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QUANTUM GRAVITY AND INFINITIES IN QUANTUM ELECTRODYNAMICS

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ABSTRACT

Using Einstein's non-polynomial modification of electrodynamics, with quantum gravitational effects taken into account, it is shown that the self-mass and self-charge infinities of lepton electrodynamics disappear in a non-perturbative treatment of gravity. The typical gravitational effects appear as $\log(\kappa m)$ terms where $\kappa m \approx 2.2 \times 10^{-22}$ is the gravitational constant. This gives for the electromagnetic self-mass of the electron the reasonable value $\delta m = \frac{4m\alpha}{6\pi} \log\left(\frac{4\pi}{\kappa m}\right) \approx 1/6$ up to first order in α. A simplified treatment of the infinity-suppression mechanism is presented.

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I. ULTRAVIOLET INFINITIES

Ultraviolet infinities made their appearance in particle physics some seventy years ago, more or less about the same time that particle theory itself began.

The first infinity arose in Lorentz's calculation of electron self-mass (δm) in classical electrodynamics. His expression for this quantity was $\delta m = e^2/R$; this is linearly infinite for a point electron ($\lim R \to 0$). The infinity difficulty persisted even when quantum electrodynamics was used. The first calculation (by Waller [1]) using Dirac's single-particle electron equation gave for δm, an infinity which was quadratic. A later calculation by Weisskopf [2] using positron theory improved the situation only in the sense that, though the infinity still survived, it was logarithmic rather than quadratic with δm given (in a second-order perturbation calculation) by

$$\frac{\delta m}{m} = \frac{6\alpha}{4\pi} \lim_{R \to 0} \log \frac{1}{Rm} + \text{finite terms} \quad . \tag{1}$$

Weisskopf was able to prove the important result that in all higher orders $\delta m/m$ may diverge as fast as $\alpha^n (\log 1/Rm)^n$. On this basis he ascribed a "critical length" to positron theory; $R_{crit} \approx \frac{1}{m} \exp(-1/\alpha) \approx 10^{-56}/m$. This would be the length*) where the theory may be expected to need fundamental modification - either "realistic" [3] (through inclusion of other forces whose influence may become important at lengths of this order and which may remove this infinity) or a purely mathematical modification; for example, a change in the Dirac Lagrangian for electrons and photons.

The subsequent history of infinities in quantum electrodynamics (and in the rest of field theory) is well known. Using Feynman's graphical techniques, Dyson [4] in a classic paper confirmed Weisskopf's result and in addition showed that, so far as the perturbation solution of quantum electrodynamics is concerned, there is just one other magnitude - self-charge (δe) - which is (logarithmically) infinite in addition to self-mass (δm) .

*) In the natural units used $\hbar = c = 1$, $1/m \approx 3.8 \times 10^{-11}$ cms.

The question which remained open after Dyson's work was whether these two infinities were indeed genuinely intrinsic to the theory or whether they were making their appearance simply because one is using in the perturbation solution an unjustified expansion round the point $e = 0$.

Basically this question has remained open. However, the recent important work of Jaffe and Glimm [5] seems to raise the suspicion that some of the infinities could well be intrinsic to the types of Lagrangian considered. Jaffe and Glimm have considered electrodynamics in two dimensions (one of space and one of time). They show that one encounters logarithmic ultraviolet infinities even in a rigorous non-perturbative solution.

Jaffe and Glimm's result for electrodynamics in two dimensions cannot of course directly be carried over to four-dimensional space-time. For one thing, the theory they are dealing with is super-renormalizable - i.e., infinities do not arise in all orders of perturbation theory but only in the second, and thus there is, for example, no possibility of some of the infinities from different orders compensating each other (for special values of coupling constant). If, however, with these authors, one makes the conjecture that the infinities of perturbation theory do carry over to the exact solution, one must seriously consider modifying Dirac's Lagrangian for electrodynamics

$$\mathcal{L}_{int} = e \, \bar{\psi} \gamma_\mu \psi A_\mu \tag{2}$$

(with ψ and A_μ representing electron and photon fields) for distances of the order of Weisskopf's critical length. One would of course desire that the modification should not be ad hoc but dictated by something deeper.

II. NON-POLYNOMIAL LAGRANGIANS

On the formal side, the type of modification - realistic or otherwise - which promises to provide a theory free of infinities was indicated in 1963 by Fradkin and Efimov.[6] . This has subsequently been developed by other authors [7]. In the writing of the Dirac Lagrangian in Eq. (2) - and in fact of all conventional Lagrangians in field theory - an unstated assumption has always been made. By construction these conventional Lagrangians are simple local polynomial functions of field variables. No-one had seriously considered non-polynomial Lagrangians as possible fundamental Lagrangians of particle theory. The chief problem with these was to guarantee that solutions to field equations could be constructed which preserved unitarity and S-matrix analyticity and reasonable high-energy behaviour. It now appears[7)] reasonably clear that there do exist classes of non-polynomial Lagrangians (and these are defined more precisely in Refs. 6 and 7) - for which unitarity-preserving analytic solutions with no infinities can be constructed. The Dirac Lagrangian (2) must be modified - realistically or otherwise - to a non-polynomial form for small distances if we wish to make self-mass and self-charge finite[*)].

III. NON-POLYNOMIAL CHARACTER OF GRAVITY-MODIFIED ELECTRODYNAMICS

Now among the realistic modifications to (2), there is the appealing universal modification where one considers not just electrons and photons interacting with each other, but the full complex of electrons, photons and gravitons. Gravitational forces begin to make themselves felt at distances of the order of the gravitational constant

$$\kappa = 2.2 \times 10^{-22}/m$$

($\kappa^2 = 8\pi G/c^4$; G is the newtonian constant). Traditionally one has assumed that,this constant being so small, gravitational effects can safely be neglected as a first approximation. Arguing, however, from the point of view of the infinities and Weisskopf's "critical length", one may surmise to the contrary and possibly expect that gravity may be vitally relevant.

Following Einstein and Weyl, one can write down the full graviton-photon-electron (vierbein) modification of the Dirac Lagrangian (2). This has been done in detail in Ref. 8. For the purposes of this note it will

*) Renormalization theory of quantum electrodynamics is based on the thesis that these two quantities are basically unobservable and therefore the fact that the theory computes them as logarithmically infinite need cause no consternation. Unfortunately, a number of self-mass and self-charge effects are observable, like electromagnetic corrections to g_A/g_V in weak interaction theory, or mass difference of charged and neutral pions.

suffice to take note of just one modification which is in fact universal to all Lagrangians in general relativity. The invariance of the action integral $\int L(x)\, d^4x$ for general co-ordinate transformations demands that every Lagrangian $L(x)$ must contain the factor

$$\det\left(\eta^{\mu a} + \kappa h^{\mu a}\right)^{-1} \quad . \tag{3}$$

Here $h^{\mu a}$ is the (vierbein) gravity field and $\eta^{\mu a} = (1,-1,-1,-1)$ is the Minkowski metric tensor. In the presence of gravitons the Dirac Lagrangian (2) must at the very least be modified to read:

$$L_{int} = \left[\det\left(\eta^{\mu a} + \kappa h^{\mu a}\right)\right]^{-1} \left(e\, \bar{\psi}\, \gamma_a\, A^a\, \psi\right) \quad . \tag{4}$$

We shall sometimes refer to this modification (multiplication by the factor $[\det(\eta^{\mu a} + \kappa h^{\mu a})]^{-1}$) as the "minimal modification". The crucial point about the minimal modification is that it makes the new Dirac Lagrangian (3) - and indeed every Lagrangian in particle physics - automatically a non-polynomial one.

Already in the earlier paper of Ref. 8, where gravitational modifications of Lagrangians of particle physics were considered in this context, it was argued, on a general power-counting basis, that the vierbein-gravity Lagrangians would be free of all infinities provided calculations are made nonperturbatively in the constant κ. The purpose of this note is to present results of such non-perturbative calculations*)[9] which are of second order in e, for self-mass and self-charge. The remarkable result of this work is that the gravitational effects appear in the form $\kappa^n \log \kappa$ and $\kappa^n (\log \kappa)^2$ - the precise form for self-mass and self-charge being

$$\left.\begin{aligned} \frac{\delta m}{m} &= \frac{6\alpha}{4\pi} \log \frac{4\pi}{m\kappa} + \text{ terms of order } \alpha \text{ and } \kappa\alpha \\ \frac{\delta e}{e} &= -\frac{2\alpha}{3\pi} \log \frac{2\pi}{m\kappa} + \text{ terms of order } \alpha \text{ and } \kappa\alpha \ . \end{aligned}\right\} \tag{5}$$

*) Our calculational technique does not preserve gauge invariance; one aspect of this is the appearance of a finite photon self-mass (to be renormalized using a finite counter-term). The gauge-invariance problem, which is of course unrelated to the problem of infinities, requires further investigation.

The gravitational length κ thus provides the natural inbuilt cut-off for electrodynamics, the infinities of the unmodified Dirac Lagrangian being simply a reflection of the unfortunate limit of gravity having been neglected ($\kappa = 0$) and the theory making a desperate attempt to expand $\log \kappa$ round $\kappa = 0$.

IV. QUANTUM GRAVITY AND SUPPRESSION OF INFINITIES, A SIMPLIFIED TREATMENT

That gravitational interaction should provide a natural suppression mechanism for infinities is a conjecture which has been made in the past by Pauli, Klein, Landau[10] and others. Their heuristic argument went something like this.*)

All particle propagators are at least as singular as $1/x^2$ on the light-cone ($x^2 = 0$) . Infinities in field theory have their origin in coincidences of such light-cone singularities (we shall see an example of this in the later part of this section). Consider now light and matter propagating in a gravitational field. This propagation is affected by the presence of the field, the light-cone itself being affected and its equation influenced by gravity. Thus far the statement is classical. Now come the quantum effects of quantum gravity with its zero-point field fluctuations. One may expect that these fluctuations would cause a smearing of the otherwise sharply defined light-cone and with it of light (and particle) propagators, producing something like a shift of the propagators from their customary form $1/x^2$ to $1/(x^2 + G)$ (G is the newtonian constant). This smearing would suffice to suppress the infinities of field theory.

The fascinating thing about this heuristic argument is that it is correct in its essentials. This will of course be shown in detail in Ref. 9, but it is perhaps instructive to show it by carrying out an idealized calculation with a simplified version of the modified Dirac Lagrangian (3) - a simplification which does not change any of the essential steps in the argument and yet is transparent enough that one can follow the infinity-suppression mechanism.

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*) We are indebted to Professor W. Thirring for telling us of this.

Replace the (vierbein) gravity field $h^{\mu a}$ by a scalar gravity field $h(x)$ and $[\det [\eta^{\mu a} + \kappa h^{\mu a}]]^{-1}$ by the factor*) $[1 + \kappa h(x)]^{-1}$. The modified electromagnetic interaction in this model reads:

$$L_{int} = [1 + \kappa h(x)]^{-1} (e \bar{\psi} \gamma_\mu \psi A_\mu) \quad . \tag{6}$$

Let us compute δm for the two cases when $\kappa = 0$ and when $\kappa \neq 0$.

<u>Case I: $\kappa = 0$</u>

The Feynman rules for computing δm are well known. δm (in second order in e) is essentially proportional to the space-time integral of the amplitude for an electron emitting a photon at point x_1 , making a transition to point x_2 and reabsorbing the photon emitted earlier. According to these rules,

$$\delta m \propto e^2 \int d^4x \; D_{ab}(x) \, \gamma_a \, S(x) \, \gamma_b \tag{7}$$

where $D_{\mu b}(x)$ and $S(x)$ are the photon and electron propagators given by their well-known expressions:

$$D_{ab}(x) = \langle A_a(x) \, , \, A_b(0) \rangle_+ = - \frac{\eta_{ab}}{x^2} \tag{8}$$

$$S(x) = \langle \psi(x) \, , \, \bar{\psi}(0) \rangle_+ = \frac{\gamma_a x^a}{(x^2)^2} + \frac{m}{x^2} + \text{less singular terms.} \tag{9}$$

(Likewise for the graviton

$$D(x) = \langle \phi(x) \, , \, \phi(0) \rangle_+ = - \frac{1}{x^2} \;) \quad . \tag{10}$$

Though the integration in (7) is linearly infinite on a simple power count basis, it is easy to see that the leading linear infinity integrates to zero because of the oddness of the integrand, and only the logarithmically infinite Weisskopf term survives. This gives:

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*) This type of modification of field theory has recently been considered by P. Budini and G. Calucci as providing a regularizing technique for removing infinities (see Ref.11).

$$\frac{\delta m}{m} \propto e^2 \lim_{R\to 0} \int_{|x|\geq R} \frac{d^4x}{(x^2)^2} \quad . \tag{11}$$

Case II: $\kappa \neq 0$

Now consider the case of gravity being present; $\kappa \neq 0$. The modified Lagrangian (6) describes (at each space-time point) the emission and absorption of electrons, photons and any number of scalar gravitons. To see this, expand $[1 + \kappa\phi]^{-1} = 1 - \kappa\phi + \kappa^2\phi^2 - \cdots + (-)^n \kappa^n\phi^n + \cdots$. The transition amplitude and the integrand in (7) must contain factors describing the propagation of these gravitons and specifically the super-propagator $G(x) = \left(1/(1+\kappa\phi(x)),\ 1/(1+\kappa\phi(0))\right)_+$ which represents the exchange of all gravitons. This super-propagator is simply computed in the following manner.

From the expansion

$$[1 + \kappa\phi]^{-1} = 1 - \kappa\phi + \kappa^2\phi^2 + \cdots$$

we obtain *)

$$G(x) = \left\langle \frac{1}{1+\kappa\phi(x)} , \frac{1}{1+\kappa\phi(0)} \right\rangle_+ = \sum (\kappa^2)^{2n}\, n!\left(\frac{-1}{x^2}\right)^n . \tag{12}$$

Now comes the important point. The series expression (12) - even though formally divergent - can be summed, using the Euler-Borel formula:

$$n! = \int_0^\infty e^{-\zeta}\, \zeta^n\, d\zeta \quad .$$

*) The $n!$ factor in the expression for

$$(\phi^n(x)\, \phi^m(0))_+ = n! \left(\frac{-1}{x^2}\right)^n \delta_{mn}$$

comes from the possibility of pairing the ϕ factors in the bracket in $n!$ distinct ways.

Inserting this in (12), we obtain

$$G(x) = \sum_{n=0} \int \left(\frac{-\kappa^2}{x^2} \zeta \right)^n e^{-\zeta} d\zeta$$

$$= \int_0^\infty \frac{x^2}{x^2 + \kappa^2 \zeta} e^{-\zeta} d\zeta \quad . \tag{13}$$

Notice the remarkable result that whereas each term on the right of the series (12) is singular at $x^2 = 0$, the singularity increasing as the order n increases, it has disappeared in the formal sum (13). One can actually see in expression (13) the expected smearing out of the singularity at $x^2 = 0$ by an amount which is κ^2 times the Borel-average of ζ .

To compute $\delta m/m$ for the case $\kappa \neq 0$ we must now modify the integrand of (7) by multiplying it with G(x) . This gives

$$\frac{\delta m}{m} = e^2 \int \frac{d^4x}{(x^2 + \kappa^2 \zeta)} e^{-\zeta} d\zeta \quad . \tag{14}$$

Once again notice that the logarithmic (Weisskopf) infinity at the lower limit $x = 0$ is no longer present*).

Graphically what has happened is easy to picture. The Weisskopf calculation corresponds to the Feynman graph (see Fig. 1) with one photon and one electron line exchanged between x_1 and x_2 . The graviton super-propagator corresponds to summing over none, one, two, three, ... graviton-exchanges. The sum must be taken over the entire multitude of gravitons; billions and trillions of them. If the series on the right of (12) is truncated at any finite value of n , a truly virulent infinity - much worse than Weisskopf's - would result. Thus to compute with a linearized version of gravity theory - as has been done by most authors in the past -

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* In this paper we are not concerned with the basically harmless infra-red infinity of the integral (14) at $x = \infty$, nor with the problems of analytic continuation of Fourier transform of G(x) from space-like to time-like regions which are discussed in papers of Ref. 7.

and to approximate $[1 + \kappa\phi]^{-1}$ by a <u>finite</u> set of terms $1 - \kappa\phi + \cdots + (-1)^n \kappa^n \phi^n$ is an unforgivable crime. Non-linear gravitation of Einstein can and must be treated non-perturbatively. When this is done the effects of gravity appear in a new role - that of "realistic" infinity suppression.

V. TOWARDS A THEORY OF COUPLING CONSTANTS

We wish to argue that whatever cosmic role classical gravity may possess, an important role of <u>quantum-gravity</u> is this realistic suppression of infinities. If this is accepted, one may perhaps make tentative beginnings towards a fundamental theory of coupling constants.

There is no denying that the couplings of two of the fundamental forces of nature - the electromagnetic (with $\alpha = 1/137$) and the gravitational (with $G m^2 = \frac{1}{8\pi} \kappa^2 m^2 \approx 5 \times 10^{-45}$) are outrageously different from each other. However, when one remembers that the intrinsic non-polynomial nature of gravity theory (through the appearance of $[\det(\eta^{\mu a} + \kappa h^{\mu a})]^{-1}$) necessarily leads to <u>leading</u> gravitational effects being of order of $\log(\kappa m) \approx 50$, (as far as self-mass or self-charge are concerned) one is approaching "effective couplings" which are neither too small nor too big. (If the Weisskopf conjecture (Ref. 2) is correct and gravitational modification in higher orders in α follows the pattern of the second order, $\delta m/m$ may be expected to equal $\sum a_n (\alpha \log \kappa m)^n$ with the "effective constant" $\alpha \log(\kappa m) \approx 50/137$.) With this effective constant one may even revive the conjecture that the electron's self-mass may be due mainly to its electromagnetic interaction with gravity providing the natural inbuilt radius of the particle.

In the second order of our calculation, $\delta m/m \approx 6/4\pi \;\; \alpha \log(\kappa m) \approx 2/11$; an eminently reasonable result*). The hope is that higher orders in $\alpha \log(\kappa m)$ may bring this quantity nearer unity. Alternatively one may start with the assumption that nearly all of electron mass is electromagnetic $(\delta m/m \approx 1)$

*) We are indebted to Professor R.P. Feynman for stressing the significance of this result towards building up a theory of fundamental constants. A similar result has recently been arrived at by F. Hoyle and V. Narlikar (private communication) using different methods.

and in turn deduce κ for a given α . From this type of consideration there possibly begins to emerge a theory of coupling constants, giving a relation between two outrageously different numbers.

As far as higher orders are concerned, power counting arguments show that no new infinities arise. The major unsolved problem is the reconciling of various gauge invariances of the theory - including the Einstein-Fock-de Donder gauge for gravity which we have used throughout - with the non-perturbative calculational technique here used. This is a difficult technical problem. A theory similar to the present one may exist for strong interaction physics also, the role of Einstein's gravity there being naturally played by strong gravity[12)] produced by the spin-two f^0 meson field with κ_{strong} (of the order of $(m_N)^{-1}$) providing the appropriate inbuilt cut-off much earlier than $\kappa_{gravity}$.

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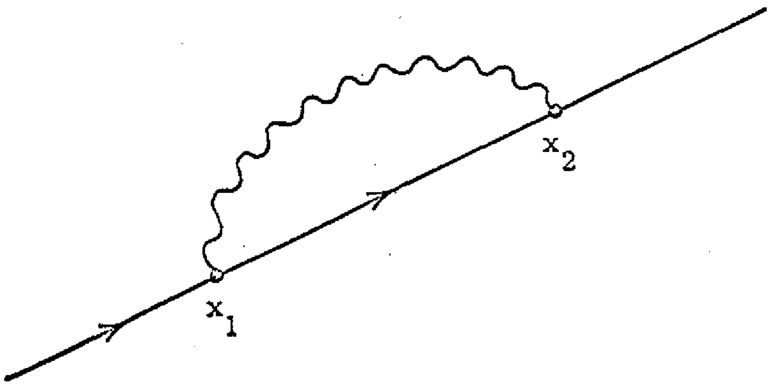

Fig. 1

Electron self-mass graph.

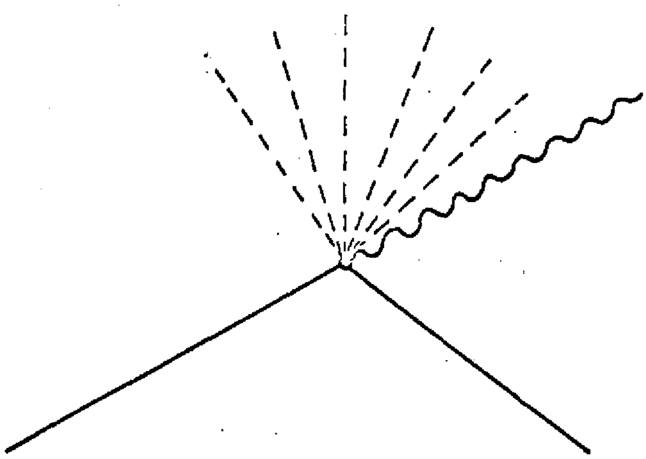

Fig. 2

Electron-graviton-photon vertex.