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CROSSING SYMMETRIC REGGE POLE MODEL FOR π-π SCATTERING *

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ABSTRACT

An interference model of π-π scattering is constructed which possesses linearly rising trajectories, Regge behaviour, resonances in all channels and is crossing symmetric. The model satisfies the Adler condition and predicts the Weinberg scattering lengths to within 10%. The calculated partial widths of the resonances associated with the leading trajectory are in reasonable agreement with experiment.

I. INTRODUCTION

In view of the successes of the Veneziano model[1)-3)] it is interesting to consider how dependent the results are upon the specific features of the model. The Veneziano model is based on the idea of "duality", i.e., that the Regge poles describe in some average sense the resonances in a given channel[4), 5)]. We shall study in the following a more conventional Regge pole model of the "interference type" in which the amplitude is described by the sum of the resonances and the Regge behaviour. The model has the following properties:

a) Regge behaviour in all channels for fixed momentum transfers $\lesssim 2\,\mathrm{GeV}^2$;

b) resonances in all channels;

c) crossing symmetry;

d) linearly rising trajectories;

e) exchange degeneracy;

f) the Adler self-consistency condition[6)].

It will appear from what follows that successful predictions of these models arise mainly from the properties c), d), e) and f) although some specific predictions of resonance widths, etc., will differ in various models. Thus we find that the basic results are largely independent of the Veneziano model *per se*, but depend more on general physical features of strong interactions. We shall consider in the present work mainly π-π scattering, ignoring possible violations of unitarity.

II. MODEL FOR π-π SCATTERING

Our amplitude has the form

$$A(s,t) = -\gamma \left[\frac{\Gamma(1-\alpha(s))}{\Gamma(\frac{1}{2}-\alpha(s))}\ \alpha(t)^{\alpha(s)} + \frac{\Gamma(1-\alpha(t))}{\Gamma(\frac{1}{2}-\alpha(t))}\ \alpha(s)^{\alpha(t)} \right], \tag{1}$$

where γ is a real constant. The π-π amplitudes for definite isotopic spin in the s channel are given by [3), 7)]

$$\begin{aligned} A^{I=0} &= \frac{3}{2}\,[A(s,t) + A(s,u)] - \frac{1}{2}\,A(t,u) \\ A^{I=1} &= A(s,t) - A(s,u) \\ A^{I=2} &= A(t,u)\ . \end{aligned} \tag{2}$$

The amplitude (1) is explicitly crossing symmetric and the structure of (2) is such that no "exotic resonances" occur in the $I = 2$ channel and there is no $J = 1$ resonance in the t channel for $I = 0$.

The trajectory $\alpha(t)$ satisfies the dispersion relation

$$\alpha(t) = \alpha(0) + t\alpha'(0) + \frac{t^2}{\pi}\int_{4m_\pi^2}^{\infty} \frac{dt'\ \mathrm{Im}\,\alpha(t')}{t'^2(t'-t-i\epsilon)}\ , \tag{3}$$

where $\mathrm{Im}\,\alpha(t) = 0$ for $t \leqslant 4m_\pi^2$. We observe that (1) vanishes for $\alpha(m_\pi^2) = 1/2$ and our model satisfies the Adler self-consistency condition [6)] for one of the external pions off the mass shell. In view of this we adopt as in the Veneziano model [3)] the linearly rising trajectory

$$\alpha(t) = \frac{1}{2} + \alpha'(t - m_\pi^2) + i\ \mathrm{Im}\,\alpha(t)\ , \tag{4}$$

where we write for convenience $\alpha' = \alpha(m_\pi^2)$. Then,

$$\alpha' = \frac{1}{2(m_\rho^2 - m_\pi^2)} = 0.88\ \mathrm{GeV}^{-2}\ . \tag{5}$$

In order to avoid high-spin "ancestors" in (1) we must require that $\mathrm{Im}\,\alpha(t)$ is small such that the linear behaviour of $\alpha(t)$ dominates as t increases from threshold. This corresponds to the narrow resonance approximation in the Veneziano model[1)-3)].

III. REGGE ASYMPTOTIC BEHAVIOUR

In discussing the asymptotic behaviour we shall use the identity

$$\Gamma(1 - z) = \pi/\Gamma(z)\sin\pi z \quad . \tag{6}$$

From (1), (2) and (4) we get in the s channel for large s, recalling that $s \to -u$, the Regge form

$$\begin{aligned}
A^{I=0} &= \frac{3}{2}\,\frac{\gamma\pi(\alpha' s)^{\alpha(t)}}{\Gamma(\alpha(t))\,\Gamma(\frac{1}{2} - \alpha(t))}\quad \frac{[1 + \exp(-i\pi\alpha(t)]}{\sin\pi\alpha(t)} \\
A^{I=1} &= \frac{\gamma\pi(\alpha' s)^{\alpha(t)}}{\Gamma(\alpha(t))\,\Gamma(\frac{1}{2} - \alpha(t))}\quad \frac{[1 - \exp(-i\pi\alpha(t)]}{\sin\pi\alpha(t)} \\
A^{I=2} &\longrightarrow 0\,.
\end{aligned} \tag{7}$$

We obtain the correct Regge behaviour in all three channels provided $t \lesssim 2\ \mathrm{GeV}^2$. The latter restriction arises in the model because for $t \leqslant 1.7\ \mathrm{GeV}^2$, we have $-1 \leqslant \alpha(t) \leqslant 1$ and the first term in (1) behaves for large s and fixed t like

$$s^{1/2}\, e^{-\alpha' s \log(\alpha(t))} \tag{8}$$

and therefore as s grows the first term in (1) is superconvergent. However, for $t \gtrsim 2\ \mathrm{GeV}^2$ the first term in (1) blows up due to the constant behaviour of the residue γ. But it is well known from Regge pole phenomenology that a constant residue function does not fit the data (e.g., π-N charge exchange scattering) for $t \gtrsim 1\ \mathrm{GeV}^2$ and therefore in our model we must replace γ by an exponentially decreasing residue function for $t \gtrsim 2\ \mathrm{GeV}^2$ as is commonly done in Regge pole analysis.

In view of the superconvergent nature of the first term in (1) for $t \lesssim 2\ \mathrm{GeV}^2$ the F.E.S.R. are satisfied by our model for large s.

We observe that our model has a ghost-killing mechanism which forces $\beta_\rho = 0$ for $\alpha_\rho = 0$. From factorization we know that a similar dip should occur in the helicity non-flip cross-section in π-N charge-exchange scattering, but the experimental situation on this issue is still unclear. The same feature occurs in the Veneziano model; a model that avoids this problem was considered by Virasoro [8].

IV. LOW-ENERGY π-π SCATTERING

In the neighbourhood of a t channel resonance for $I = 1$ and $\alpha = 1$, we obtain from (1) and (2) for large s:

$$A^{I=1} = \frac{\gamma}{2\sqrt{\pi}}\ \frac{(-1)\ (\alpha(s) - \alpha(u))}{\mathrm{Re}\,\alpha'(t - m_\rho^2 + i\,\mathrm{Im}\,\alpha(\mathrm{Re}\,\alpha')^{-1})}$$

$$\approx \frac{\gamma}{2\sqrt{\pi}}\ \frac{(-1)\ \cos\theta_t\ (m_\rho^2 - 4m_\pi^2)}{\mathrm{Re}\,\alpha'(t - m_\rho^2 + i\,\mathrm{Im}\,\alpha(\mathrm{Re}\,\alpha')^{-1})} \quad . \tag{9}$$

This should be compared with the field theory or dispersion theory expression for the exchange of a ρ meson

$$A^{I=1} = \frac{2\gamma_{\rho\pi\pi}^2\,(-1)\ \cos\theta_t\ (m_\rho^2 - 4m_\pi^2)}{t - m_\rho^2 + i\Gamma_\rho m_\rho} \quad . \tag{10}$$

Comparing (9) and (10) we get

$$\mathrm{Im}\,\alpha(\mathrm{Re}\,\alpha')^{-1} = \Gamma_\rho m_\rho$$

$$\gamma = 4\sqrt{\pi}\,\gamma^2_{\rho\pi\pi} \quad . \tag{11}$$

If the dominant decay mode is $\rho \to 2\pi$, then the width Γ_ρ is given by

$$\Gamma_\rho = \frac{1}{12}\left(\frac{\gamma^2_{\rho\pi\pi}}{4\pi}\right)\frac{(m_\rho^2 - 4m_\pi^2)^{3/2}}{m_\rho^2} \quad . \tag{12}$$

The Weinberg amplitude [9] takes the form

$$M_{\beta\delta,\alpha\gamma} = \frac{1}{F_\pi^2}[\delta_{\alpha\gamma}\,\delta_{\beta\delta}\,(s - m_\pi^2)$$

$$+ \delta_{\alpha\beta}\,\delta_{\gamma\delta}\,(t - m_\pi^2) + \delta_{\alpha\delta}\,\delta_{\beta\gamma}(u - m_\pi^2)] \tag{13}$$

and

$$A^{I=0} = \frac{1}{F_\pi^2}\,(3s + t + u - 5m_\pi^2)$$

$$A^{I=1} = \frac{1}{F_\pi^2}\,(t - u) \tag{14}$$

$$A^{I=2} = \frac{1}{F_\pi^2}\,(t + u - 2m_\pi^2) \quad .$$

If we expand (1) round the point $s = t = u = m_\pi^2$ and consider only the linear approximation, we find

$$A(t,u) \approx \sqrt{\frac{\pi}{2}}\,\gamma\,\alpha'(t + u - 2m_\pi^2) \tag{15}$$

and from (14) and (15) we get

$$\gamma = \sqrt{\frac{2}{\pi}}\,\frac{1}{\alpha' F_\pi^2} \quad . \tag{16}$$

Identifying (11) and (16) we have

$$\gamma^2_{\rho\pi\pi} = \frac{m_\rho^2 - m_\pi^2}{\sqrt{2}\pi\, F_\pi^2} \quad . \tag{17}$$

If we use the Goldberger-Treiman value of pion decay $F_\pi = 0.086\, M_p$ we find from (17) that $\gamma^2_{\rho\pi\pi} = 20$ and in terms of (12) this predicts $\Gamma_\rho = 80$ MeV , which is not bad when compared with the recent colliding beam experimental value $\Gamma_\rho = 111 \pm 11$ MeV [10]. Eq. (17) differs from the KSRF relation [11] by a factor $\sqrt{2}/\pi$.

We shall now calculate the S wave π-π scattering lengths from (1) and (2) using the values $\alpha(0) = 0.483$ and $\alpha(4m_\pi^2) = 0.552$ obtained from (4). The scattering lengths are defined by

$$a_I = A^I(4m_\pi^2, 0, 0)/32\pi\, m_\pi \quad . \tag{18}$$

Using the value $\gamma = 135$ obtained from (16), we get

$$a_0 = 0.21 m_\pi^{-1} \quad , \quad a_2 = -0.055\, m_\pi^{-1} \tag{19}$$

and

$$a_0/a_2 = -3.8 \quad . \tag{20}$$

These results differ by no more than 10% from the Weinberg scattering lengths [9]. The result (20) is not surprising as it depends mainly on the Adler condition and crossing symmetry. But the results (19) depend on the way the model is scaled in terms of the Weinberg amplitude.

We see that at this point we have obtained the basic low-energy π-π results derived by Lovelace [3] from the Veneziano model.

V. DECAY WIDTHS OF RESONANCES

Let us now see what partial decay widths are predicted by our model.

The Breit-Wigner amplitude for a definite spin J is

$$A_J = 16\pi \frac{\sqrt{s}}{k} \frac{\Gamma_R m_R (2J+1) P_J(\cos\theta)}{s - m_R^2 + i\Gamma_R m_R} \tag{21}$$

so that near the resonance the "residue" is given by

$$16\pi \frac{\Gamma_R m_R^2}{k_R} (2J+1) P_J(\cos\theta) \quad . \tag{22}$$

From (1) and (2) we find that the residue near the resonance with $I = 1$ and $\alpha = 1$ on the leading trajectory is

$$\text{Res. } A^{I=1}_{\alpha=1} = \frac{\gamma}{2\sqrt{\pi}} (m_\rho^2 - 4m_\pi^2)(-1) \cos\theta \tag{23}$$

and near the $I = 0$, $\alpha = 2$ resonance

$$\text{Res. } A^{I=0}_{\alpha=2} = \frac{9}{16} \frac{\gamma}{\sqrt{\pi}} \alpha'(m_f^2 - 4m_\pi^2)^2 (-1) \cos^2\theta \quad . \tag{24}$$

The ratio of the coefficients of the leading powers of $\cos\theta$ is then given by

$$\frac{8}{9} \frac{m_\rho^2 - 4m_\pi^2}{\alpha'(m_f^2 - 4m_\pi^2)^2} \tag{25}$$

whereas from (21) we find

$$\frac{A_{J=1}}{A_{J=0}} = \frac{2}{5} \frac{\Gamma_\rho(m_\rho^2/k_\rho)}{\Gamma_f(m_f^2/k_f)} \quad . \tag{26}$$

Equating (25) and (26) gives

$$\frac{\Gamma_\rho}{\Gamma_f} = \frac{20}{9} \frac{(m_f^2/k_f)}{(m_\rho^2/k_\rho)} \frac{(m_\rho^2 - 4m_\pi^2)}{\alpha'(m_f^2 - 4m_\pi^2)^2} \tag{27}$$

which yields $\Gamma_\rho/\Gamma_f = 0.89$. For a ρ width $\Gamma_\rho = 100$ MeV this predicts $\Gamma_f = 112$ MeV compared with the experimental value[12)] $\Gamma_f = 145 \pm 20$ MeV. This is an improvement on the Veneziano model in which the ratio goes the other way, $\Gamma_\rho/\Gamma_f = 1.2$, and yields $\Gamma_f = 83$ MeV.

For the ratio of the width of the first recurrence of the leading trajectory ($J^P = 3^-$) to the ρ meson we get

$$\frac{\Gamma_g}{\Gamma_\rho} = \frac{9}{112} \frac{(m_\rho^2/k_\rho)(m_g^2 - 4m_\pi^2)^3 \alpha'^2}{(m_g^2/k_g)(m_\rho^2 - 4m_\pi^2)} \tag{28}$$

and $\Gamma_g/\Gamma_\rho = 1.12$ yielding $\Gamma_g = 112$ MeV compared with the quoted experimental value $\Gamma_g = 120 \pm 30$ MeV. This is also an improvement on the prediction of the Veneziano model for which $\Gamma_g = 34$ MeV (if we normalize to $\Gamma_\rho = 112$ MeV this becomes $\Gamma_g = 38$ MeV). The widths of all the resonances associated with the leading trajectory in our model are positive.

Consider now the daughter resonances. In this case the daughter trajectory is defined by $\alpha^D = \alpha - n$ and $\alpha^D = J$ for the daughter resonance. We find for the daughter of the ρ meson called the σ (or ϵ) meson the ratio $\Gamma_\sigma/\Gamma_\rho = 5.34$ and this gives $\Gamma_\sigma = 534$ MeV predicting a broad S wave resonance under the ρ meson which does not appear to be inconsistent with the present experimental data. Next we consider the first daughter of the f meson usually identified with the ρ' with $J^P = 1^-$ and $I = 1$. We find that

$$\frac{\Gamma_{\rho'}}{\Gamma_\rho} = \frac{3}{2} \frac{(m_\rho^2/k_\rho)}{(m_f^2/k_f)} \left(\frac{m_f^2 - 4m_\pi^2}{m_\rho^2 - 4m_\pi^2}\right) [1 - \alpha'(m_f^2 - 4m_\pi^2)]\,. \tag{29}$$

For a slope $\alpha' = 0.88\ \text{GeV}^{-2}$ this predicts a negative width of the ρ' since $\Gamma_{\rho'}/\Gamma_\rho = -0.88$ or $\Gamma_{\rho'} = -88$ MeV. Thus, not all the daughter widths in our model are positive, which indicates a violation of unitarity.

The same phenomenon occurs in the Veneziano model[7], since the S wave daughter of the f meson is for physical π-π scattering $\Gamma_{f''} = -11$ MeV. But in our model the S wave daughter of the f meson is predicted to be positive $\Gamma_\rho/\Gamma_{f''} = 0.80$, giving $\Gamma_{f''} = 124$ MeV.

We see that from (29) the ratio $\Gamma_{\rho'}/\Gamma_\rho$ is fairly sensitive to the value of α' and for $\alpha' \approx 0.7\ \mathrm{GeV}^{-2}$ we have $\Gamma_{\rho'} \approx 0$. It is perhaps not surprising that since we attempt to fit all the properties of the model with one trajectory that some error in this trajectory creates an unphysical result. In addition it must be remembered that we have neglected all secondary and lower-order terms in (1). If these were included in our model then it is probable that the daughter widths could all be made positive - still retaining our predictions for the leading trajectory.

In view of the fact that we have used the same linear trajectory in our model as was used by Lovelace[3] it follows that we obtain the mass sum rules

$$m_{K^*}^2 - m_K^2 = m_\rho^2 - m_\pi^2 \tag{30}$$

and

$$m_{A_1}^2 - m_\rho^2 = m_\rho^2 - m_\pi^2 \ . \tag{31}$$

These sum rules do not depend in any essential way on the specific model adopted for the scattering amplitude.

VI. CONCLUSIONS

The results obtained from our model - which is effectively like the conventional Regge pole model made crossing symmetric and with the Adler condition thrown in - strongly indicate that the successes of models of this kind, including the Veneziano model, arise mainly from general properties like crossing symmetry, exchange degeneracy, linearly rising trajectories and the Adler condition and not from the specific form the model takes. They also indicate that one should not defend the concept of duality on the basis of agreement of such models with experiment since an interference model appears to do equally well in this respect. It would be interesting to calculate further predictions from the model for other processes because some of the specific predictions are not the same as the Veneziano model, as was found to be the case for partial decay widths of resonances.

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