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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

COMPUTATION OF RENORMALIZATION CONSTANTS

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ADDENDUM

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A D D E N D U M

The following should be added to Reference 16 (page 32):

P. Budini and G. Calucci, Nuovo Cimento 70A, 419 (1970);

Abdus Salam, "Renormalization constants and interrelation of fundamental forces", Lecture at the Kiev Conference 1970 (ICTP, Trieste, preprint IC/70/106).

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COMPUTATION OF RENORMALIZATION CONSTANTS

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INTRODUCTION

The theme of this session is computation of the traditionally infinite renormalization constants. These constants are usually expressed (in Källèn-Lehmann formulation) as integrals of spectral functions and their moments. They therefore represent magnitudes as measurable or as unmeasurable as the corresponding form factors. For theories possessing internal symmetry, we in fact know more about them. For example, we believe that bare m_{π^+} equals bare m_{π^0} ; bare g_A equals bare g_V and bare m_e (possibly) equals bare m_ν (= 0).

In computing renormalization constants, one is dealing with products of singular distribution functions. Basically the problem is to extract good physics from the bad mathematics. This now seems possible because of two advances.

1) Advances in mathematics of generalized functions: I have in mind the Gel'fand-Shilov and related work in defining Fourier transforms of products of distributions like:

$$\left(\frac{1}{x^2}\right)^{z_1} \otimes \left(\frac{1}{x^2}\right)^{z_2} \quad \text{and} \quad \partial_\mu \left(\frac{1}{x^2}\right)^{z_1} \otimes \partial_\nu \left(\frac{1}{x^2}\right)^{z_2} \quad \text{for integer } z\text{'s.}$$

This uses analytic continuation methods (continuation in the variables z_1 and z_2 from the region $0 < \text{Re } z_1 , z_2 , z_1 + z_2 < 1$) and represents a major and as yet unappreciated advance in the mathematics of our subject.

2) Advances in field theory: The realization that:

a) all Lagrangians of physical interest are intrinsically non-polynomial in character (if all else fails, inclusion of gravity makes them so);

b) the proofs (to be presented here to-day) that localizable non-polynomial Lagrangians are as respectable in a strict field-theoretic sense as the polynomial ones;

c) the realization that analytic regularization methods mentioned above are absolutely tailor-made for such Lagrangians, yielding for a variety of these finite and (as Prof. Lehmann will tell us, with one further physical ansatz) unambiguous values for the renormalization constants.

The session will be divided into three parts.

Part I: Analytic regularization methods and their applicability to non-polynomial Lagrangians.

Part II: Effects of including quantized tensor gravity with the electrodynamics of leptons - i.e. the finite and gauge-invariant computation of electron's self-charge and self-mass in the quantum theorist's version of curved space and time. (The preservation of gauge invariance is a new result.)

Part III: The speculative suggestion that F-mesons couple to the hadronic stress tensor in the same (non-polynomial) manner as Einstein's gravitons do to leptons (i.e. the postulate of the two-tensor theory of gravity), and the possibility of using this non-polynomiality to regularize renormalization constants in strong-interaction physics.

I shall briefly mention in these opening remarks some of the newer contributions, speculations, and the unanswered questions. (In a lecture like this, one can be unashamedly speculative in order to emphasise lines of possible further work.)

Before doing this, however, it may be useful to make a list of some of the non-polynomial Lagrangians important in physics.

Non-polynomial Lagrangians of physical interest include the following:

A) CHIRAL SU(2) x SU(2) LAGRANGIAN FOR STRONG INTERACTIONS

A typical example is the π-meson Lagrangian in its different parametric versions:

$$\mathcal{L} = \mathrm{Tr}\ \partial_\mu S\ \partial_\mu S^+$$

where

$$S = \frac{1 + i\lambda_W \, \tau \cdot \pi}{1 - i\lambda_W \, \tau \cdot \pi} \qquad \text{(Weinberg-Schwinger co-ordinates)}$$

or

$$S = \exp(i\lambda_G \, \tau \cdot \pi) \qquad \text{(Gürsey co-ordinates)} \quad .$$

Here λ, (λ_W or λ_G), which we call the minor coupling constant, has dimensions of inverse length; (empirically $\lambda \approx m_\pi^{-1}$). An important open question for field theory in general is this: Are on-shell S-matrices for these two versions equal; particularly as the Gürsey (exponential) form on the face of it appears to define a localizable chiral theory of π-mesons and the Weinberg (rational) form a non-localizable one.[FN1] *

B) INTERMEDIATE-BOSON MEDIATED WEAK LAGRANGIAN

A typical example is a neutral W-meson of mass m interacting with quarks (Q) of mass M , with

$$\mathcal{L}_{int} = f\,\bar{Q}\,\gamma_\mu(1+\gamma_5)\,Q\,W_\mu \quad \text{and} \quad m^2 f^2 \approx G_F \quad \text{(the Fermi constant)} .$$

The essential non-polynomiality of the theory is concealed in the derivative coupling of the spin-zero daughter of the physical spin-one particle which is described by the four-vector field W_μ . To make this manifest, write $W_\mu = A_\mu + \frac{1}{m}\partial_\mu B$ in the well-known Stückelberg form and transform the quark-field $Q' = \exp(if\gamma_5 B/m)Q$. The transformed $\mathcal{L}_{int}$ equals

$$\mathcal{L}_{int} = f\,\bar{Q}'\,\gamma_\mu(1+\gamma_5)\,Q'\,A_\mu + M\,\bar{Q}'\left(\exp\left[\frac{2if\gamma_5 B}{m}\right] - 1\right)Q' \quad .$$

The constant $f/m \approx \sqrt{G_F}$ plays the role of the minor coupling constant in the second term of this Lagrangian. The important point I wish to stress is this: A derivative coupling (of the B-field in this case) can look deceptively polynomial in form; by suitable field transformations its essential non-polynomiality (with the characteristic property that, in Feynman's language, a whole host of lines emanate from a single vertex) can be made manifest.

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* Footnotes are denoted [FN1] ..., and are given on pp.33-37.

C) EINSTEIN'S TENSOR GRAVITY AND GRAVITY-MODIFIED MATTER FIELDS

The conventional Lagrangian for gravity is

$$L_{\text{Einstein}} = \kappa^{-2} g^{\mu\nu} \left(\Gamma_{\mu\rho}^{\ \ \lambda} \Gamma_{\nu\lambda}^{\ \ \rho} - \Gamma_{\mu\nu}^{\ \ \lambda} \Gamma_{\lambda\rho}^{\ \ \rho} \right) \Big/ \sqrt{-\det(g^{\mu\nu})}$$

where

$$\Gamma_{\mu\nu}^{\ \ \lambda} = \frac{1}{2} g^{\lambda\rho} (\partial_\mu g_{\nu\rho} + \partial_\nu g_{\mu\rho} - \partial_\rho g_{\mu\nu}) \quad .$$

If $g^{\mu\nu}$ is the fundamental field, the covariant quantity $g_{\mu\nu}$ is intrinsically non-polynomial and vice versa. The simplest example for matter-field in curved space-time is the spin-zero field:

$$L_{\text{matter}} = \frac{g^{\mu\nu} \partial_\mu \phi \, \partial_\nu \phi}{\sqrt{-\det g^{\mu\nu}}} \quad .$$

L_{matter} is also non-polynomial. The quantity $g^{\mu\nu}$ (the metric tensor of classical physics) is conventionally parametrized (when space-time at infinity is Minkowskian) in the form

$$g^{\mu\nu} = \eta^{\mu\nu} + \kappa\, h^{\mu\nu} \quad , \quad \eta^{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \quad .$$

Here κ^2 , the coupling constant of the theory, equals $8\pi G_N$ (G_N is the Newtonian constant) ($\approx 10^{-44}/m_e^2$) , i.e. $\kappa^{-1} \approx 10^{18}$ BeV. An alternative (and by the mathematicians the more favoured) parametrization is given by

$$g^{\mu\nu} = \left[\exp\left(\kappa\, \gamma_{ab}\, h^{ab} \right) \right]^{\mu\nu} \quad ,$$

where γ_{ab} are 4 x 4 pseudosymmetric matrices. (Note that for this "exponential" parametrization $\det g^{\mu\nu} = \exp(\kappa\, h_a^a)$. When we come to consider spin-$\frac{1}{2}$ particles, Einstein's tensor can be treated as the fundamental field. Instead one must work with vierbein gravity $L^{\mu a}$ whose relation to $g^{\mu\nu}$ is given by $g^{\mu\nu} = L^{\mu a} L_a^{\nu}$.

P A R T I

There are three speakers in this part of the session.

Prof. J.G. Taylor, in his lecture, will survey the methods which have been developed to compute S-matrix elements in non-polynomial Lagrangian theories, to any desired order in the major and all orders in the minor coupling constant. (To remind you, for $L_{int} = g : \exp(\kappa\phi) - 1 :$, we call g the major and κ the minor constant.) He will exhibit the inbuilt infinity-suppression mechanism for these theories and also give a beautiful new proof of the unitarity of this solution. Prof. Lehmann, in his lecture, discusses the very important problem of possible distribution-theoretic ambiguities in the definition of time-ordered products in localizable theories and their elimination. This follows on the work by Lehmann and Pohlmeyer who have shown that the procedures developed by Filippov, Volkov, Salam, Strathdee and others, do guarantee analyticity and unitarity of localizable theories to an arbitrary order in the major constant. The demonstration that the analyticity and unitarity behaviour of localizable non-polynomial theories is as good as that for the conventional polynomial ones is to me tremendous news. Efimov had already given one proof; a more rigorous confirmation from Lehmann and Pohlmeyer (and for unitarity from Taylor) is extremely welcome. Mathematically, at any rate, the theories we are dealing with are as respectable - or as ordinary and normal, with no special mystery about them - as one could desire. Prof. Lehmann will be followed by Dr. N. Christ, who discusses the ambiguity problem from a different point of view.

I shall give here a very brief survey of the ideas which will be presented in more detail in later lectures.

A) GEL'FAND-SHILOV METHOD AND INFINITY SUPPRESSION

i) The problems

Given a localizable Lagrangian like

$$L_I = g(\exp(\kappa\phi) - 1) \quad , \tag{1.1}$$

we wish typically to compute the superpropagator

$$S(x) = \left(T\, L_I(\phi_{in}(x)\ \ L_I(\phi_{in}(0))\right) \quad . \tag{1.2}$$

Formally,

$$S(x) = \sum_{n=1}^{\infty} \frac{(\kappa^2)^n}{n!} \langle T\, \phi_{in}^n(x)\, \phi_{in}^n(0) \rangle \quad . \tag{1.3}$$

We specialize to zero-mass particles where

$$\left(T\, \phi_{in}(x)\, \phi_{in}(0) \right) = D(x) = \frac{-1}{x^2} \quad , \tag{1.4}$$

with the Fourier transform (FT) proportional to $1/p^2$. In evaluating a term like $(T\phi^2(x)\, \phi^2(0))$, the first problem is the meaning to be ascribed to $\phi^2(x)$. The conventional procedure uses (1.4) to define a <u>normal product</u> $:\phi^2:$ from the relation $\phi^2(x) = :\phi^2(x): + D(0)$. Here $D(0)$ is the infinite renormalization constant $\lim_{x\to 0} 1/x^2$. One now shows that

$$(T : \phi^2(x) : : \phi^2(0) :) = \frac{2!}{(x^2)^2} \tag{1.4'}$$

up to a distribution-theoretic ambiguity of the form $b\, \delta^4(x)$ (b-ambiguity). This simplest of situations already poses the three problems which lie at the heart of our discussion:

a) <u>Normal ordering:</u> is there any physics concealed in $D(0)$ and being discarded with it by the normal-ordering procedure?

b) $(1/x^2)^2$ is a product of singular distributions $1/x^2 \otimes 1/x^2$. Is there a natural definition for its Fourier transform?

c) The role of the "ambiguity constant" b .

Conventional renormalization theory treats problems b) and c) as parts of one problem; in Fourier space, a faltung is used to write

$$\int \frac{1}{x^2} \otimes \frac{1}{x^2}\, e^{ipx}\, d^4x = \frac{1}{(2\pi)^4} \int \frac{d^4k}{(p-k)^2\, k^2} \quad .$$

The integral on the right-hand side exhibits (a logarithmic) infinity. A subtraction procedure is devised to separate this from the integral and the constant b is adjusted to compensate this infinity.

This faltung method and infinity separation become prohibitively complicated when we consider objects like

$$(T : \phi^n_{in}(x) : : \phi^n_{in}(0) :)$$

represented by a cocoon-like graph with n-lines

with its (n-1) divergent subintegrations in momentum space. This was one reason why non-renormalizable theories with polynomial Lagrangians (e.g., $\mathcal{L}_{int} = g\phi^5$ or $g\phi^6$, etc.) were soon abandoned. Even a subtraction procedure was hard to define.

ii) Gel'fand method

Non-polynomial Lagrangian theories, on the other hand, offer, through the Gel'fand-Shilov procedures,[1] a different approach, where we separate problems b) and c). (Basically this happens because a superpropagator in such theories is a sum of a series of singular function

$S(x) = \sum \frac{1}{n!}\left(\frac{-\kappa^2}{x^2}\right)^n$. This sum is far less singular, when $x \to 0$ from an appropriate direction, than each single term of the series. (Roughly speaking, $\exp(-\kappa^2/x^2) \to 0$ when x^2 is space-like and κ^2 is negative. Analytic continuation then fills in for other directions and other κ^2.)

To be more precise, let us return to (1.4'). We wish to compute the FT of $(1/x^2)^2$; more generally of $D^n(x) = (1/x^2)^n$. Gel'fand and Shilov remark that since the FT of $(1/x^2)^z$ is a well-defined classical mathematics object whenever $0 < \mathrm{Re}\, z < 2$, and is proportional to $(\Gamma(2-z))/\Gamma(z) \times (1/p^2)^{2-z}$, the FT of $(1/x^2)^n$, with n lying outside this region, may be defined by an appropriate analytic continuation of this function in the variable z . (Contrast the elegance of this definition with the clumsiness of the conventional faltung procedure with its multiple divergent loop subintegrations. We make the word "appropriate" more precise in a minute.)

The Gel'fand-Shilov method was discussed in physics literature by Güttinger[2] as early as 1966 and, in an equivalent formulation, by Gustafson[3] even earlier. Bollini and Giambiagi[4] were perhaps the first to use it purposefully for rewriting conventional renormalization theory. Its power and value, however, become apparent particularly when we use it together with non-polynomial Lagrangians, because here the somewhat vague concept of "appropriate analytic continuation" in the variable z becomes dove-tailed with the analyticity properties of the super-propagator $S(x)$ in the variable $(\kappa^2 D(x))$.

To give the bare bones of the method, consider the superpropagator for the Lagrangian $L_I = (: g^2 e^{\kappa\phi} : - 1)$. This is an entire function of $\kappa^2 D(x)$:

$$S(x) = g^2 \sum_{n=1}^{\infty} \frac{1}{n!} \left[\kappa^2 D(x)\right]^n \quad . \tag{1.5}$$

First write its Sommerfeld-Watson transform:

$$S(x) = \frac{g^2}{2\pi i} \int \frac{dz}{\Gamma(z+1)} \frac{1}{\tan \pi z} \left[\kappa^2 D(x)\right]^z \quad . \tag{1.6}$$

The conditions under which transition from (1.5) to (1.6) is justified are stated in the papers of Volkov, Salam, Strathdee, Lehmann and Pohlmeyer.[5] There are b-ambiguities in writing (1.6) which are discussed below. The contour as usual encloses the positive real axis from $\mathrm{Re}\, z < 1$ to infinity. Second, rotate the contour to lie parallel to the imaginary axis – in this particular case along $0 < \mathrm{Re}\, z < 1$. The Gel'fand-Shilov condition for "classical" Fourier transforming is met, and we write

$$\tilde{S}(p) = \int_{0 < \mathrm{Re}\, z < 1} (\kappa^2)^z \left(\frac{1}{p^2}\right)^{2-z} \frac{\Gamma(2-z)\, dz}{\Gamma(z)\, \Gamma(z+1) \tan \pi z} \quad . \tag{1.7}$$

The integrand has a single pole at $z = 1$ (corresponding to the $-\kappa^2/x^2$ term in $S(x)$) and double poles at $z = 2, 3, 4, \ldots$. These give rise to characteristic terms proportional to $(\kappa^2 p^2)^r \log(\kappa^2 p^2)$. This logarithmic dependence of the Green's function on the minor coupling constant is a hallmark of non-polynomial Lagrangian theories.

iii) Infinity suppression

To see that this logarithmic dependence represents infinity suppression, consider a mixed theory with $L_{int} = g\, \chi^3 \exp(\kappa\phi)$. The superpropagator equals

$$S(x) = g^2 \sum_{n=3} \frac{D^{n+3}\,(\kappa^2)^n}{n!}$$

with the Sommerfeld-Watson transform

$$\frac{g^2}{2\pi i} \int_{\mathrm{Re}\,z<3} \frac{(\kappa^2)^z\, D^{z+3}}{\tan \pi z\, \Gamma(z+1)} \qquad (1.8)$$

For the Gel'fand condition to be met, the contour must be shifted to the left of $\mathrm{Re}\ z + 3 < 2$. This is perfectly possible since the integrand is not singular at $z = -1$. The FT $\tilde{S}(p)$ is easily evaluated and contains terms proportional to g^2/κ^2, $g^2 \log(\kappa^2 p^2)$, $g^2 \kappa^2 p^2 \log(p^2\kappa^2)$, etc. Clearly g^2/κ^2 is the relic of the quadratic infinity in a polynomial theory given by the Lagrangian $g\chi^3 = \lim_{\kappa\to 0} (g\chi^3)\, e^{\kappa\phi}$; likewise $\log(\kappa^2 p^2)$ is the relic of the logarithmic infinity. We recover these infinities in the limit $\kappa \to 0$. To put it another way, $(\kappa)^{-1}$ is the inbuilt, realistic, regularizing cut-off in the non-polynomial theory $(g\chi^3 \exp(\kappa\phi))$.

B) FINITE VERSUS RENORMALIZABLE LAGRANGIANS

Now it is not always the case that every infinity can be regularized. Consider the Lagrangian $L_{II} = (e^{\kappa\phi} - 1 - \kappa\phi - ((\kappa^2\phi^2)/2!))$. Here the superpropagator $S_{II}(x)$ has the same expression as in (1.6); the contour of integration, however, lies along $2 < \mathrm{Re}\ z < 3$. We cannot interchange z-integration with the FT since the Gel'fand-Shilov condition $\mathrm{Re}\ z < 2$ is not met. We must write $S_{II}(x) = S_I(x) - (\kappa^2 D^2)/2!$ before doing so. While $S_I(x)$ can be Fourier-transformed by the methods above, the $(-\kappa^2 D^2)/2!$ term sticks out like a sore thumb (ST). One may regularize it using any available method; there is no reason, however, for the effective cut-off to depend on κ. We shall call the Lagrangian L_I finite and, in contrast, L_{II} renormalizable, since L_{II} needs a subtraction constant of the conventional variety. At least one physical quantity cannot be computed within the theory so far as L_{II} is concerned. The ideal theory would of course be the one where there are no uncomputable, renormalization constants whatever.

C) AMBIGUITIES

6 Let us now turn to the ambiguity problem.[6] Even the finite theories in the sense defined in the last subsection suffer from these. Their origin is distribution-theoretic; the distribution $(T : \phi^n(x) : : \phi^n(0) :)$ is ambiguous up to terms of the type $\sum b_n (\partial^2)^{n-2} \delta(x)$. Alternatively, one can see these b-ambiguities in the Sommerfeld-Watson formulation. In passing from (1.5) to (1.6) and (1.7) we have written down an extrapolated function merely from a knowledge of its value at integer points. To be more accurate we should have written in, in (1.6), the factor $\left[(1/(\tan\pi z)) + b(z)\right]$ rather than just $1/\tan\pi z$.

As I said earlier, Lehmann and Pohlmeyer's crucial contribution is to show that there exists a simple physical criterion - and this applies to all orders in the major coupling constant - using which one can eliminate these distribution-theoretic ambiguities from localizable theories. The basic idea is that for finite localizable theories the b-dependent (and ambiguous) contributions in (1.7) can be distinguished from those with $b = 0$ through their behaviour in p-space for large $|p^2|$. The b-dependent terms do not fall in any direction in the $|p^2|$ plane, while the b-independent terms do and thus define a minimal-singularity superpropagator $\tilde{S}(p)$. The same criterion was also used earlier by Filippov[7] for the second-order superpropagator to eliminate ambiguities and used by all workers in this field. Lehmann and Pohlmeyer give a general formulation valid for all higher-order superpropagators. They show further that this distinction between b-dependent and b-independent terms cannot be made for polynomial theories; the non-polynomials are far superior to polynomials in this regard.

D) NON-LOCALIZABLE LAGRANGIANS OF RATIONAL VARIETY

Let us now turn to the case of normally ordered non-localizable theories, e.g. $L_{III} = : g/(1 + \kappa\phi) :$ Here

$$S_{III}(x) = g^2 \sum_{n=0}^{\infty} \Gamma(n+1)(\kappa^2 D(x))^n \quad . \tag{1.9}$$

Contrast this with the superpropagator $S_I(x)$. Considered as a series in $\kappa^2 D(x)$, (1.9) in contrast to (1.5) has zero radius of convergence. Efimov and Fradkin[8] suggested defining the superpropagator as a Borel sum of (1.9). Formally write

$$S_{III}(x) = \sum_{n=0}^{\infty} \int_0^{\infty} e^{-\zeta} (\kappa^2 D(x) \zeta)^n d\zeta \qquad (1.10)$$

$$= \int_0^{\infty} \frac{1}{1 - \kappa^2 D(x) \zeta} e^{-\zeta} d\zeta \quad . \qquad (1.11)$$

For the definition of the physical superpropagator, Efimov and Fradkin adopted a principal value definition of the integral (1.11) in the ζ-plane. From this definition (or equivalently from (1.10)) one can go on to use Gel'fand and Shilov methods for Fourier transforming, precisely as in (1.7).

Quite clearly the Borel ansatz introduces here a new and additional source of ambiguity, the Borel ambiguity, which just does not exist for localizable theories. One unfortunate aspect of this is that the Lehmann-Pohlmeyer ansatz for dealing with b-ambiguities is useless; both the b-ambiguity terms as well as the principal value terms in the FT of (1.11) fall equally fast in the $|p^2|$ plane for large $|p^2|$. A second difficulty has been noted by S. Fels[9)] who shows that there is no guarantee that the requisite field-theoretic positive-definiteness of the principal value propagator (1.11) can be guaranteed.

Now Strathdee, Delbourgo and myself have proved a theorem which might save the situation. Sparked by some remarks of Fels, what we have shown is this. The trouble for non-localizable theories lies in normal ordering. If $\mathcal{L}_{III}$ is not normally ordered - i.e., we leave $D(0)$'s in formally as an undetermined parameter in the theory - then the high-energy behaviour of the momentum-space superpropagator is drastically altered. To be precise, we show that for non-localizable theories of the rational variety (with Lagrangians of the type $\sum a_i \frac{1}{1 + \kappa_i \phi}$) the non-normal-ordered series expansions for superpropagators converge, when considered as a series in $D(x)$. This radius of convergence is given by $D(0)$.

If $D(0) = \infty$, the radius of convergence is infinitely large, no principal value ansatz is needed to define (1.11) and the superpropagators visibly satisfy positive definiteness criteria. This is nice; unfortunately, as Fried noted[10], if we do set $D(0) = \lim_{x^2 \to 0} \left(\frac{-1}{x^2}\right) = \infty$, nearly all matrix elements will vanish. Clearly $D(0)$ must be renormalized. If we have physical reasons for believing that $D(0)$ should be finite (for example, if a scalar field[FN2] in the theory possesses a non-zero expectation value $\langle\phi\rangle \neq 0$, which can be expressed as function of $D(0)$, then the superpropagators would possess finite radii of convergence. In p-space this would mean that for non-normally-ordered rational theories, the two-point spectral function $\rho(p^2)$ would behave like $\exp\sqrt{|p^2|}$ – i.e., we would have a just localizable situation. The proof of our results is simple and can easily be constructed. A useful remark for its construction is the following: given a Lagrangian $L(\phi)$, one obtains the corresponding normally-ordered Lagrangian from it by a simple operator identity,

$$:L(\phi): = \exp\left(+\frac{D(0)}{2}\frac{\partial^2}{\partial\varphi^2}\right) L(\phi) .$$

Our conclusion is that normal ordering is a crime[FN3] for non-localizable theories. Rational Lagrangians are just localizable provided $D(0)$ is renormalized to a finite non-zero value; they become non-localizable only when they are normally ordered[11] (equivalently when $D(0)$ is renormalized to zero).

We have not yet completed the investigation of whether the Lehmann ansatz for isolating b-ambiguities applies to just-localizable theories, nor can we say whether the inbuilt cut-off for these theories is still $(\kappa)^{-1}$ or if it is modified through the appearance of the new constant $D(0)$. Clearly there is a certain amount of physics concealed in $D(0)$ whenever we work with non-localizable theories.

E) FINITE NON-LOCALIZABLE THEORIES

Let us now turn to enumerate the cases when non-polynomial theories are finite as opposed to being just renormalizable (i.e., still needing a finite number of constants unspecified and uncomputable within the theory). Unfortunately all work so far on this problem[12] (papers by

Efimov, Fradkin, Delbourgo, Koller and Salam, J.G. Taylor and Keck) has concentrated on normally-ordered rational Lagrangians of the type $\mathcal{L} = : g \frac{\phi^n}{(1+\kappa\phi^w)} :$ To summarize the results of these papers:

a) $\mathcal{L}$ is finite if $n - w < 2$.

b) $\mathcal{L}$ is renormalizable provided $2 \leqslant n - w < 4$, i.e., there exist a finite number of ST's which cannot be computed in terms of g and κ , but whose infinities can be absorbed into a renormalization of physical constants. Note the surprising inequality $n - w < 4$. One may naively have expected renormalizable theories to range over $n - w \leqslant 4$.

c) For mixed theories like

$$L = : g \frac{\bar{\psi}\psi A}{(1+\kappa\phi)^w} : \qquad (1.12)$$

where the Dyson weight of the polynomial part of L (i.e. $\bar{\psi}\psi A$) is $\leqslant 4$ (Dyson weight of a scalar field A is 1 , of spinor field ψ is 3/2), the theory is renormalizable for all $w > 0$.

d) We believe that if normal ordering is dropped (i.e., D(0) is kept as an undetermined constant or alternatively D(0) is re-expressed in terms of an uncomputable magnitude $\langle\phi\rangle$) the Lagrangian (1.12) gives a finite theory, provided $w > 2$. For the borderline case $w = 2$, the only ST's are those which correspond to vacuum-to-vacuum graphs.

e) For exponential (localizable) Lagrangians the situation is more favourable towards finiteness[13]. Lagrangians of the type $: g\phi^n e^{\kappa\phi}:$ are finite. The result is trivial for superpropagators in the second order. For higher-order superpropagators the proof of this assertion presents no difficulties whenever $n \leqslant 2$. For $n > 2$ the result is deduced by differentiating $(\exp \kappa\phi)$ with respect to κ.

f) Less easy to prove is the finiteness of the mixed case $: g \chi^n e^{\kappa\phi} :$ for higher superpropagators. For $n \leqslant 2$ the proof goes through easily. For $2 < n \leqslant 4$ we believe these theories are finite, but a proof complete in details has not yet been constructed. It would appear as if $e^{\kappa\phi}$ behaves like a rational Lagrangian with an arbitrarily large weight w ,

$$\left(\exp \kappa \phi = \underset{w \to \infty}{\ell t} \frac{1}{(1 - \frac{\kappa\phi}{w})^{w}}\right) .$$

so far as its strong smoothing effect on infinities is concerned.

F) CALCULUS OF DERIVATIVES AND GENERALIZED NORMAL ORDERING

We turn finally to the derivative-containing Lagrangians which constitute the majority of the physically interesting cases. In my Miami lecture of 1970, it was remarked that even though, looked at naively, differentiating $1/x^2$ seems to increase its singularity, **one's experience is that the increase is not as lethal as multiplying by a corresponding number of powers of $(1/x^2)$.**

Stated differently, a factor like $\partial_\mu \partial_\nu \phi$ in a Lagrangian is gentler, so far as its infinity-inducing properties are concerned, than a factor like $\delta_{\mu\nu}\phi^3$ though, on the face of it, both seem to possess the same Dyson weight $(w = 3)$. This observation - deduced empirically from practical calculations - has important consequences for finiteness versus renormalizability of derivative-containing Lagrangians of the chiral or the gravitational type. For these the free Lagrangian and the interaction Lagrangian are inextricably mixed in. As a simple example consider

$$L_{total} = \frac{(\partial\varphi)^2}{(1+\kappa\varphi)^2} + m^2\varphi^2 \quad .$$

Assuming each derivative possesses a weight unity, the Dyson weight of L_{total} equals 2 . However, when the free Lagrangian is split off from L_{total} - and we must do this to start our perturbation calculations - the weight of the resulting L_{int} appears to have jumped to 4 . What is the true weight? Is the theory finite or merely renormalizable? To put it another way, if we write $\mathcal{L}_{int} = \mathcal{L}_{total} - (\partial\varphi)^2$, do the two derivatives in the second term really produce new ST's?

To answer these questions, it seems essential that we make an extension of the Gel'fand-Shilov calculus of generalized function to include derivatives. This can be done as follows: The formula

$$\partial_\mu \left(\frac{1}{x^2}\right)^{z_1} \boxtimes \left(\frac{1}{x^2}\right)^{z_2} = \frac{z_1}{z_1 + z_2}\ \partial_\mu \left(\frac{1}{x^2}\right)^{z_1 + z_2} \qquad (1.13)$$

is true for all x, when $0 < \mathrm{Re}\ z_1, z_2, z_1 + z_2 < 2$. For z_1 and z_2 outside this range, the two sides may differ at $x = 0$ up to terms containing $\delta(x)$, its derivatives and also factors containing $(D(0))^r$. We believe this formula and its generalizations offer the correct extension of Gel'fand-Shilov calculus when derivatives occur. To evaluate the left-hand side write it in the form on the right and make the appropriate continuations from the range $0 < \mathrm{Re}\ z_1, z_2, z_1 + z_2 < 2$.[FN4] The important point about (1.13) is that the number of derivatives on both sides is conserved. This ensures that the overall singularity from the lines $(1/x^2)^{z_1+z_2}$ is not enhanced more on one side of (1.13) by the differentiations than on the other. In other words, derivatives and singular functions are somehow different, distribution-theoretically (I am not enough of a distribution-theorist to formalize this) and this distinction must be maintained. To see the power and raison of this ansatz, consider the simplest generalization of (1.13) given by

$$\partial_\mu \left(\frac{1}{x^2}\right)^{z_1} \boxtimes \partial_\nu \left(\frac{1}{x^2}\right)^{z_2} =$$

$$= \frac{z_1 z_2}{(z_1 + z_2)(z_1 + z_2 + 1)} \left(\partial_\mu \partial_\nu + \frac{1}{2(z_1 + z_2 - 1)} \partial^2 \delta_{\mu\nu}\right)\left(\frac{1}{x^2}\right)^{z_1 + z_2}. \qquad (1.14)$$

$$0 < \mathrm{Re}\, z_1 z_2,\ z_1 + z_2 < 1 .$$

The terms on the right contain two derivatives just as on the left. Now consider using (1.14) in a conventional photon self-energy calculation (zero-mass electrons with propagators $S(x) = \not\partial(1/x^2)$):

$$\Pi_{\mu\nu}(x) = \mathrm{Tr}\ \gamma_\mu \not\partial \left(\frac{1}{x^2}\right)\ \gamma_\nu \not\partial \left(\frac{1}{x^2}\right)$$

$$= 2\left(\partial_\mu \partial_\nu - \delta_{\mu\nu}\, \partial^2\right)\left(\frac{1}{x^2}\right)^2 . \qquad (1.15)$$

(We have continued the relation (1.14) outside the region of definition. This of course always needs care.) Now (1.15) correctly exhibits the transverse character of photon self-energy. Using even the most obtuse and old-fashioned of computing procedures, the FT of $\Pi_{\mu\nu}(x)$ equals

$$2(p_\mu p_\nu - \delta_{\mu\nu} p^2) \int \frac{d^4k}{(p-k)^2 k^2} \quad .$$

This shows no sign whatsoever of generating any photon self-mass. The photon self-mass in conventional calculations is indecently manufactured by forgetting the ansatz regarding conservation of derivatives and by writing $\Pi_{\mu\nu}(x)$ in the form:

$$\Pi_{\mu\nu}(x) = 2\, \partial_\mu \partial_\nu \left(\frac{1}{x^2}\right)^2 - 16\ \delta_{\mu\nu} \left(\frac{1}{x^2}\right)^3 \quad . \qquad (1.16)$$

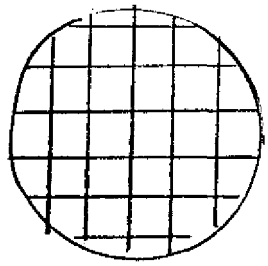
That is, in the second term on the right of (1.15), $\partial^2(1/x^2)^2$ is replaced by $8/(x^2)^3$ in most standard treatments. Since $(1/x^2)^3$ is more singular than $(1/x^2)^2$, it is alleged that the theory is giving a (quadratically infinite) photon self-mass!

Admittedly this care with derivatives pays off most when derivatives occur at those vertices where external lines impinge. But I am labouring this point because I feel that the power of the analytic methods of Gel'fand and their extension to derivative couplings is unappreciated and that half of our difficulties in renormalization theory are due to bad mathematics. If one accepts that no faltung theorems will be used to compute FT's of product of distributions like $(1/x^2)^{z_1} \otimes (1/x^2)^{z_2}$, then none should be used [FN5] for $\partial_\mu(1/x^2)^{z_1} \otimes \partial_\nu(1/x^2)^{z_2}$.

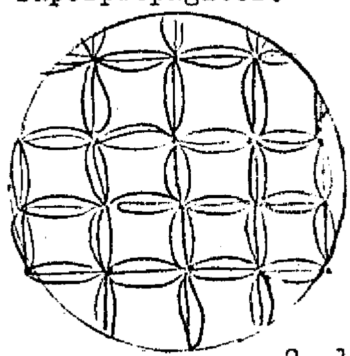
G) HIGH-ENERGY BEHAVIOUR OF LOCALIZABLE THEORIES ON THE MASS SHELL

One more problem for non-polynomial Lagrangians on the pure field-theory side is this. Glaser, Martin and Epstein[14] in a fundamental paper have shown that all <u>localizable</u> theories should give Froissart-bounded high-energy dependence of S-matrix elements. The Volkov-Lehmann expansion[5] in the major constant does not achieve this. In this expansion any given order behaves like $\exp(|p^2|^\alpha)$ $\alpha < \frac{1}{2}$. I believe this difficulty will be resolved when summations over the major coupling constants are performed. We already know that for conventional polynomial theories this type of summation - performed either directly by summing diagrams or by using Padé approximants or carried through indirectly using the Bethe-Salpeter integral equation -

alters the high-energy behaviour of individual diagrams. Specifically we know that even though individual diagrams behave like s^{α} (α constant), for the sum of ladder diagrams one finds a drastic change; one obtains the Regge behaviour $s^{\alpha(t)}$. Likewise I believe that a sum of a chain of superpropagators is likely to satisfy Glaser, Martin, Epstein results and to give Froissart-bounded high-energy dependence, though no proof for this has been constructed. A corroborating indication for this is perhaps provided by the work of Oleson, Nilsson, Susskind, Sakita, Virasoro and others,[15] who consider fishnet diagrams like the one shown:



They show that if the fishnets are made of basic propagators of the form $\exp(\kappa^2 p^2)$ rather than of the Feynman form $1/p^2$, the whole fishnet exhibits a Veneziano-Regge (Froissart-bounded) high-energy behaviour. Now $\exp(\kappa^2 p^2)$ is just the type of behaviour we obtain from an individual superpropagator (from non-localizable theories). It would seem from this that the Veneziano and the non-polynomial Lagrangian developments may come together through graphs like the one shown where each basic rung of the fishnet represents a superpropagator:



The authors mentioned above show that $(\kappa^2)^{-1}$ in their formula gives the slope of the Regge trajectory. This then is another possible physical role of the minor coupling constant. (Reflect what this means for gravity.)

In this part of the session Dr. J. Strathdee will present work published from Trieste on Einstein's gravity and its role in infinity suppression in lepton electrodynamics.[16] In my remarks I shall emphasise in particular three aspects of work we have done subsequent to the published papers.

1) We can now arrange the calculations so that gauge invariance is preserved.

2) We can see more clearly why it is the tensor rather than scalar gravity which is responsible for infinity suppression. This accords with one's physical intuition (emphasised to me particularly by Prof. Weisskopf) in that infinity suppression should come as a consequence of the light cone fluctuations which are peculiar to tensor gravity and its (Schwarzschild-like) metrical aspects.

3) To exhibit this distinction between scalar and tensor gravity, we need a better comprehension of the role of equivalence theorems for field transformations. Since we have seen in Part I that localizable Lagrangians are ordinary, decent, unassuming types of Lagrangians, and since micro-causality holds for operators in localizable theories, we can take over Borchers' results and assume that S-matrix equivalence theorems hold for localizable theories. This will necessitate using exponential parametrization for gravity and not the usual rational parametrization.

4) As shown in Part I(E), exponential non-polynomiality is extremely potent in smoothing infinities. This will mean our problem in gravity-modified theories will not be making sure whether infinities can be regularized or not; it will rather be making sure if gauge invariance can be preserved.

A. GAUGE-INVARIANT CALCULATIONS IN TENSOR GRAVITY

As Dr. Strathdee will show, the gravity-modified Lagrangian for quantum electrodynamics may be written in the form

$$L_{total} = L_{gravity} + \frac{L^{\mu a}\,\bar{\psi}\gamma_a(\partial_\mu - ieA_\mu)\psi + m\bar{\psi}\psi}{\det L} + \frac{g^{\mu\mu'}\,g^{\nu\nu'}\,F_{\mu'\nu'}\,F_{\mu\nu}}{\det L} \quad . \tag{2.1}$$

$L^{\mu a}$ is the vierbein gravity field which in exponential parametrization will be written as $\exp(\kappa\beta_{\lambda c}h^{\lambda c})$. $\beta_{\lambda a}$ are 4 x 4 pseudosymmetric matrices and $h^{\lambda c}$ are the physical (quantized fields) describing tensor quanta. ∇_μ is the covariant derivative; $F_{\mu\nu}$ equals $\partial_\mu A_\nu - \partial_\nu A_\mu$, while the Einstein field $g^{\mu\nu}$ equals a bilinear product of vierbein fields $L^{\mu a} L^{\nu}_{\ a}$. Note that $\det L = \exp[\mathrm{tr.}(\kappa h)]$. The simplicity of this expression makes work with exponential parametrization even simpler than with the rational parametrization we used in our previous papers so far as $\det L$ is concerned. Scalar gravity Lagrangian can be recovered from (2.1) by substituting

$$\left.\begin{aligned} L^{\mu a} &= \exp(\kappa\phi)\ \eta^{\mu a} \quad \text{where} \quad \eta^{\mu a} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix} \\ g^{\mu\nu} &= \exp(\kappa\phi)\ \eta^{\mu\nu} \\ \det L &= \exp(4\kappa\phi) \end{aligned}\right\} \tag{2.2}$$

L_{total} reduces to the form:

$$= L_{gravity} + [\bar\psi\gamma_\mu(\partial_\mu - ie_0A_\mu)\psi]\ \exp(-3\kappa\phi) + m_0\bar\psi\psi\ \exp(-4\kappa\phi) + F_{\mu\nu}F_{\mu\nu}\ . \tag{2.3a}$$

Note that, as physically expected, the photon field does not interact with scalar gravity.

Let us now make a field transformation

$$\psi' = \exp(-3/2\kappa\phi)\psi\ . \tag{2.3b}$$

This will completely eliminate $\kappa\phi$ coupling from the Lagrangian from all except the electron mass term. In the limit $m_0 = 0$ scalar gravity and (massless) electrons also get uncoupled.

Now if δm and δe were strict physical mass-shell quantities one would unhesitatingly have said that, like all mass-shell S-matrix elements, δm and δe as computed using (2.3a) cannot involve the constant κ in the limit that $m_0 = 0$. Therefore scalar gravity would have no regularizing role for massless electrons.[FN6] (For massive electrons the situation may be better. We have, however, so far been unable to show that the mass term in (2.3a) is potent enough to regularize all infinities.

Fortunately true gravity is tensor and even though the transformation

$$\psi' = (\det L)^{-\frac{1}{2}} \psi \tag{2.4}$$

does remove[FN7] the $(\det L)^{-1}$ factor from the kinetic energy and electric current terms of the electron Lagrangian, there is no uncoupling of gravity either from the electron or (more important) from the photon. To make matters uniform, let us also transform

$$A'_\mu = (\det L)^{-\frac{1}{2}} A_\mu \quad . \tag{2.5}$$

The resulting Lagrangian reads:

$$L_{total} = L_{gravity} + L^{\mu a} \bar{\psi}' \gamma_a [\nabla_\mu - ieA'_\mu (\det L)^{1/2}]\psi' + m_0 \bar{\psi}'\psi' + g^{\mu\mu'} g^{\nu\nu'} F'_{\mu\nu} F'_{\mu'\nu'} \quad . \tag{2.6}$$

Here

$$F'_{\mu\nu} = (\partial_\mu A'_\nu - \partial_\nu A'_\mu + A'_\nu \partial_\mu \log\sqrt{\det L} - A'_\mu \partial_\nu \log\sqrt{\det L}) \quad . \tag{2.7}$$

We shall call (2.4) and (2.5) the <u>standard</u> transformations and (2.6) the standard form of the Lagrangian. The advantage of the standard form is that the free kinetic energy terms contained in (2.6) do not have powers of det L multiplying them.

Now in our earlier published papers we effectively considered just those graphs for electron and photon self-energies which are given by a second-order iteration of

$$L_I = -ie L^{\mu a} \bar{\psi}' \gamma_\mu \psi' A'_\mu (\det L)^{1/2}, \ (\det L = \exp \mathrm{tr}.\kappa h) \quad . \tag{2.8}$$

These look like  for electron self-energy and  for photon self-energy. It is clear

electron line
photon line
graviton line

that to preserve gauge invariance it is essential to include further graphs arising from the photon-graviton terms.

$$L_{II} = (g^{\mu\mu'} g^{\nu\nu'} - \eta^{\mu\mu'} \eta^{\nu\nu'}) F'_{\mu\nu} F'_{\mu'\nu'} \quad . \tag{2.9}$$

In the published paper we did not compute these graphs (on account of the hard work involved) and the result was manifestly non-gauge invariant.

Now it is possible to secure gauge invariance - and still guarantee infinity suppression (with, as we have verified, exactly the <u>same</u> results as obtained before so far as the $(\alpha \log \kappa^2 m^2)$ terms in the computed expressions for δm and δe are concerned) by slightly modifying our earlier procedure.

Carry through the electron field transformation (2.4) but not the photon transformation (2.5). This gives:

$$L_{total} = L_{gravity} + L^{\mu a} \bar{\psi}' \gamma_a (\nabla_\mu - ieA_\mu) \psi' + m_0 \bar{\psi}'\psi' + g^{\mu\mu'} g^{\nu\nu'} F_{\mu\nu} F_{\mu'\nu'} (\det L)^{-1} \quad . \tag{2.10}$$

To make the gauge-invariant infinity suppression mechanism transparent, we shall make an approximation to (2.10). (We later estimate the effects of this approximation.) Neglect κ-dependent terms in $L^{\mu a}$ and $g^{\mu\nu}$ and in the covariant derivative ∇_μ (i.e., replace these factors by $\eta^{\mu a}$ and $\eta^{\mu\nu}$ and ∂_μ). The gravity fields h^{11}, h^{22}, h^{33} and h^{44} of course survive in $\det L = \exp(\kappa h^{aa})$. The resulting approximation to (2.10) reads:

$$L_{total} = L_{gravity} + \bar{\psi}' \gamma_\mu (\partial_\mu - ieA_\mu) \psi' + F_{\mu\nu} F_{\mu\nu} \exp(\kappa h^{aa}) + m_0 \bar{\psi}'\psi' \quad . \tag{2.11}$$

Essentially there are now just four fields (h^{aa}) in the theory which couple not with the electron but (manifestly) gauge-invariantly <u>with the photon</u>. Further, the electron-charge current $\bar{\psi}'\gamma_\mu\psi'$ is conserved both when the ψ' operators are interaction as well as Heisenberg operators. This is crucially different from the case of (2.6). It is this circumstance which will preserve gauge invariance at all steps while at the same time guaranteeing infinity suppression by providing the requisite <u>non-polynomiality</u> through the modified photon terms in (2.11).

The photon self-energy graphs given by (2.11) up to order e^2 and to all orders in κ are shown in Fig.1 while electron self-energy graphs are shown in Fig.2. (There are the irrelevant external-photon-modifying graphs in Fig.1 which we have not shown.)

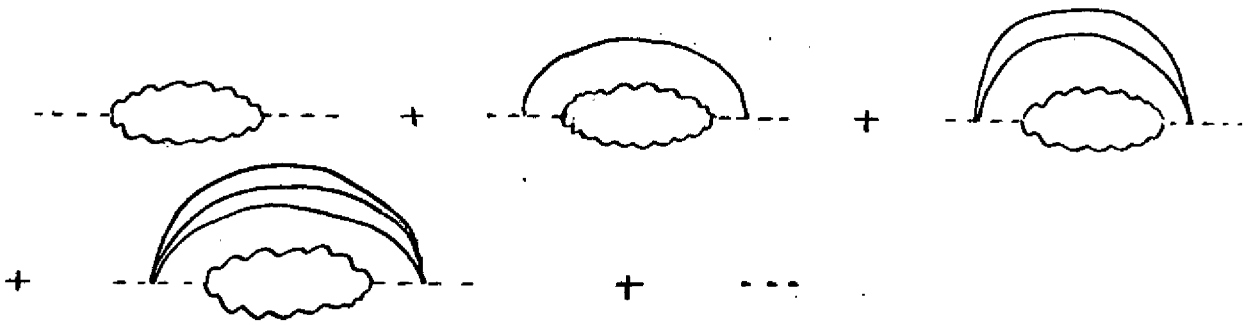


Fig.1

Photon self-energy graphs.

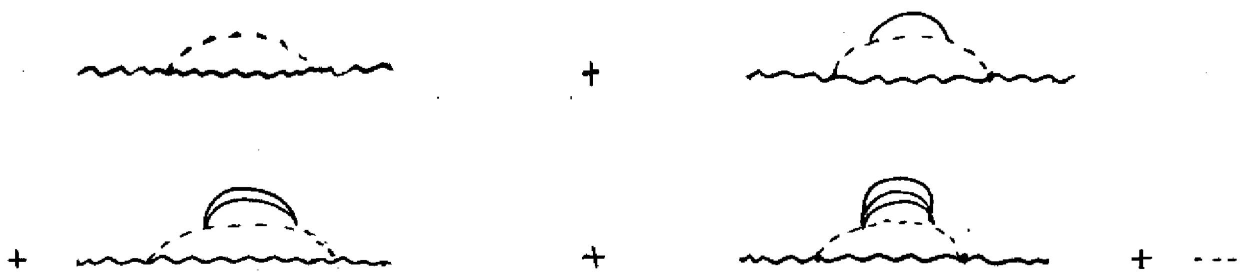
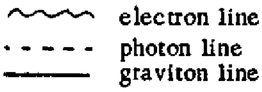


Fig.2

Electron self-energy graphs.



In a separate paper it will be shown that non-polynomial summation techniques work beautifully for these chains of graphs. The crucial step is to notice the sore thumb(ST)graph  identically equals the graph  where the crosses indicate the operation of the term $F_{\mu\nu} F_{\mu\nu}$, Thus the ST diagrams  and  quite generally form parts of the chains indicated in Fig.1 and Fig.2. The results are gauge invariant while numerically $\delta m/m$ (for example) exactly equals (in accordance with what equivalence theorems would assert) the previously obtained value $3/(4\pi)\ \alpha \log \kappa^2 m^2$ (up to terms of order $\alpha, \alpha\kappa^2, \ldots$). One can show that the effect of including the terms which were neglected in the approximation leading to (2.11) (i.e., replacing $L^{\mu a}$ by $\eta^{\mu a}$, etc.) alters only terms of these higher orders, $\alpha, \alpha\kappa^2, \ldots$, without affecting the leading contribution. The proof is simple but will not be given here.

B. TENSOR GRAVITY AND CURVED SPACE-TIME

As I said before, Weisskopf with his penetrating physical intuition had earlier made the point (private communication) that (crudely speaking it is the $(\det L)^{-1}$ factor which modifies the photon Lagrangian which should be mainly responsible for infinity suppression even more than the $(\det L)^{-1}$ factor associated with the electron.[FN8] Physically this should be so, since Weisskopf expected that infinity suppression in electrodynamics would be connected with the fact that the frequency of a standing photon wave in the curved space-time around an electron possesses an upper limit k_{max} given by $1/(\kappa^2 m)$. It is this cut-off in (virtual) photon frequencies which in Weisskopf's view is really providing the inbuilt cut-off for quantum electrodynamics which we have exploited.

In December last year, Prof. Leonard Schiff formalized for me the Weisskopf argument. We have just heard of Schiff's sudden death yesterday from heart failure. This is devastating news for all who knew him, admired him and valued his friendship. He was one of the clearest-thinking and most penetrating men I ever met. I would like to recall the argument he wrote out for me.

A Schwarzschild Universe round the electron is given by the metric

$$(ds)^2 = (dt)^2 \left(1 - \frac{2m}{r}\right) - \frac{(dr)^2}{1 - \frac{2mG}{r}} - r^2\left[(d\theta)^2 + \sin^2\theta(d\varphi)^2\right] \quad , \qquad (G = 8\pi\kappa^2) \; .$$

With $\theta = \pi/2$ and $u = \frac{1}{r}$ the path of a light ray is given by

$$\frac{d^2u}{d\varphi^2} + u = 3mu^2 \; .$$

For u = constant and $u = u_0 = \frac{1}{3m}$ we obtain a circular path which the ray travels in time $t = t_0 = \frac{2\pi r_0}{\sqrt{1 - \frac{2m}{r_0}}} = 6\sqrt{3}\pi m$. Thus the proper angular frequency of orbital revolution of a circularly orbiting photon is $\frac{1}{3m}$, i.e. the maximal frequency of Weisskopf k_{max} equals $\frac{1}{3mG}$. If this k_{max} is substituted in the old Weisskopf formula

$$\frac{\delta m}{m} \approx \alpha \int_0^{k_{max}} \frac{d^3k}{|k^2|}$$

we recover our result for $\frac{\delta m}{m}$.

C. RELATION BETWEEN THE FINE STRUCTURE CONSTANT AND THE NEWTONIAN CONSTANT

One final remark about the formula

$$\frac{\delta m}{m} \approx \frac{3}{4\pi} \alpha \log u^2 m^2 \; . \qquad (2.12)$$

This term is the first term in an expansion which according to Weisskopf's paper of 1939 should go like

$$\frac{\delta m}{m} \approx \sum a_n (\alpha \log \kappa^2 m^2)^r \quad , \quad r \leqslant n \qquad (2.13)$$

when higher orders in α are included. [FN.9] The important point is that the effective parameter in this expansion is $(\alpha \log \kappa^2 m^2)$. This number has the suprising value near to unity ($\alpha \log \kappa^2 m^2 \approx \frac{100}{137}$). In an earlier paper we suggested that nature probably intended the formulae (2.12) and (2.13) to be read backwards, that is to say, we might start with the assumption that all (or nearly all electron self-mass is electromagnetic in origin, so that $\delta m/m \approx 1$. This may, in converse, determine $\alpha \log(\kappa^2 m^2)$ and possibly help in understanding why this number is empirically so close to unity.

The problem of infinities in electrodynamics arose with Lorentz's classical electron theory some seventy years ago. Waller investigated these infinities using Dirac's one-particle theory in 1930 and found that they persisted even after quantization. The modern formulation of the problem dates back to the famous 1934 paper of Weisskopf. There may be other solutions of the infinity problem; it has been suggested, for example, that summations over the major constant α will regularize these or with a non-unitarity-violating indefinite metric the infinities disappear. This may indeed be so. All we wish to point out is that there is in nature this powerful realistic regularizing effect of tensor gravity; that its effect is not small ($\approx \log \kappa^2 m^2 \approx \alpha^{-1}$) and that the regularized answers are such that we visibly recover the old infinities when we take the limit $\kappa \to 0$. The first calculations we published were finite but non-gauge invariant. We also worked with one particular non-localizable parametrization of gravity ($g^{\mu\nu}$ as the basic field, with $g_{\mu\nu}$ as the non-polynomial subsidiary quantity). We did not understand ambiguities; neither the distribution-theoretic ones nor the Borel ambiguities. This is changed now. With Lehmann's ansatz and his work and with the localizable (exponential) parametrization of gravity, we are working with a field theory which is no longer mysterious - one may call it orthodox. We are permitted to make field transformations at will. As we have shown, this allows us to exhibit explicitly the gauge-invariance of gravity-modified finite electrodynamics. We believe at least one complete solution of the very long-standing infinity problem now exists within the context of a nearly orthodox theory. We humbly wish that it may be taken note of.

PART III

A. The two-tensor theory of gravity

Since $\kappa_g^{-1} \approx 10^{18}$ BeV, the cut-off provided by Einstein's gravity is unlikely to be of significance in strong interaction physics. We need a universal non-polynomial strong coupling (analogous to the gravitational) which may provide an inbuilt cut-off around a nucleon mass. This part of the session will be devoted to an examination of the hypothesis that Einstein's gravity theory itself may be modified in such a manner that leptons interact directly with Einstein's tensor $g^{\mu\nu}$, while a new tensor $f^{\mu\nu}$ (representing a massive 2^+ unitary singlet) couples strongly to the hadronic stress tensor with coupling strength κ_f.[17] For the Lagrangian of the f-particle we adopt the Einstein form. A covariant mixing term between f and g tensors is postulated which guarantees that one combination $\tilde{f}(= f-g)$ of the two tensors represents a massive 2^+ particle (mass m_f) while another combination $\tilde{g} = \frac{\kappa_g^2 f + \kappa_f^2 g}{\kappa_g^2 + \kappa_g^2}$ describes massless gravitons.

Dr. C.J. Isham, in his lecture, will introduce the subject. As I said, our primary motivation in postulating this particular theory of strong phenomena was to build into it a cut-off round $(\kappa_f)^{-1} \approx m_f \approx 1$ BeV in order to regularize strong physics infinities. Since, however, F-dominance of the strong stress tensor could also be built into the theory, it is of course pertinent to examine this particular hypothesis.[FN10] Drs. B. Renner, H. Jones and K. Raman will speak on this subject. As is usual in strong interaction physics, their conclusions appear to be: "nothing wrong with the hypothesis; things are optimistic, but don't forget there is nothing ever clear-cut in strong physics either". I wish to make just one remark in connection with Dr. Jones' talk. Making the hypothesis that the relevant F-meson in the two-tensor theory is a yet undiscovered object, lying possibly on the Pomeron trajectory (at a mass near 1700-2000 MeV with the presently accepted value for Pomeron slope), we have tried to understand s-channel helicity conservation (empirically associated with Pomeron exchanges) as a consequence of the Pomeron trajectory possessing a stress tensor coupling at its 2^+ recurrence.

We succeeded in proving that a close connection between these ideas does exist; however, the relevant expression for the hadronic stress tensor must be the "minimal" one. (The minimal stress tensor is essentially the free-particle stress tensor.)

Now this fits nicely in with what Gell-Mann has told us in his session of the conference. It appears that an ansatz is emerging from studies of short-distance behaviour which states that physics follows the <u>free-particle field operators</u> (as contradistinguished from free-particle S-matrix elements) in the context of current algebras, Regge couplings or short-distance behaviour. This Gell-Mann ansatz translated to our situation would lead right away to the "minimal" stress tensor as the relevant tensor appearing in our Lagrangian and the one coupling with the F-meson.

B. BLACK HOLES IN THE F-GRAVITY FIELD

Even though the theory was invented for the task of suppressing infinities, I would like to take F-gravity seriously in its own right as modifying conventional gravity for short distances. Prof. S. Deser will be speaking in the session on this subject. As one of the prominent men among that small, select (but popularly considered somewhat crazy) band who move felicitously between gravity theory and particle physics, it is welcome to have his assurance that the two-tensor gravity theory makes no childish blunders (like violating the equivalence principle) in describing conventional gravitational phenomena. It has, however, pecularities of its own.

1) If we consider two hadronic particles it is clear (on account of f-g mixing) that in the linear approximation the static potential between them will be given by

$$V_{hh}(r) \propto - \frac{\kappa_f^2}{\kappa_f^2 + \kappa_g^2} \left[\kappa_f^2 \frac{e^{-m_f r}}{r} + \kappa_g^2 \frac{1}{r} \right] .$$

The corresponding potential between two leptons is

$$V_{\ell\ell}(r) \propto - \frac{\kappa_g^2}{\kappa_f^2 + \kappa_g^2} \left[\kappa_g^2 \frac{e^{-m_f r}}{r} + \kappa_f^2 \frac{1}{r} \right] .$$

There are, of course, no surprises here. The surprise comes for the hadron-lepton gravitational potential at distances $\approx 10^{-13}$. This is given by

$$V_{\ell h} \propto - \frac{\kappa_f^2 + \kappa_g^2}{\kappa_f^2 + \kappa_g^2} \left[- \frac{e^{-m_f r}}{r} + \frac{1}{r} \right] .$$

Note that the $1/r$ singularity has disappeared. Layers of leptonic and hadronic matter do not attract (gravitationally) as strongly as had been assumed when they approach closer than 10^{-13}cms; alternate layers would produce partial shielding of conventional gravity.

2) One may expect that the non-static gravitational potential between hadron and hadrons V_{hh} would contain a short-range repulsive component on account of the spin-two f-graviton exchanges in second and higher orders. We have made no computations of this repulsive gravity so far and would appreciate help in finding out what the situation in higher orders in F-meson-nucleon coupling is.

3) For interactions of ordinary high-frequency gravitons with large concentrations of matter, we may expect to see surface effects familiar in the analogous $\rho-\gamma$ mixing theory of hadronic electrodynamics. A high-frequency g-graviton would convert into an f-graviton, through the mixing term, which on account of the short-range character of its force would be absorbed predominantly at the surface of a (large) mass of matter, rather than penetrate into the inner layers. Very crudely speaking, gravitational effects may be expected to show $M^{2/3}$ rather than an M-dependence where M is the mass of the large object. This weakening of gravity may have consequences in respect of the onset of gravitational collapse phenomena. (For the analogous $\rho-\gamma$ mixing, photons of energies around 20 BeV empirically appear to exhibit a $Z^{0.9}$ dependence in their interactions with massive nuclei of charge Z).

4) So far we have spoken of conventional collapse. Consider F-gravity now in its "metrical" aspects. For regions inside hadronic matter where g-gravity may perhaps be neglected (to a good approximation), we are dealing with space-time of strong curvature. Let us for a moment take this aspect of F-gravity seriously. The conventional (Schwarzschild) formulae which give collapse radii in Einsteinian gravity will of course need to be revised for the f-g mixing situation - and I shall indicate some possible modifications below.[FN11] But, taking the formulae as they stand,

we may estimate κ_f in the same manner as Eddington's famous estimate of κ_g. Eddington assumed that the Universe is a Schwarzschild sphere (inside which we live and beyond which nothing from our side escapes). The radius of the Universe R_u, its mass m_u and the gravitational constant κ_g ($G_g = 8\pi\kappa_g^2$) would then be related by the formula:

$$R_u \approx M_u G_g \quad . \tag{3.1}$$

To the extent that R_u and M_u are known, G_g (and κ_g^2) come out roughly right ($\kappa_g^2 \approx 10^{-44} m_e^{-2}$) from this formula.[FN12] Let us now assume that hadrons are (nearly) collapsed objects in the F-gravity field. A typical hadron is the F-meson itself. Writing $R \approx \frac{1}{m_f}$ in (3.1) we would obtain a relation like

$$\frac{1}{m_f} \approx m_f \kappa_f^2 \ , \quad \text{i.e.,}\ \kappa_f^2 \approx m_f^{-2}$$

which is again roughly of the right magnitude for the strong coupling constant of f-mesons.[FN11]

What I am trying to say is that even if the black-hole physics does not prove fruitful for the macro-universe I expect it to possess applications in the micro-universe of hadronic matter. Some of you may be familiar with the black-hole jargon of "event horizons," "trapped surfaces", "anti-event horizons" (which in some cases can annihilate event horizons and give what I have been calling nearly-collapsed black holes). One of the beautiful formulae which have been recently discovered states that the condition for a trapped surface (and eventually a black hole) being formed for an object of mass M with angular momentum L and charge Q is given by

$$m^2\kappa^2 > \frac{L^2}{m^2\kappa^2} + Q^2 \quad .$$

For an electron, the right-hand side is 10^{-44}, the left-hand side is 10^{+44}. There is no danger that the leptons could ever collapse. But consider typical hadrons in F-gravity field. Here both sides are of the order of unity. Replace, as a good particle physicist always will, the charge Q^2 by I-spin $I(I+1)$ or by unitary-symmetry Casimir operators and you can see how mass formulae we are familiar with:

$$M^4 \kappa^4 = j(j + 1) + \kappa^2 M^2 \, I(I + 1)$$

may emerge from (F)-gravity. There are untapped riches here and a promise of a new language for the particle physicist.

CONCLUSIONS

1) The best thing which happened at this conference is the assurance that localizable non-polymonial theories are perfectly respectable field theories - in fact more or less orthodox, rather tame from an axiomatic point of view. Not only that; they are superior to polynomial theories in that they permit (with Lehmann's ansatz) an unambiguous and finite computation of renormalization constants.

2) For far too long,particle physicists have neglected gravity. Our excuse has been the smallness of the coupling constant. We have now learnt that this neglect distorts space-time and is one origin of the conventional infinities of quantum electrodynamics. The logarithmic infinities, for example, are simply a reflection of our setting $\kappa_g = 0$ in $\alpha \log \kappa_g^2 m^2$. Since localizable gravity is intrinsically non-polynomial, one finds that the first place where gravity comes in is through the logarithm of the Newtonian constant rather than through effects of order $\alpha \kappa^2$. Amazingly enough this number ($\log \kappa_g^2 m^2$) mearly equals α^{-1}, a fact we have tried to understand by postulating that (almost) all electron self-mass is possibly (gravity-modified) electrodynamic in nature.

3) With F-mesons and with the postulation of the two-tensor theory of gravity, nuclear physics would appear to be another name for strong gravity (this is provided isotopic and unitary spin phenomena are added from outside). Considering that he spent the later years of his life searching for a unification of the forces of nature, I am sure that if the Old Man were alive he would feel happy with this rather direct and amusing unification of concepts in the strongest and the weakest of forces. I feel we have left gravity theory too long to cosmologists. I would hope that at the next Rochester Conference, in addition to strong, weak and electromagnetic sessions, we have, as we had to-day, a fourth session devoted to gravity and particle physics. I hope also that by then our Chairmen or some other experimental physicist will have established the quantum nature of gravitational radiation assumed throughout this lecture. Thank you.

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FOOTNOTES

[1] On the basis of their microcausality properties, Jaffe classifies field theories as localizable or non-localizable. Examples are:

A) Jaffe-localizable

$$L_{int} = g : \phi^n \exp(\kappa\phi) : \qquad \text{(pure non-polynomial)}$$

or

$$L_{int} = g : (\bar{\psi}\psi A) \exp(\kappa\phi) : \qquad \text{(mixed Lagrangian)}$$

The dots : : denote normal ordering.

B) Jaffe-non-localizable

$$L_{int} = g : \phi^n (1 + \kappa\phi)^{-w} :$$

or

$$L_{int} = g : (\bar{\psi}\psi A)\ (1 + \kappa\phi)^{-w} : \qquad (w > 0)$$

Basically Jaffe's distinction rests on the high-energy behaviour of the two-point spectral function $\rho(p^2)$; it falls faster than $\exp\sqrt{|p^2|}$ for the localizable case and slower for the non-localizable. Jaffe shows that localizable theories are microcausal; the non-localizable ones are not. Also, by a rigorous theorem of Glaser, Epstein and Martin, S-matrix elements (on the mass shell) should exhibit Froissart high-energy boundedness for the localizable case. Far less is known about the non-localizable Lagrangians, apart from Steinmann-Taylor results that, notwithstanding the possible breakdown of microcausality, the LSZ construction of the S-matrix can still be carried through - as also the proofs for CPT and spin-statistics theorems.

Efimov and, following him, we at Trieste in our work have used seemingly non-localizable versions of chiral and gravitational theories (Weinberg-Schwinger parametrization for the chiral case, and the commonly used parametrization $g^{\mu\nu} = \eta^{\mu\nu} + \kappa h^{\mu\nu}$ for gravity). There is nothing in our work which would not permit us to use the alternative exponential field-parametrizations for both these cases, thus remaining formally in the localizable class and guaranteeing for ourselves all the promised good things - in particular Froissart-boundedness of the on-shell S-matrix elements. If equivalence theorems hold, presumably the seemingly non-localizable versions of chiral theories or gravity are in fact localizable through a host of cancellations taking place.

[2] For gravity, for example, $\langle g_{\mu\nu} \rangle$, which is related to the cosmological constant, may be expressed as a function of $D(0)$. I am indebted to Profs. Gell-Mann and Sexl for this remark.

[3] That normal ordering is a crime for gauge theories has been known for a long time. This is because, for naive normal ordering, chiral and gauge transformations are hard to define and gauge properties of the theory are destroyed.

[4] We shall call formulae (1.13) and (1.14) "generalized normal-ordering formulae" for derivative-containing situations. We believe that the use of these formulae eliminates those $D(0)$'s which arise from consonance of derivatives (i.e. from terms like $D(x)\,\partial^2 D(x)$). The $D(0)$'s which arise from the non-derivative parts of the Lagrangian can presumably be eliminated, using standard normal ordering so that the two procedures together perhaps give a complete scheme for elimination of all $D(0)$'s for localizable theories of the gauge variety. This needs further investigation.

[5] It is worth stressing that we have no special insights - at least none has been developed so far - for situations like

$$\frac{\partial}{\partial x_\mu}\left[\frac{1}{(x-y)^2}\right]^{z_1} \otimes \frac{\partial}{\partial x_\nu}\left[\frac{1}{(x-u)^2}\right]^{z_2} \qquad y \neq u$$

This is the same as for the Gel'fand-Shilov case where conventional faltung must be used to evaluate FT of

$$\left[\frac{1}{(x-y)^2}\right]^{z_1} \otimes \left[\frac{1}{(x-u)^2}\right]^{z_2}. \qquad y \neq u$$

[6] There could be two reasons for doubting this assertion; <u>first</u> could the inclusion of the mass term change this? <u>Second</u> since δm and δe, in an exact theory, are expressed as integrals of spectral functions, do the equivalence theorems really apply? My personal feeling is that they do. We know, for example, that both δm and δe share with strict mass-shell quantities the property of gauge invariance (unlike z_2). However, the only decent way of checking this would be to compute. Unhappily (and notoriously) direct verifications of equivalence theorems need summations of infinite sets of chain diagrams. (Recall the effort needed to prove equivalence theorems using perturbation expansions for the Dyson-Nelson Lagrangians, $\bar{\psi}\,\gamma_\mu \gamma_5\, \partial_\mu \phi\, \psi$ and $\bar{\psi}'\, \exp(i\gamma_5 \phi)\psi'$). And the problem is complicated still further by the necessity for preserving gauge invariance at each stage of the calculation and also coping successfully with the normal-ordering problem (compare Footnote 4).

[7] Note (2.4) is <u>not</u> the same transformation as (2.3b) where we took $\psi' = (\det L)^{-3/8}\psi$.

[8] Mathematically, the factor $(\det L)^{-1}$ for the electrons can be transformed away using (2.4) but this cannot be done for the photon. This, of course, is an over-simplification of the tensor gravity situation where the non-polynomiality associated with $L^{\mu a}$ (manifest in exponential parametrization) is by itself equally potent (even without the $(\det L)$ factor) in suppressing infinities. It is important once again to emphasise that irrespective of what chains of diagrams are summed, one can show that the final regularized magnitudes for $\delta m/m$ and $\delta e/e$ do not alter up to terms of order $\alpha \log \kappa^2 m^2$. This is to say that non-polynomialities from different terms do not give additive effects to the regularization of the infinities in the basic ST diagrams  and  .

[9] S. Perveen has verified that, in the fourth order in e, the infinity regularization does proceed in the expected manner, that is to say $(\alpha \log \omega)^2$ is regularized by gravity to $(\alpha \log \kappa^2 m^2)^2$. [S. Perveen, J. of Physics A, General Physics, 3, 625 (1970).]

[10] It should be emphasised that there is no compelling logical necessity for the interpolating Heisenberg field F to have a physical particle associated with it nor for the ansatz that such a particle pole should completely dominate the stress tensor, though it is nicer if this is so.

[11] My conjecture is that the f-g mixed equations when solved for static solutions would yield instead of the characteristic Schwarzschild formula

$$g_{rr} = \frac{1}{1 - \frac{2MG}{r}\, g}$$

something more like

$$f_{rr} = \frac{1}{1 - 8\pi M \left(\frac{\kappa_g^2}{r} + \frac{\kappa_f^2 e^{-m_f r}}{r} \right)}$$

for a hadronic source particle of mass M acting on a hadronic test particle. This would give a generalized Schwarzschild radius for an F-field singularity more like:

$$R \simeq \frac{1}{m_f} \log (\kappa_f^2 M m_f) .$$

Note the weak dependence of R on the mass of the source particle M. Even using this formula, $\kappa_f^2 m_f^2$ comes out to be of the right order of magnitude with typical hadrons considered as nearly collapsed objects with anti-event horizons reaching out to event-horizons.

[12] I would gratefully like to acknowledge conversations on this subject with Prof. S. Chandrashekar.

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