0_{k\ell} = \left.\frac{\delta^{k+\ell} \mathcal{L}(\varphi)}{\delta\varphi^{k+\ell}}\right|_{\varphi=0} .$$

Graphically the integral equation represents



For the case of the Lagrangian $g(e^{\kappa\phi} - 1)$

$$F^0_{k\ell} = g\,\kappa^{k+\ell}$$

and

$$F_{k\ell}(s) = g\left(\kappa^{k+\ell} + \sum_n \kappa^{k+n} \frac{D_n(s)}{\Gamma(n+1)} F_{n\ell}(s)\right)$$

which solves to

$$F_{k\ell}(s) = g\,\kappa^{k+\ell} \left[1 - \sum_n \frac{g\,\kappa^{2n} D_n(s)}{\Gamma(n+1)}\right]^{-1} .$$

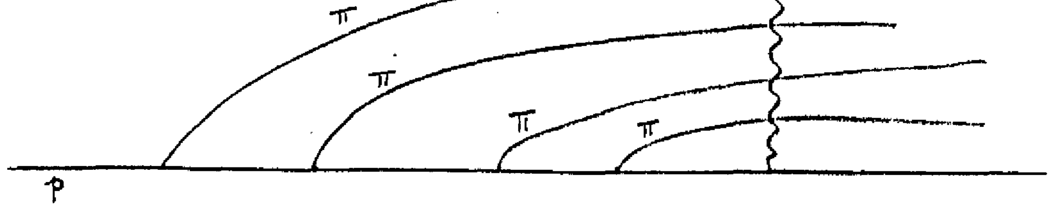
Using the Sommerfeld-Watson transformation method one sums the series in the square bracket. It is here that the usual Meijer's G-function makes its appearance. For large s we obtain

$$F_{k\ell} = g\,\kappa^{k+\ell} \frac{1}{1 - c \exp\left\{\frac{\kappa^2 s}{16\pi^2}\right\}^{1/3}} .$$

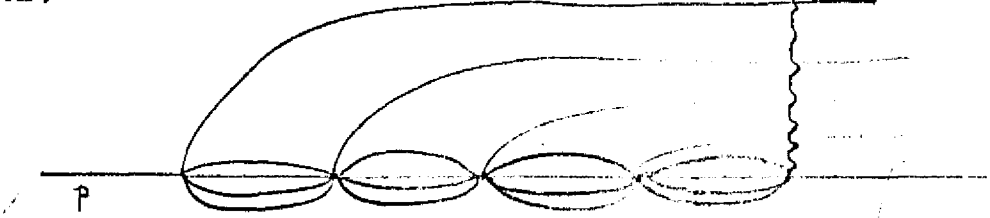
---

5) A legitimate question which one may ask is this; if the propagator satisfies the Lehmann-Källen bound in these theories and behaves like a free propagator for large s, how can one reconcile this statement with the finiteness of matrix elements in such theories? To answer this, one must stress that it is a fallacy to bring to the non-polynomial theories the ideas and techniques we have inherited from polynomial Lagrangian theories. The technique of insertion of self-energy graphs in free lines, exploited so successfully by Dyson, in polynomial theories, simply does not hold here. Every graph in a non-polynomial theory is a jumble of elementary super-propagators like those in Fig.1 or 3, (just one superpropagator joining every two points). No (self-energy) insertions can be isolated in these graphs. Further, all calculations are made in the Symanzik region where momenta are space-like and even the infinitely long linear chains of the type considered above (admittedly a bad approximation) behave like $\exp - |\kappa^2 t|^{1/3}$. It is these convergence factors which provide the inbuilt finiteness of the S-matrix elements.

6) For example, the Drell et al. graphs in deep inelastic domain



When modified (in a non-polynomial chiral theory) to look;



will lead in a natural manner to a cut-off in $|p_T|$, needed by Drell et al. to exhibit the scaling behaviour of deep inelastic matrix elements.

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7) We have no explanation of the muon mass.

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8) This passage seems to have been deleted in the Preface to the second edition in the ICTP Library.

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