IC/68/87

INTERNATIONAL ATOMIC ENERGY AGENCY

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

FUNDAMENTAL THEORY OF MATTER; A SURVEY OF RESULTS AND METHODS

ABDUS SALAM

1968
MIRAMARE - TRIESTE

IC/68/87

INTERNATIONAL ATOMIC ENERGY AGENCY

INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

FUNDAMENTAL THEORY OF MATTER; A SURVEY OF RESULTS AND METHODS

Abdus Salam

MIRAMARE - TRIESTE
September 1968

Lecture delivered during the International Symposium on Contemporary Physics (June 1968); to be published in the Proceedings of the Symposium (IAEA, Vienna).

FUNDAMENTAL THEORY OF MATTER; A SURVEY OF RESULTS AND METHODS

INTRODUCTION

Our systematic knowledge of stable and semi-stable forms *) (particles and resonances, lifetimes $\geqslant 10^{-24}$ secs) in which matter seems to exist, extends at present to forms created in laboratory collision experiments with proton beams of energies less than 30 BeV. A like situation holds in respect of the fundamental forces which govern the behaviour of these forms of matter; our systematic empirical knowledge extends no further than these same relatively low energies. I wish to give a rapid survey of what we believe are some of the "truths" and "insights" about fundamental laws of physics which, though abstracted from this low-energy data, may, hopefully, survive in a future theory.

The course that theory of matter would take in future centuries - its particle aspect and the hierarchy of forces between particles - was forecast in a remarkable prediction made by Isaac Newton:

"Now the smallest particles of matter may cohere by the strongest attractions, and compose bigger particles of weaker virtue; and many of these may cohere and compose bigger particles whose virtue is still weaker, and so on for divers successions, until the progression end in the biggest particles on which the operations in chemistry and the colours of natural bodies depend, and which by cohering compose bodies of a sensible magnitude.

"There are, therefore, agents in nature able to make the particles of bodies stick together by very strong attractions. And it is the business of experimental philosophy to find them out."

In Newton's day, the only virtue which particles of matter were known to possess was gravitational. Subsequently, we have learnt that there are at least three other virtues. These are: 1) strong; 2) electromagnetic; 3) weak; and, possibly, 4) super-weak.

---

*) Lifetime $\tau \sim \dfrac{\hbar}{\text{width}\,\Gamma} \sim \dfrac{6.6 \times 10^{-22}\ \text{MeV secs}}{\text{width in MeV}}$. Thus 10^{-24} secs lifetime corresponds to resonances of width ~ 100 MeV.

There are two important points to be made about these fundamental forces:

1. They have vastly different strengths; typically the ratios are,*)

strong	:	E.M.	:	weak	:	super-weak	:	gravitational
1	:	10^{-2}	:	10^{-5}	:	10^{-8}	:	10^{-34}

2. Even more important; there is a sharp selectivity about the other forces in contrast to the universality of gravitation. Thus,

(a) strong forces divide matter sharply into hadrons (strongly interacting matter) and leptons (with no strong forces);

(b) E.M. forces divide matter into electrically charged and uncharged;

(c) weak forces divide matter into "left-spinning" and "right-spinning". They act selectively between "left-spinning" matter in a sense I shall define more precisely later.

As in the familiar case of electrodynamics, it turns out that this selectivity is best expressed by assigning to different varieties of matter a number of fully or approximately conserved charges (strong charges, E.M. charge, weak charges). The assignment of charges serves two roles:

1. Kinematic; since the charges defined are (fully or partially) conserved, they serve to classify single particle states.

---

*) These are ratios (of squares) of the well-known dimensionless coupling constants, normalized (where necessary) with proton mass. There has been the conjecture (due, I believe, to Dirac) that the strengths of these forces and presumably also their relative ratios may have varied with the age of the universe (G. Gamow, Phys. Rev. Letters 19, 759 (1967)). Evidence to show that this is not likely for some of the constants,has been adduced recently (F.J. Dyson, Phys. Rev. Letters, 19, 1291 (1967); A. Peres, ibid. 19, 1293 (1967); J.N. Bahcall and M. Schmidt, ibid. 19, 1294 (1967); and S.M. Chitre and Y. Pal, ibid. 20, 278 (1968)).

2. **Dynamic; as for the case of the Coulomb force the long-range part** of physical forces is determined by the magnitudes of the corresponding charges.

Since the concept of strong, weak and E.M. charges is so crucial to particle physics, nearly half of my talk will be devoted to it. Indeed, if I were asked to list the major recent "truths" in particle physics, the list would be somewhat like this:

1. A clearer comprehension of the charge concept, both for classifying particles and for the dynamical role it plays. In particular, the discovery that there is a unifying principle running through strong, weak and E.M. forces, in that these forces share the same basic charges and their corresponding currents (scaled differently, of course, in strength).

2. Recognition of the essential correctness of basic laws of relativistic quantum mechanics (RQM) up to the energies available at present. Quantum mechanics was invented for systems of typical dimensions $\approx 10^{-8}$ cm, typical energies measured in eV. Non-relativistic quantum mechanics continued to work in the nuclear domain with typical lengths $\approx 10^{-12}$ cm and MeV energies - a vast extrapolation from the situation it was invented for.*) Relativistic quantum theory - and the strongly operative word is relativistic - seems to hold down to 10^{-14} cm and BeV energies. With quantum theory, we appear to have built ourselves a house with no doors and no windows and with walls so high that (in Jost's phrase) it is hard to know if it is a house or a prison we have inherited. It will be clear as I go on how tight and restrictive the structure of relativistic quantum mechanics is; for example, it does not appear to permit an easy mixing of space-time degrees of freedom with the "internal" degrees of freedom represented by the charges.

3. Renormalized quantum electrodynamics (Dirac-Maxwell theory of electrons and muons) which accords to the highest experimental accuracy achieved at present. Since Professor F. Low will review

*) There never was a conference held in the nineteen-thirties, where some of the founders of quantum theory did not express doubts about this extrapolation. The fundamental length at which quantum mechanics was expected to break down has continually diminished in size as time has gone on.

it in the next hour, I shall not discuss this most beautiful of theories which makes every one of us purr with elation.

4. Developments in weak interaction theory, initiated by the discovery of their extraordinary space-time reflection properties, which were first correlated through the recognition of the crucial role of the two-component (left-handed) nature of the neutrino, and through its later generalization, to the concept of chirality (handedness) for all matter. Prof. T.D. Lee will be covering this soon after this lecture and I shall not discuss this beautifully connected development.

The plan of my lecture will be as follows. First we go over the Rosenfeld table (Appendix A) of stable and semi-stable particles and resonances, introducing at the same time the various classifying charges; next describe the theoretical apparatus used, setting down also the canons of relativistic quantum mechanics (RQM); finally, turn back to the charges in their dynamical role and consider other dynamical symmetries not associated - to our present knowledge - with charges. We end with speculations whether these symmetries foreshadow discovery of newer forms of matter interacting with super-strong forces.

PART I

CLASSIFICATION OF PARTICLES AND CHARGES

A. POINCARÉ DESCRIPTION

Throughout we make the assumption that laws of physics are translation- and space-time rotation symmetric. This assumed Poincaré symmetry of space-time implies in turn conservation of energy, momentum (P_μ) and angular momentum $(J_{\mu\nu})$ *), respectively**)

---

*) Conservation law of angular momentum is verified empirically, for example, in $0^+ \to 0^+$ internal conversion in electro-transitions in Ge^{22}; Sunyar (G. Feinberg and M. Goldhaber, Proc. Nat. Academy, **45**, 1301 (1959)) found a ratio $\gamma/e^- < 1/1000$. Since γ-rate for a $1^\pm \to 0^+$ transition would be $\approx 10^6$ faster than e^- rate for $0^+ \to 0^+$ transition, one concludes that the amplitude for the admixture of spin 1 state with spin zero state is less than 1 part in 10^{-4}.

**) The argument connecting symmetries and conservation laws, though well known, is worth repeating. A symmetry in quantum theory is represented by a unitary transformation U relating a given state of the system to the one obtained by the symmetry operation. Infinitesimally, let

$$U = 1 + i\alpha_e X_e \quad (J_e \ \text{hermitian}) .$$

A symmetry is exact if U commutes with the full Hamiltonian of the system; it is a partial symmetry if it commutes with only a part (hopefully the dominant part) of the Hamiltonian. Thus for an exact symmetry,

$$[U , H] = 0 = [X_e , H] ,$$

i.e., $$\frac{dX_e}{dt} = 0$$

so that X_e is a time-independent operator; its eigenvalues do not change with time and serve to classify one particle states.

If the symmetry is only partial, i.e., if the Hamiltonian consists of two pieces, $H_{inv.} + gH_{non-inv.}$, $dX_e/dt \neq 0$; i.e., X_e is time-dependent, its time rate of change being proportional to the symmetry breaking parameter g.

It is our aim in the sequel to specify a set of quantum operators whose eigenvalues serve to classify single-particle states. A complete characterization of those involving the Poincaré operators P_μ and $J_{\mu\nu}$ ($\mu = 0,1,2,3$), was given by Wigner as follows:

Consider the rest frame of the particle $P_i = 0$, $i = 1,2,3$. Single-particle states are labelled according to:

(i) Rest mass (eigenvalues of the operator P_0).

(ii) Spin. The three spin operators *) J_i corresponding to space rotations close on the algebra of SU(2);

$$[J_i, J_j] = i\,\epsilon_{ijk}\,J_k \qquad i = 1,2,3.$$

Thus from standard group theory, quantum mechanical states (in the rest frame) can be labelled with the spin eigenvalues J and J_3 of the two (Casimir) invariants of SU(2) - i.e., of the operators

$$\underline{J}^2 = J_1^2 + J_2^2 + J_3^2 = J(J+1) \quad \text{and} \quad J_3 \quad ,$$

where **)

$$J = 0, \tfrac{1}{2}, 1, \ldots\ldots.$$
$$J_3 = 0, \pm\tfrac{1}{2}, {}^{\pm 1}_{\;\;0}, \ldots\ldots.$$

---

*) J_1, J_2, J_3 are the same as J_{23}, J_{31}, J_{12} of the set $J_{\mu\nu}$.

**) If the precise value of J_3 is not specified, the spin label J denotes collectively a $(2J+1)$ dimensional spin multiplet of particles. In subsequent work, spin J will be specified by indicating the multiplicity; thus multiplicities $\underset{\sim}{1}, \underset{\sim}{3}, \underset{\sim}{5}, \ldots$ will in indicate spins $0, \frac{1}{2}, 1, \ldots.$, respectively.

(iii) Particle-antiparticle duality

One of the gifts of local relativistic quantum theory and Poincaré symmetry is the assertion that all particles possess antiparticles *) (with the same mass, same lifetime and opposite charge). If experiment shows that this symmetry does not hold, either the Lorentz-invariance or the locality of the theory must be abandoned.**)

Summarizing, the Poincaré classification of physical states proceeds with the following ingredients: ***) (Table 2)

---

*) The reason for this is subtle; real Poincaré invariance in a local theory (defined more precisely in Part IIB) implies - it so happens - full complex Poincaré invariance ($U = 1 + i\alpha_\ell X_\ell$; α_ℓ, X_ℓ complex). Now a part of the complex Poincaré group connected to real Poincaré is space-time reflection $\underline{x} \rightarrow -\underset{\sim}{x}$ (parity or P-operation) and $t \longrightarrow -t$ (time reflection, T). This, together with C (conjugation of particles with antiparticles), defines the CTP symmetry which converts incoming particle states to the corresponding outgoing antiparticle states. CTP symmetry is thus an in-built part of a Poincaré-Lorentz-symmetric local relativistic quantum mechanics.

**) Present limits on mass and lifetime equalities of π^+ and π^-, for example, set (not too stringent) limits on the experimental validity of CTP $\left(\frac{I_{\pi^+}}{I_{\pi^-}} - 1 = .064 \pm .069\right)$.

***) The impatient reader will get a reasonable notion of the concepts in the subject by reading only the tables.

TABLE 1

Invariance or symmetry
represented by unitary transformation

$$U = 1 + i\alpha_\ell \chi_\ell ;$$

χ_ℓ observables.
If symmetry is exact,

$$[U,H] = 0 = [\chi_\ell ,H]$$

then

$$\frac{d}{dt} \chi_\ell(t) = 0 \quad .$$

If

$$\frac{d}{dt} \chi_\ell(t) \neq 0$$

symmetry is broken.

TABLE 2

Poincaré description

1) Rest mass
2) Spin

Spin algebra SU(2) $\Longrightarrow$

$$[J_i, J_j] = i\epsilon_{ijk} J_k$$

$$J = 0,\ \tfrac{1}{2},\ 1,\ \ldots\ldots$$

$$J_3 = 0,\ \pm\tfrac{1}{2},\ \begin{matrix}\pm 1\\ 0\end{matrix},\ \ldots\ldots$$

Multiplets of dimensionality $\longrightarrow$ $\underset{\sim}{1}$, $\underset{\sim}{2}$, $\underset{\sim}{3}$, ...

3) Particle $\overset{CTP}{\longleftrightarrow}$ antiparticle duality.

B. CLASSIFICATION CHARGES

The spin operators J_i (and their group-theoretic properties) provide prototypes for other classification charges. We now go over these.

1. Electric charge (Q)

Empirically the electric charge Q ,

(a) is conserved $\dot{Q} = 0$; $\tau(\text{electron}) > 2 \times 10^{21}$ yrs.;

(b) possesses (like J_3) the eigenvalues 0, ± 1, ± 2, ... in units of electron's charge. (In algebraic language Q is the generator of an internal symmetry group U(1).) All known charges exist in units of electron's charge;

(c) the most remarkable manifestation of this is the charge equality of electron and proton verified to better than 1 part in 10^{20} .

2. Strong (hadronic) charges

Unlike the electric case with but one type of charge known,*) there exist a variety of strong charges which manifest themselves for strongly interacting particles, the so-called hadrons. These are baryon charge (B) , hypercharge (Y), isotopic charges ($\underline{I}$) and unitary charges ($\underset{\sim}{F}$).

(a) Baryon charge B

Like electric charge, baryon charge B carried by protons, neutrons, etc. (see Table 4 for more baryons),

(i) is absolutely conserved **)

$$\tau_{\text{free proton}} > 10^{21} \text{ yrs.},$$

$$\tau_{\text{bound protons}} > 4 \times 10^{23} \text{ yrs.},$$

---

*) There could, for example, exist particles with magnetic charges (monopoles) but at present there is no evidence for these.

**) These are estimates of Reines, Cowan and Goldhaber (see G. Feiberg and M. Goldhaber, ibid.) who looked for possible proton decays in large hydrogeneous scintillation counters C_7H_8 .

(ii) possesses (like J_3) eigenvalues

$$0, \pm 1, \pm 2, \ldots$$

in units of proton's baryonic charge.
A remarkable (but not understood) empirical relation seems to exist connecting spin J and baryon charge B; empirically $(J - \frac{1}{2}B)$ takes integer but not half-integer values.*)

(b) Hypercharge Y

The discovery of hypercharge Y for strongly interacting particles is the outstanding post-war discovery of experimental hadron physics. Like Q and B, Y is associated with the algebra of group structure U(1) (eigenvalues $Y = 0, \pm 1, \pm 2, \ldots$). Unlike Q and B, particles of large Y value have not so far been reported; the largest firmly-established value of Y is -2 for Ω^-. Whereas Q and B are (to all intents and purposes) absolutely conserved, with Y we enter the domain of partially-conserved charges $\dot{Y} \neq 0$. A K^0-meson with $Y = +1$ decays in 10^{-10} secs into a pair of π-mesons with $Y = 0$, i.e., a unit of hypercharge disappears (into the vacuum) in 10^{-10} secs. Since the characteristic times involved in strong interactions are of the order of $10^{-23} - 10^{-24}$ secs ($\triangle E \approx$ several hundred MeVs), it is clear that this relatively slow hypercharge violation ($\tau \sim 10^{-10}$ secs) is irrelevant (to one part

*) This empirical relation has led to a certain confusion of terminology. To clarify: All strongly interacting particles are called hadrons. Among these are half-integer (Fermi) as well as integer-spin (Bose) particles. Empirically all baryons interact strongly and are therefore hadrons. From the relation $J - \frac{1}{2}B$ = integer, clearly zero baryon charge $B = 0$ implies integer (including zero) spin. Such particles are called mesons. A deuteron $(J = 1, B = 2)$ is a hadron, a boson, a baryon, but not a meson. When a particle is called baryon without qualification, the normal implication is baryon charge $B = 1$ and (from $J - \frac{1}{2}B$ = integer) half-integer spin.

in a trillion)*) in strong interaction physics.**)

(c) Isotopic charges

The concept of the three isotopic charges I_i (which generate, like the spin-operators J_i, the algebra SU(2)) has been familiar since the 1930's:

$$\left[I_i(t)\,,\,I_j(t)\right] = i\epsilon_{ijk}\,I_k(t) \quad .$$

(i) Like spin, multiplets of isotopic charge are labelled with two numbers:

$$|\underset{\sim}{I}| = 0,\ \tfrac{1}{2},\ 1,\ \tfrac{3}{2},\ \ldots$$

$$I_3 = 0,\ \pm\tfrac{1}{2},\ \left.\begin{matrix}\pm 1\\ 0\end{matrix}\right\},\ \begin{matrix}\pm\frac{3}{2}\\ \pm\frac{1}{2}\end{matrix}\ ,\ \ldots$$

with dimensionalities $\underset{\sim}{1}$, $\underset{\sim}{2}$, $\underset{\sim}{3}$, $\underset{\sim}{4}$, ...

(ii) Like hypercharge, isotopic charge is not conserved

$$\dot{\underset{\sim}{I}} \neq 0$$

if forces other than the strong forces are taken into account. As is well known from nuclear physics, the isotopic symmetry is broken by electromagnetism. Thus,

$$\frac{d}{dt}\,|\underset{\sim}{I}| \propto g^2_{\mathrm{E.M.}} \quad .$$

---

*) The lack of hypercharge conservation proceeds from that part of the Hamiltonian which corresponds to weak forces;

$$\dot{Y} \propto g^2_{\mathrm{weak}} \quad .$$

This is qualitatively clear since the ratio of hypercharge-violating decay times ($K^0 \to 2\pi$) to the hypercharge-conserving decays ($\rho^0 \to 2\pi$) is (apart from phase space factors) $\left|g^2_{\mathrm{weak}}/g^2_{\mathrm{strong}}\right|^2 \sim 10^{-11}$.

**) This is perhaps an appropriate place to make the essentially semantic distinction between semi-stable particles and resonances. When a hadron decays through E.M. or weak forces (like $K^0 \to 2\pi$), we have called it a (semi-stable) particle, the word resonance being reserved for hadrons decaying through strong forces themselves (like $\rho \to 2\pi$).

A quantitative measure of $|\underset{\sim}{I}|$ non-conservation is the mass difference among members of the same I-multiplet; typically for $I = 1$,

	Mass (MeV)	I_3
Σ^+	1189	1
Σ^0	1192	0
Σ^-	1197	-1

so that $\Delta m < 1$ part in 10^2 $\left(\frac{\Delta m}{m} \sim \frac{g^2_{E.M.}}{g^2_{strong}}\right)$.

(iii) For hadron physics there exists the empirical relation*) which expresses electric charge Q as a linear sum of hyper- and one of the isotopic charges

$$Q = I_3 + \tfrac{1}{2} Y \quad .$$

<u>This is the first example of charges shared between two different types of interactions, strong and E.M.</u>

(d) <u>Unitary charges</u>

A remarkable synthesis of isotopic charges and hypercharge was achieved when it was recognized, between 1959 and 1961, that these four charges were part of a larger set of eight, the so-called unitary charges F_i ($i = 1, 2, 3, \ldots, 8$) which close on the algebra of the SU(3) group

$$\left[F_i(t), F_j(t)\right] = i\, f_{ijk}\, F_k(t)$$

provided one identifies F_1, F_2, F_3 with I_1, I_2, I_3 and Y with $2/\sqrt{3}\ F_8$. The charge-sharing relation for hadrons, mentioned above, now reads

$$Q = F_3 + \frac{1}{\sqrt{3}} F_8 \quad .$$

The multiplets of unitary charge correspond to the representations of the SU(3) algebra; these typically possess dimensionalities $\underset{\sim}{1}$, $\underset{\sim}{3}$, $\underset{\sim}{\overline{3}}$, $\underset{\sim}{8}$, $\underset{\sim}{10}$, $\underset{\sim}{27}$, $\underset{\sim}{35}$, ...

---

*) In all such numerical sharing relations, and throughout this lecture, the charges are specified after scaling in their "natural" units.

Since the algebras of the hypercharge $(U_Y(1))$ and isotopic charge $(SU_I(2))$ are contained in the algebra of SU(3), each SU(3) multiplet contains in a specified manner a number of $U_Y(1)$ and $SU_I(2)$ multiplets. Listed below are some of the known $\underset{\sim}{8}$'s and $\underset{\sim}{10}$'s of SU(3) - the existence of the latter *) being splendidly confirmed by the discovery of the Ω^-. The fundamental triplet representations $\underset{\sim}{3}$ and $\underset{\sim}{\bar{3}}$ of SU(3) from which, according to standard group theory, all other representations can be made **) by repeated multiplications, have been named quark and antiquark representations. Their isotopic, hypercharge and charge content (based on the empirical formula $Q = I_3 + \frac{1}{2}Y$) would be as follows:

TABLE 3

		I_3	Y	$Q = I_3 + \frac{1}{2}Y$
Quark	$\underset{\sim}{3}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{2}{3}$
		$-\frac{1}{2}$	$\frac{1}{3}$	$-\frac{1}{3}$
		0	$-\frac{2}{3}$	$-\frac{1}{3}$
Antiquark	$\underset{\sim}{\bar{3}}$	$-\frac{1}{2}$	$-\frac{1}{3}$	$-\frac{2}{3}$
		$\frac{1}{2}$	$-\frac{1}{3}$	$\frac{1}{3}$
		0	$\frac{2}{3}$	$\frac{1}{3}$

*) A puzzle on par with the puzzle of non-existence of particles with $|Y| > 2$ is the non-appearance in particle spectrum of higher SU(3) multiplets $\underset{\sim}{27}$, $\underset{\sim}{35}$, ...

**) e.g.,

$$\underset{\sim}{3} \times \underset{\sim}{\bar{3}} = \underset{\sim}{8} + \underset{\sim}{1}$$

$$\underset{\sim}{3} \times \underset{\sim}{3} \times \underset{\sim}{3} = \underset{\sim}{1} + \underset{\sim}{8} + \underset{\sim}{8} + \underset{\sim}{10} \ .$$

Note the fractional charges one may expect quarks and anti-quarks to possess; no physical particles with these charges are known to exist to date. The accompanying chart (Table 4) summarizes some of the better-known hadrons in Rosenfeld's table.

The well-identified SU(3) multiplets with their spins J and parities P, are:

(i) Boson octets $J^P = 0^-, 1^-, 1^+, 2^+,$

(ii) baryon octet $J^P = \frac{1}{2}^+,$

(iii) baryon 10-fold $J^P = 3/2^+$.

Fig.2 is an important plot of still higher spin particles, some already identified, others conjectured on the assumption that all hadrons form SU(3) multiplets of $\underset{\sim}{8}$ and $\underset{\sim}{10}$'s (octets and decuplets). The plots give spin versus (mass)^2. Notice the important empirical result: these plots (known as Regge trajectory plots) are essentially straight lines and are continually rising. One of the most important unknowns in hadron physics is how far in the mass scale may one expect the rising of these trajectories, how high values of spin J are obtained in the particle spectrum; is there an "ionization limit" as there indeed is if we make a like Bohr plot of J versus M^2 for the hydrogen atom levels.

Returning to unitary symmetry, SU(3) (like the isotopic SU(2)) is a broken symmetry; unitary charge is only partially conserved. A measure of dF/dt is provided by the mass difference among members of the same multiplet; e.g., Table 4 lists the mass differences of successive members of the 10-fold. There is a (nearly constant)*) mass increase of ~145 MeV (equal-spacing rule) when we go from Y = 1 to Y = -2. A rough estimate of dF/dt is provided by $\Delta m/m \sim 1/10$

*) We shall not have time here to consider this SU(3) symmetry breaking medium-strong force in any detail. An important clue towards its group-theoretic specification is provided by the equal-spacing rule for masses mentioned above, which is most simply explained if we assume that the part of the Hamiltonian breaking SU(3) transforms like hypercharge. A certain amount of group theory is involved in deriving this result which is not immediate.

TABLE 4

Eight-folds					Ten-fold	
Mesons B = 0				Baryons B = 1	B = 1	
J^P	0^-	1^-	1^+	$\frac{1}{2}^+$	$\frac{3}{2}^+$	Mass (MeV)
I = 1 Y = 0	π^+, π^0, π^-	$\rho^+ \rho^0 \rho^-$	A_1^+, A_1^0, A_1^-	$\Sigma^+ \Sigma^0 \Sigma^-$	$\Delta^{++}, \Delta^+, \Delta^0, \Delta^-$	1236
I = ½ Y = 1	$K^+ \; K^0$	$K^{*+} \; K^{*0}$	K_A^+, K_A^0	p n	$Y_1^+ \; Y_1^0 \; Y_1^-$	1385
I = ½ Y = -1	$\bar{K}^0 \; K^-$	$\bar{K}^{*0} \; K^{*-}$	$\bar{K}_A^0, K_A^-$	$\Xi^0 \; \Xi^-$	$\Xi^{*0} \; \Xi^{*-}$	1530
I = 0 Y = 0	η^0	ϕ	D	Λ	Ω^-	1672

Electric charges indicated as superscripts; J^P = $\text{spin}^{\text{parity}}$.

Summary of Rosenfeld's table for hadrons this rather large number giving a measure of the (medium-strong) coupling strength of SU(3)-breaking forces relative to the strongest SU(3)-symmetric forces. The nature of these symmetry breaking forces (unlike the case of isotopic symmetry where we know that the symmetry is broken by electromagnetism) is one of the important unresolved problems.

To summarize, we recognize a succession - a hierarchy - of strong charges and associated symmetries; SU(3) unitary charges, SU(2) isotopic charges, hypercharge and (in a category by itself) baryon charge. If all but the strongest forces in nature are neglected, these charges are fully conserved. Their lack of exact conservation is a reflection of the existence of medium-strong, electromagnetic and weak forces, respectively.

(e) Left and right unitary charges

Even if SU(3) triplets (quarks) do not exist, in so far as they constitute the fundamental representation of SU(3), all other representations, as stated before, can be made as mathematical composites from them.

To construct both fermions as well as bosons as composites, quarks must carry spin $\frac{1}{2}$. This circumstance makes it possible to extend the notion of unitary charge in a most fruitful direction. A spin-$\frac{1}{2}$ particle has two polarization directions for a particle in motion, spin along and opposite to its direction of motion; the so-called left spin and right spin.
For a particle of rest mass zero (particles travelling at the speed of light) these two polarization states are completely independent states.*) There is no rest frame for a massless particle in which a rotation may transform right-spinning particles to left-spinning ones. There are thus not just three, but six zero mass quarks; three spinning right; three spinning left. Corresponding to each spin polarization, there are two distinct types of unitary charges defining two independent algebras, $SU(3)_{left}$ and $SU(3)_{right}$. The symmetry represented by $SU(3)_L \times SU(3)_R$ is of course a badly broken one, because it holds only in the idealized limit of zero mass for the fundamental representation (the quark). We shall see later that this symmetry (also called chiral symmetry) is not a good kinematical symmetry for classifying particle states; surprisingly, however, it turns out to be an excellent symmetry in its dynamical aspects, at least for low-frequency phenomena (Part III).

*) The archetypal example is the neutrino (see under leptons). Left (spinning) neutrinos exist; right (spinning) neutrinos do not. This (two-component) aspect of the neutrino gave rise to the concept of left and right (chiral) charges.

(f) Nature of the internal symmetries ($U_Q(1)$, $U_B(1)$, $U_Y(1)$, $SU_I(2)$, $SU_F(3)$)

The spin quantum numbers have their origin in Poincaré-Lorentz symmetry of space and time. The other charges Q, B, Y, $\underset{\sim}{I}$, F, in so far as they are dissociated from space-time structure, presumably represent, in some sense, symmetries associated with internal degrees of freedom. But are these charges really that dissociated from space-time? I shall come back to this problem in Part IIE of the lecture.

TABLE 5

Summary of classifying charges for hadrons

Charges	Algebra	Typical multiplets	
Electric Q	U(1)		$\dot{Q} = 0$
Baryonic B	U(1)		$\dot{B} = 0$
Hypercharge Y	U(1)		$\dot{Y} \propto g^2_{weak}$
Isotopic $\underset{\sim}{I}$	SU(2)	$\underset{\sim}{1}$, $\underset{\sim}{2}$, $\underset{\sim}{3}$, ...	$\dot{I} \propto g^2_{\text{electromagnetic}}$
Unitary F^i	SU(3)	$\underset{\sim}{3}$, $\underset{\sim}{8}$, $\underset{\sim}{10}$, ...	$\dot{F} \propto g^2_{\text{medium-strong}}$
Left & Right Unitary	$SU_L(3) \times SU_R(3)$	$(3,\bar{3}) + (\bar{3},3)$,...	

Sharing of charges $Q = I_3 + \frac{1}{2}Y = F^3 + \frac{1}{\sqrt{3}} F^8$.

3. Leptons

So far we have dealt with hadrons. Consider now leptons — particles without strong interactions. The known lepton spectrum is meagre in contrast to the richness of the hadron spectrum.*) There appear to be but four leptons differentiated from each other by two types of charge. These are:

(a) μ^-, ν_μ,

(b) e^-, ν_e.

Both ν_μ and ν_e are left-spinning objects; μ^-, ν_μ each carry unit muonic charge; e^-, ν_e each carry unit "electronic" charge. Both these leptonic varieties of charge are (individually) conserved. Leptons exhibit E.M., weak and gravitational interactions. One of the ununderstood empirical facts is the remarkable identity of muonic and electronic interactions - roughly speaking, the equality of the two types of leptonic charges - notwithstanding the different masses **) of these particles. This is somewhat analogous to the surprising numerical equality of the electron's and proton's electric charges mentioned earlier.

4. Weak charges

Both hadrons and leptons interact weakly. In analogy to strong charges, there are weak charges, defined more precisely in Part III, where we shall see that (apart from scale) these are related, for leptons, to the (left) leptonic charges introduced above and, for hadrons, to certain combinations of $SU_L(3)$ charges.

*) Recent systematic search at Stanford has failed to reveal any other leptons with masses less than a BeV with fairly low production cross-sections. (A. Barna et al., Phys. Rev. Letters 18, 360 (1967).)

**) The large muonic mass (nearly as large as that of the pion) has always led to the suspicion that muons may possess strong charges as well as weak ones.

C. DISTINGUISHED PARTICLES AND ELEMENTARITY

It has been remarked that all hadrons could be considered as (mathematical) composites of three quarks; together with the four leptons above, one could say that this set of seven objects - the last remnants of the elementarity idea - constitutes in some sense a distinguished set (of particles or fields) from which all other particles could be made.

If one wished to substitute for the unobserved quarks a set of particles observed physically, numerous other choices are possible. One economical set would be the Sakata set p, n, Λ - but the SU(3) symmetry would be harder to build in with this as the basic set. To build in SU(3) credibly, the best hadron set is still the familiar old favourite, the octet of baryons N, Λ, Σ, Ξ, of which the nucleon is a member. This, however, would be only one of the many choices possible.

The arbitrariness of such a choice brings us back full cycle to the dilemma of elementarity. The historical tradition of physics - and the view found profitable when dealing with leptons - lies along an identification of a distinguished - an aristocratic - set of particles (as few in number as possible) of which all others are made. The other viewpoint could be that there is no distinguished set at all, at least not for hadrons, that there is full democracy in hadron physics and that it is more profitable to consider all hadrons as composites of each other. We shall consider these two contrasting views of elementarity - aristocratic vs. democratic - in more detail in Part II.

PART II

RELATIVISTIC QUANTUM MECHANICS

In this part of the lecture we discuss the structure of relativistic quantum mechanics (RQM). The ideas I shall be wrestling with are some of the prettiest, also some of the profoundest, in the whole range of physics. I have tried very hard to achieve a clear exposition but the sheer number of concepts makes the task nearly impossible. Perhaps an initial listing here of the topics may help.

Sec. A: discuss relativistic kinematics.

Sec. B: ingredients of two approaches to RQM, field theory and S-matrix theory.

Sec. C: set down the accepted canons of RQM as abstracted from the two approaches.

Sec. D: list some far-reaching consequences of the canons.

Sec. E: describe the attempts to reproduce the hadron-spectrum within the framework of RQM.

A. KINEMATICAL CONSIDERATIONS

In Part I, we introduced the personnae of the cast, the particles and resonances (see in particular, Appendix A). All members of this cast (except protons, neutrons and electrons of which normal matter is composed) were created and discovered in cosmic rays or accelerator-beam collisions. Appendix B is a note which Dr. G.H. Stafford of the Rutherford Laboratory, Harwell, has kindly prepared for me; this lists typical beams and beam intensities of the present generation of accelerators. Since collision experiments using these beams are our sole systematic means of the study of semi-stable particles, resonances, their masses, their decay widths, as well as the details of the interaction processes, it is important to familiarize oneself with the kinematical and theoretical constructs employed. This

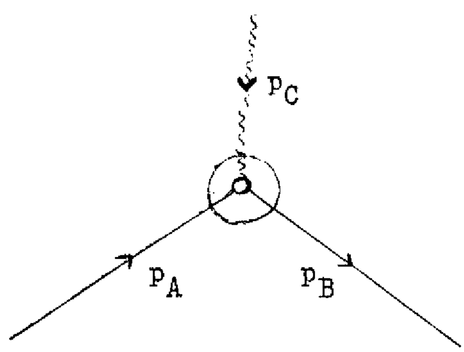
(Sec. A) is perhaps the dullest part of the lecture; I need however, the notation we introduce here. Anyone familiar with it may read just the summary at the end of the section and pass on.

1. The masses of semi-stable particles and resonances

One of the major - and in contrast to non-relativistic quantum mechanics - essentially unsolved problems of particle theory is the prediction of the particle spectrum - the masses, spins, paritites and other charges of semi-stable particles and resonances. The problem has been attacked both from the "aristocratic" and "democratic" points of view of Part IC and we return to it in IIB.*)

2. Decay widths, coupling constants and form factors

The second task of the theory is to give a description of decay widths. The kinematical tool for two-body decays is the so-called <u>three-point function or the vertex function</u> $F(p_A, p_B, p_C)$,



---

*) Internal symmetries discussed in Part I simplify the problem to the extent that if one member of a multiplet is discovered, and if we have some idea of the symmetry breaking forces, we can make a reasonable guess at the masses of the other members of the multiplet.

PART II

RELATIVISTIC QUANTUM MECHANICS

In this part of the lecture we discuss the structure of relativistic quantum mechanics (RQM). The ideas I shall be wrestling with are some of the prettiest, also some of the profoundest, in the whole range of physics. I have tried very hard to achieve a clear exposition but the sheer number of concepts makes the task nearly impossible. Perhaps an initial listing here of the topics may help.

- Sec. A: discuss relativistic kinematics.
- Sec. B: ingredients of two approaches to RQM, field theory and S-matrix theory.
- Sec. C: set down the accepted canons of RQM as abstracted from the two approaches.
- Sec. D: list some far-reaching consequences of the canons.
- Sec. E: describe the attempts to reproduce the hadron-spectrum within the framework of RQM.

A. KINEMATICAL CONSIDERATIONS

In Part I, we introduced the personnae of the cast, the particles and resonances (see in particular, Appendix A). All members of this cast (except protons, neutrons and electrons of which normal matter is composed) were created and discovered in cosmic rays or accelerator-beam collisions. Appendix B is a note which Dr. G.H. Stafford of the Rutherford Laboratory, Harwell, has kindly prepared for me; this lists typical beams and beam intensities of the present generation of accelerators. Since collision experiments using these beams are our sole systematic means of the study of semi-stable particles, resonances, their masses, their decay widths, as well as the details of the interaction processes, it is important to familiarize oneself with the kinematical and theoretical constructs employed. This

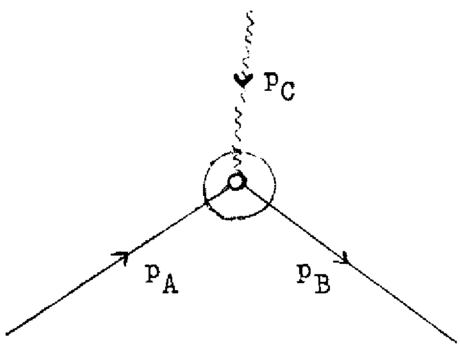
(Sec. A) is perhaps the dullest part of the lecture; I need however, the notation we introduce here. Anyone familiar with it may read just the summary at the end of the section and pass on.

1. The masses of semi-stable particles and resonances

One of the major - and in contrast to non-relativistic quantum mechanics - essentially unsolved problems of particle theory is the prediction of the particle spectrum - the masses, spins, paritites and other charges of semi-stable particles and resonances. The problem has been attacked both from the "aristocratic" and "democratic" points of view of Part IC and we return to it in IIB.*)

2. Decay widths, coupling constants and form factors

The second task of the theory is to give a description of decay widths. The kinematical tool for two-body decays is the so-called three-point function or the vertex function $F(p_A, p_B, p_C)$,



*) Internal symmetries discussed in Part I simplify the problem to the extent that if one member of a multiplet is discovered, and if we have some idea of the symmetry breaking forces, we can make a reasonable guess at the masses of the other members of the multiplet.

PART II

RELATIVISTIC QUANTUM MECHANICS

In this part of the lecture we discuss the structure of relativistic quantum mechanics (RQM). The ideas I shall be wrestling with are some of the prettiest, also some of the profoundest, in the whole range of physics. I have tried very hard to achieve a clear exposition but the sheer number of concepts makes the task nearly impossible. Perhaps an initial listing here of the topics may help.

- Sec. A: discuss relativistic kinematics.
- Sec. B: ingredients of two approaches to RQM, field theory and S-matrix theory.
- Sec. C: set down the accepted canons of RQM as abstracted from the two approaches.
- Sec. D: list some far-reaching consequences of the canons.
- Sec. E: describe the attempts to reproduce the hadron-spectrum within the framework of RQM.

A. KINEMATICAL CONSIDERATIONS

In Part I, we introduced the personnae of the cast, the particles and resonances (see in particular, Appendix A). All members of this cast (except protons, neutrons and electrons of which normal matter is composed) were created and discovered in cosmic rays or accelerator-beam collisions. Appendix B is a note which Dr. G.H. Stafford of the Rutherford Laboratory, Harwell, has kindly prepared for me; this lists typical beams and beam intensities of the present generation of accelerators. Since collision experiments using these beams are our sole systematic means of the study of semi-stable particles, resonances, their masses, their decay widths, as well as the details of the interaction processes, it is important to familiarize oneself with the kinematical and theoretical constructs employed. This

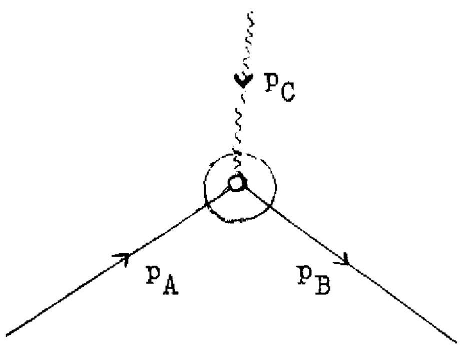
(Sec. A) is perhaps the dullest part of the lecture; I need however, the notation we introduce here. Anyone familiar with it may read just the summary at the end of the section and pass on.

1. <u>The masses of semi-stable particles and resonances</u>

One of the major - and in contrast to non-relativistic quantum mechanics - essentially unsolved problems of particle theory is the prediction of the particle spectrum - the masses, spins, paritites and other charges of semi-stable particles and resonances. The problem has been attacked both from the "aristocratic" and "democratic" points of view of Part IC and we return to it in IIB.*)

2. <u>Decay widths, coupling constants and form factors</u>

The second task of the theory is to give a description of decay widths. The kinematical tool for two-body decays is the so-called <u>three-point function or the vertex function</u> $F(p_A,p_B,p_C)$,



*) Internal symmetries discussed in Part I simplify the problem to the extent that if one member of a multiplet is discovered, and if we have some idea of the symmetry breaking forces, we can make a reasonable guess at the masses of the other members of the multiplet.

Denote the four momenta of the three objects A,B,C involved as p_A, p_B, p_C. From energy momentum conservation $p_B = p_A + p_C$. The general vertex function is a function of the spins and momenta of the three objects A,B,C which interact. From Lorentz-invariance it can be written in the form of a product of spin factors χ^i and the so-called <u>invariant factors</u> F^i. The invariant functions

1) contain no spin,

2) are functions of the Lorentz-scalars which can be formed from the momenta p_A, p_B, p_C. The spin factors χ^i are kinematical objects; the dynamics is expressed by the functions F^i.

$$V = \sum_i \chi^i \text{(spins)}\ F^i\ (p_A^2, p_B^2, p_C^2) \quad .$$

Consider the following special cases:

(i) A,B,C represent three stable physical particles of masses m_A, m_B, m_C,

$$(p_A^2 = m_A^2 \quad p_B^2 = m_B^2 \quad p_C^2 = m_C^2) \quad .$$

The limiting values $F^i(m_A^2, m_B^2, m_C^2)$ of the invariant function F^i are known as the <u>coupling constants</u>.

(ii) If $m_A^2 > (m_B + m_C)^2$, the particle A is unstable. the vertex function in this case directly gives the decay amplitude *) $A \to B + C$.

---

*) If A,B and C all belong to SU(2) (or SU(3)) multiplets, it is clear that the amplitude above is related by simple group theoretic Clebsch-Gordan factors to an amplitude $A' \to B' + C'$, where primed objects are other members of the multiplets. It is the experimental verification of this type of kinematic relation which, in general, gives the empirical SU(2) (or SU(3)) assignments of particles.

(iii) When A and B are the same particle , $p_A^2 = p_B^2 = m^2$ and C is a (virtual) photon (i.e., electromagnetic field frequency), the F^i's are functions of just one variable, the momentum transfer $t = (p_C)^2 = (-p_A + p_B)^2$ and give the so-called electromagnetic form factors.*)

3. The scattering amplitude

The kinematical construct here is the so-called four-point function. (Fig.4) Consider four interacting objects with momenta p_A, p_B, p_C, p_D. From energy-momentum conservation

$$p_A + p_B + p_C + p_D = 0.$$

Like the vertex function, the four-point function $T(p_A, p_B, p_C, p_D)$ can be written in the form

$$T(p_A, \ldots) = \sum_i \chi^i(\text{spins})\, F^i(p_A^2, p_B^2, p_C^2, p_D^2, s, t)$$

where

$$s = (p_A + p_B)^2$$

$$t = (p_A + p_C)^2$$

---

*) The Fourier transforms of these functions in configuration space represent the spatial extension of charge, magnetic and other E.M. moment densities, for the particle A. One of the most beautiful of recent experimental results is (Fig.2) that apart from a scale change, the magnetic and electric charge form factors of the proton and the magnetic form factor of the neutron - and thus the spatial charge and magnetic moment densities - are identical when plotted as functions of momentum transfer up to $|(p_A - p_B)^2|$ around $(4\text{BeV})^2$. We shall see later (Part IV) how this identity of charge and magnetic densities finds its readiest explanation in terms of dynamical symmetries (like $\tilde{U}(12)$) higher still in heirarchy than the ones considered in Part I.

The F^i's are the invariant functions of the six independent Lorentz scalars which can be formed from the momentum vectors p_A, p_B, p_C, p_D (of which but three are independent, since $p_D = -(p_A + p_B + p_C)$.

We shall mainly - though not exclusively - be interested in the situations where the four objects are physical particles <u>on their mass shells</u>, i.e., when $p_A^2 = m_A^2$, $p_B^2 = m_B^2$, $p_C^2 = m_C^2$, $p_D^2 = m_D^2$. Consider the following special cases, all represented by Fig.4:

a) <u>Three body decays</u>: if $p_A^2 = m_A^2 > (m_B + m_C + m_D)^2$, A is unstable and the amplitude represents its three-body decay $A \to B + C + D$.

b) <u>Scattering</u>

<u>Channel I</u>. Let momenta p_A and p_B be incoming; p_C and p_D outgoing. The variables s and t on which $F^i(s,t)$ depend are well-known physical variables; thus*)

$$s = (p_A + p_B)^2 = E_c^2 = 4(k_c^2 + m^2)$$

$$t = (p_A + p_B)^2 = -2k_c^2 (1 - \cos\theta_c) ,$$

i.e., s equals the square of c.m. energy (E_c) and t is the momentum transfer. It is important to note that scattering occurs only when in the (s,t) plane,

$$s \geqslant (m_A + m_B)^2 > 0$$

and

$$t \leqslant 0 .$$

<u>Channel II</u>. The same Fig.4 could represent a second physical situation, where p_A and p_C are incoming momenta and p_B and p_D outgoing. In this case

$$(\text{c.m. energy})^2 = t = (p_A + p_C)^2 \geqslant (m_A + m_C)^2 > 0$$

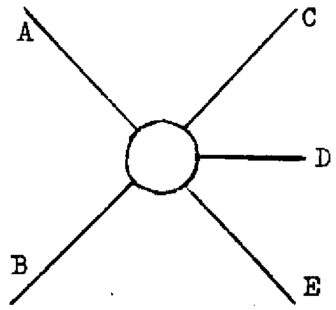
$$\text{momentum transfer} = s \leqslant 0$$

*) Assume for simplicity all particles have equal masses ($m_A = m_B = m_C = m_D$).

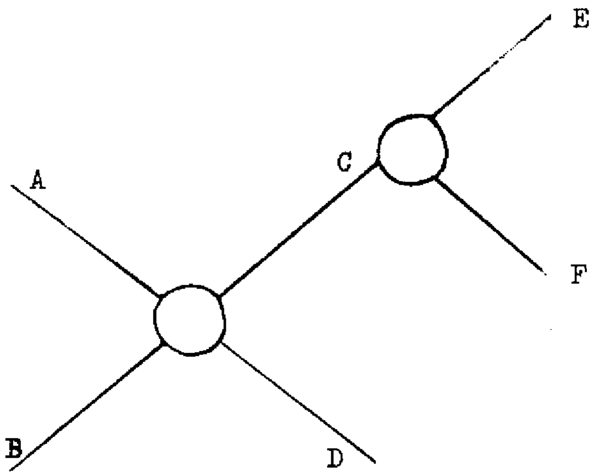
The role of s and t are reversed *) for the two channels.

4.

One could go on to the <u>five-point</u> and higher functions



which represent production processes. In the study of these processes one has tended to make a two-stage approximation. For example, most work on the five-point function assumes that it is dominated by the sequence of processes:



$$A + B \longrightarrow C + D$$
$$\quad\quad\quad\quad\quad\quad \llcorner\!\!\longrightarrow E + F$$

Here C is an intermediate resonance which is first formed decaying subsequently $C \rightarrow E + F$. This approximation has been surprisingly successful in analysis of data.

---

*) Note that Channel I and Channel II can be distinguished either by specifying which particles are incoming and which outgoing, or by specifying the regions in the (s,t) plane where $s > 0$ and $t < 0$ and vice-versa.

Summarizing: The theoretical problems investigated at present in particle physics are the following:

1) The mass spectrum of the particles, their spins, parities and internal charges - a problem towards whose theoretical understanding we have had the least success.

2) A coherent description of three-body coupling constants, decay constants and momentum transfer dependence of form factors.

3) The variation of scattering amplitudes $F^i(s,t)$ as functions of energy s and momentum transfer t in Channel I (with the roles of s and t reversed in Channel II). By and large we have been more successful in describing decay constants and the behaviour of scattering amplitudes than of particle spectra.

B. INGREDIENTS OF THE THEORY *)

Historically, relativistic quantum mechanics (RQM) was more or less contemporaneous with the epic days of Einstein's field theory of gravitation, which itself had before it the great model of another field theory - the electrodynamics of Maxwell and Faraday. Inevitably, the first realization of RQM was carried through in terms of a local field concept, with the following ingredients:

*) This section describes rather complicated ideas, perhaps not too well. The harassed reader may pass directly on to Sec. C. (Postulated canons of relativistic quantum mechanics.)

1. Local fields

Associate with each particle a local *) field operator A(x) defined at all space-time points x; the electromagnetic fields $\underline{E}(x)$ and $\underline{H}(x)$ are the archetypal examples.

2. Elementarity vs. compositeness

If A is composite of B and C, the field A(x) equals a polynomial product of B(x), C(x) and their derivates.

3. Scattering amplitudes

The amplitude for scattering of $A + B \rightarrow C + D$, and in particular the invariant functions $F^i(s,t)$**) which describe scattering (see IIA), can be written in terms of matrix elements of products of field operators associated with A,B, etc., and their derivatives.

---

*) By locality we mean the commutation relation postulate that

$$[A(x) , A(y)] = 0 \qquad (C.R.1)$$

whenever x and y are space-like to each other. This relation - called the causality relation - guarantees, in a vague manner, that field influences do not propagate with velocities greater than light. If ever a fundamental length ℓ needs to be introduced, the best place for it may possibly be through a modification of C.R.1, for example, to a form,

$$[A(x) , A(y)] = 0$$

when $(x-y)^2 \leq \ell^2$. The fact that $\ell = 0$ in (C.R.1) is (vaguely) an indication of no fundamental length in RQM.

**) What are the consequences of locality? First, as we noted in Part I, locality plus real Lorentz symmetry imply complex Lorentz symmetry and, in particular, CTP symmetry. Thus CTP operation and particle-antiparticle duality is the first gift of locality. Second, one can show that the causality postulate

$$[A(x) , B(y)] = 0 \quad , (x-y)^2 < 0$$

(cont.)

(Footnote cont.)
is powerful enough to guarantee that the amplitudes for a scattering process - a function like $F^i(s,t)$ - is an analytic function of complex s and t, in a certain domain of s and t. This is an astonishingly powerful result. What this domain in (s,t) space is, we shall discuss later. The important feature to note here is that no detailed dynamics, no precise law of force, no compositeness or structure relation like $(C(x) = A(x)\ B(x))$, etc., has gone into the deduction of analyticity of $F^i(s,t)$.

It is indeed no exaggeration to say that the local relativistic field concept is one of the most fruitful concepts invented by man. It is not just that the concept is an ideal vehicle for implementing RQM; it would be truer to say that the concept, once formulated, came to acquire a life of its own; it led inexorably to the creation of a canon of RQM. Years later, attempts were made by a process of abstraction to state these canons independently of the local field concept. The emphasis shifted to the quantities of direct physical interest - the elements of the scattering matrix themselves - the invariant functions $F^i(s,t)$ of the last section. Something had then to be substituted for the locality postulate of the field idea. Since the two major deductions from the locality postulate were (see preceding footnote),

1) CTP-symmetry,
2) analyticity of $F^i(s,t)$,

it was but natural to propose that these two (in a still stronger version) may be elevated to the status of basic canons of RQM - rather than enter through the back door via the field-theoretic version of RQM. This is the second - the so-called scattering-matrix, (S-matrix) - approach to relativistic quantum mechanics.

To summarize, then, there are at present two theoretical constructs - two methodologies - embodying RQM,

1) the historic method of local field theory,

2) the (maximally) analytic S-matrix method.*)

We now describe these approaches in somewhat greater detail.

B.1. Field theories

There are two distinct classes of field theories corresponding to the two notions of elementarity - aristocratic (Class I) or democratic (Class II) - discussed in Part I.

Class I - theories of distinguished fields

Accept from the outset the existence of a set of distinguished fields of which all other fields can be constructed compositively. An example is quantum electrodynamics of leptons and photons, where:

a) To each lepton and to the photon, a separate field is assigned, for example the Dirac field $\psi(x)$ for electrons and Maxwell field $A_\mu(x)$ for photons.

---

*) Whereas field theory patiently deduces what the analyticity domain of $F^i(s,t)$ should be in any given configuration of scattering particles using (C.R.1) as its main tool, the S-matrix approach starts with the postulate that $F^i(s,t)$ is as analytic as it possibly can be, with singularities only at those values of s and t which can be associated with the physical particle spectrum. We shall define this maximal analyticity concept more precisely later; note here the amazing circumstance that the postulated domains of the S-matrix approach have found confirmation - if anything - from the detailed (and very involved) calculations within the field-theoretic framework, at least on the so-called physical Riemann sheet of the complex (s,t) surface. When differences - so far minor - do arise, one may take one's choice if to believe in the firm and connected - though sometimes heavy - logical development from field theory or in the attractive simplicity of what one may consider is reasonable.

b) A distinguished form of Hamiltonian *) is written down; its interaction part equals

$$e\ j_\mu(x) \quad A_\mu(x)$$

where $j_\mu(x) = i\bar{\psi}(x)\gamma_\mu\psi(x)$ is the electron current. This Hamiltonian is distinguished, not just because Maxwell postulated it, but, more perhaps, since it satisfies a number of additional dynamical constraints (gauge invariance) with important physical significance. The current $j_\mu(x)$ is conserved $(\frac{\partial}{\partial x_\mu}\ j_\mu(x) = 0)$; e is the coupling strength. **)

This is the prototype of all Class I theories. Though the one "bound state" in this theory, positronium, may be represented by a separate field, there is no need to postulate field equations for it. At any rate, the field equations for the distinguished electron and photon fields form a closed set; they do not contain any piece depending on the positronium field. Clearly this theory was specially favoured for the following reasons:

---

*) The Hamiltonian completely determines the field equations and equal-time commutation relations of fields. As is well known, matrix elements of (appropriate) products of field operators define Green's functions of the theory, while operator field equations provide infinite sets of interconnecting relations among them.

**) As is well known, the smallness of the coupling parameter e allows for a complete perturbation solution (the Feynman solution). As is also well known, some of the integrals in the solution turn out to be infinite - of the form $\int^\infty \frac{dx}{x}$. The famous renormalization theorem of Dyson demonstrated that these infinities affect only self-mass and self-charge (making it impossible, within the theory, to compute m_0/m and e_0/e - the ratios of "bare" to "physical" constants).

a) **The number of particles the theory describes is small.**

b) The theory inherited from Maxwell a distinguished nearly unique Hamiltonian with a small coupling parameter e.

In hadron physics with its multitudes of particles, a repetition of any of these fortunate circumstances could not be expected to recur. Field theories of Class II are thus the more appropriate.

Class II theories

(a') Associate with each particle a local field operator and assume that all particles are composites of each other.

(b') To proceed with detailed dynamics we need field equations. By the compositeness assumption, every particle interacts locally with every other. It is thus profitless to try to conjecture unique (infinite) sets of equations of motion; there is just too much arbitrariness.

(c') A part of the dynamics, however, can be specified by postulating specific equal-time commutation relations*) (C.R.2) of the form:

$$[A(\underline{x},t)\ ,\ B(\underline{x}',t)] = C(\underline{x},t)\quad \delta(\underline{x}-\underline{x}')\ . \qquad (C.R.2)$$

---

*) Equal-time commutation relation between a field and its time derivative

$$[\phi(x,t)\ ,\ \dot{\phi}(\underline{x}',t)] = i\delta(\underline{x}-\underline{x}') \qquad (C.R.3)$$

(analagous to the relation $[q(t)\ ,\ p(t)] = i$ between the position operator q(t) and its conjugate momentum p(t)) are special cases of (C.R.2). In a theory where distinguished sets of fields exist, and one knows the composition law, a single relation like (C.R.3) would suffice to give the entire set of relations (C.R.2).

For several classes of operators A and B, and in particular when A and B represent current operators associated with internal symmetry groups of Part I, plausible conjectures are possible from the structure of the symmetry algebras; we shall study examples in part III. *)

---

*) A commutation relation through its very structure cries out for saturation procedures to be applied. By this we mean the following: If $|n\rangle$ denotes a quantum mechanical state, the completeness relation asserts that $\sum_n |n\rangle\langle n| = 1$. A commutation relation like

$$[A , B] = C .$$

on saturation, gives rise to the sum rule:

$$(\alpha|C|\beta) = \sum_n \left[(\alpha|A|n)\ (n|B|\beta) - (\alpha|B|n)\ (n|A|\beta)\right] .$$

It is this class of sum rules obtained from Class II field theories which we shall study mainly in Part III for cases when A, B and C are charge operators corresponding to the internal symmetries considered in Part I. The important point to emphasise in such cases is that equal-time commutation relations and presumably the sum rules obtained from them are exact statements even when the symmetry is broken.

The next table (Table 6) summarizes the theoretical approaches.

SUMMARY OF THEORETICAL METHODS

I. Field theoretic approaches

Class I. Assume the existence of a distinguished set of fields of which other fields are made compositely. Assume a distinguished form of Hamiltonian for these special fields. A complete pertubration solution to the scattering matrix may be constructed if the coupling parameter is small; the only undetermined parameters in the theory are the masses and coupling constants of the distinguished fields. All other parameters - like masses of bound states if any are in principle obtainable from (a non-perturbative) expression for the scattering matrix.

Class II. Associate each particle democratically with a local field. If we assume that all particles are composites of each other, no local field equations can be written with profit. The locality postulate.however, determines the analytic structure of the scattering amplitudes. A part of the dynamics is specified by postulating equal-time commutation relations,which in turn give rise to testable sum rules.

II. The analytic S-matrix method

Give up the field concept. Postulate maximum analyticity of scattering amplitudes in place of locality of fields. Assume that any singularities of the scattering matrix are intimately related to and are determined by the spectrum of the physically observed particles. In its democratic approach to all particles, this approach is more akin in spirit to field theories of Class II.

TABLE 6

C. POSTULATED CANONS OF RELATIVISTIC QUANTUM MECHANICS

In this section we set down what one has come to accept as the canons of relativistic quantum mechanics. As stated earlier, some of these were inexorable consequences of a local field theoretic formulation of RQM; others were postulated more readily and plausibly from the logic of the S-matrix approach. The most astonishing aspect of these latter postulates has been that in no important case has a deeper study from the field-theoretic method uncovered situations where they are contradicted.

a) Unitarity of the S-matrix; or the law of conservation of probabilities

This basic law - a non-linear statement -

$$\sum_n (a|S|n)\ (n|S^\dagger|b) = \delta_{ab}$$

constitutes one of the most powerful constraints on physical theory that we know of. *) In hadron physics, if there is one principle which it is criminal to approximate too drastically, it is this. One of the most useful relations arising from unitarity is the so-called optical theorem connecting the total two-body collision cross-section to the imaginary part of the elastic forward scattering amplitude

$$\mathrm{Im}\, F(s,\ \theta = 0) = k_c\ \sigma_T$$

b) Crossing symmetry or substitution law

Recall that in Part IA we stated that locality (of field theory) plus Lorentz invariance implies the existence of a CTP operation which converts an incoming particles to an outgoing antiparticle and vice-versa. Following and generalizing from this, the <u>crossing property</u> of RQM states that, given a reaction

---

*) In electrodynamics, in a perturbation expansion, it becomes essentially a linear relationship and considerably loses its restraining power.

$$A + B \rightarrow C + D$$

and the scattering amplitudes $F^i(s,t)$ for it, <u>the same function $F^i(s,t)$ also describes the amplitude for the process where we interchange one of the incoming particles with an outgoing anti-particle</u>. In detail, the same set of functions $F^i(s,t)$ describes both

$$A + B \rightarrow C + D$$

and

$$A + \bar{C} \rightarrow \bar{B} + D \quad ,$$

where $\bar{C}$ and $\bar{B}$ are antiparticles of C and B, respectively. Likewise for

$$A + \bar{D} \rightarrow \bar{B} + C \quad .$$

Note, as remarked earlier, that when we speak of the <u>same</u> functions $F^i(s,t)$ describing reactions in

Channel I $\quad A + B \rightarrow C + D$

and

Channel II $\quad A + \bar{C} \rightarrow \bar{B} + D$,

we are considering different regions of the (s,t) plane, since for Channel I the physical region is $s > 0$, $t < 0$, while for Channel II, it is $s < 0$, $t > 0$. An analytic continuation must therefore be carried through before we can read off from the knoweldge of $F(s,t)$ for Channel I, the values of the amplitude for Channel II. We consider this continuation further on; remark here, however, the economy brought about the crossing relations; there exists just <u>one</u> master function for all related channels. These relations give RQM a power that non-relativistic theory never possessed.

CANONS OF RELATIVISTIC QUANTUM MECHANICS

1. <u>Crossing</u>

incoming particle $\xrightarrow{CTP}$ outgoing antiparticle

I. $A + B \longrightarrow C + D$

II. $A + \bar{C} \longrightarrow \bar{B} + D$

III. $A + \bar{D} \longrightarrow \bar{B} + C \implies$ Reactions I,II,III described by the same master function $F(s,t)$.

2. <u>Unitarity</u> $\implies$ optical theorem

$$\text{Im } F(s,0) = k\ \sigma_{total}$$

3. <u>Analyticity</u>

of $F(s,t)$ for complex s and t, except for singularities (poles, branch points) determined by the physical particle spectrum.

TABLE 7

c) <u>Singularities of the S-matrix</u>

Before any analytic continuation from Channel I to II can be carried out, we need to know the singularity structure of the S-matrix.

SINGULARITIES OF THE S-MATRIX

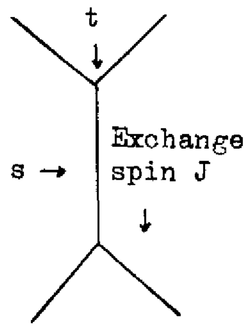
1) Yukawa poles

(c.m. energy)^2 = s

momentum transfer = t

$$F(s,t) = g^2 \frac{P_J(\theta_t)}{t - m_J^2}$$

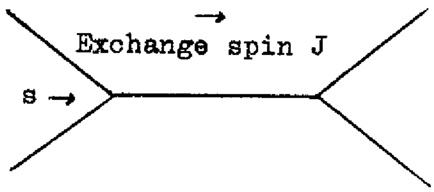
$$\cos\theta_t = 1 + \frac{2s}{t - 4m^2}$$



2) Resonance poles

$$F(s,t) = g'^2 \frac{P_J(\theta_s)}{s - m_J^2}$$

$$\cos\theta_s = 1 + \frac{2t}{s - 4m^2}$$



3) Branch points

at two-particle, three particle, ... thresholds in all the relevant channels

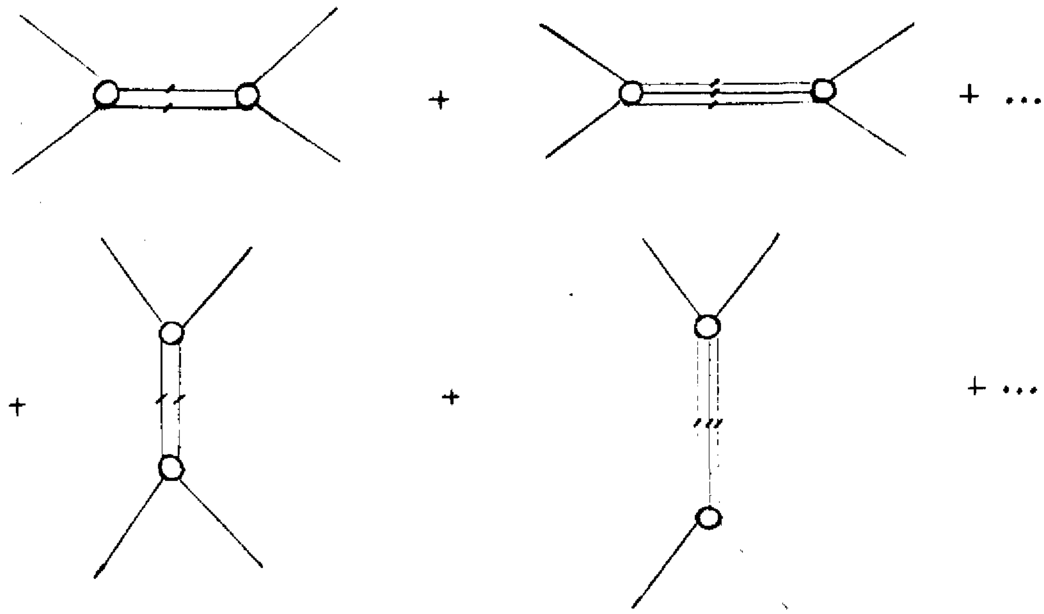


TABLE 8

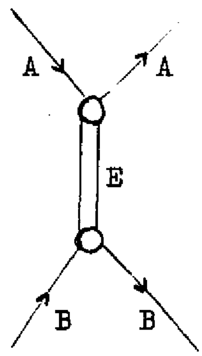
i) Poles

One of the basic postulates of S-matrix theory (shared by local field theory) is that the poles of the S-matrix correspond to physical particles exchanged in a reaction and conversely. There are two well-known types of poles:

1) The Yukawa poles

Poles in *) (complex) momentum-transfer (t)-plane; their contribution to the scattering matrix equals

$$\frac{g_{AAE} \quad g_{BBE}}{t - m_E^2}$$

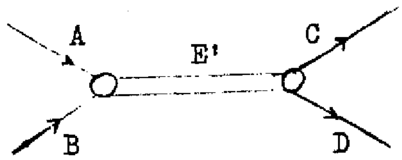


Here m_E is the mass of the Yukawa particle exchanged and g_{ACE} and g_{BDE} the coupling constants at the two vertices.

2) The Breit-Wigner poles

These are poles in complex energy (s)-plane

$$\frac{g_{AE'B} \quad g_{E'CD}}{s - m_E^2}$$



---

*) The Yukawa pole contributions (also called the Born terms in potential scattering) determine in configuration space the "potential" produced by the exchanged particle. Thus, every particle E that is exchanged, produces its share of "force" between the interacting particles A and B. The strength of this force depends, of course, on the coupling parameters g_{AAE} and g_{BBE} and its range on the mass m_E of the particle exchanged (the smaller the mass, the longer the range). In hadron physics, where every coupling constant is (nearly) equal to every other, the concept of a "fundamental force" produced by a "fundamental exchange" becomes nebulous. This is another way to restate the dilemma of BI(b').

3) **Case of spin**

It is easy to show that if the exchanged particles E or E' carry spin, the pole contributions modify to the forms:

$$gg' \cdot \frac{P_J(\theta_t)}{t - m_E^2} \qquad\qquad \cos\theta_t = 1 + \frac{2s}{t - 4m^2}$$

$$gg' \; \frac{P_J(\theta_s)}{s - m_E^2} \qquad\qquad \cos\theta_s = 1 + \frac{2t}{s - 4m^2} \quad .$$

Note the crossing symmetry typified by interchange of s and t.

To determine the other singularities of the scattering matrix, unitarity proves to be the crucial tool.

ii) Branch points

It is impossible in a general lecture like this to go over details of how the pole structure, together with the quadratic unitarity relation $SS^+ = 1$, forces the branch point singularity structure of the scattering amplitudes. This will be covered in more detail in Professor J. Eden's lecture. I shall simply state the postulated result from maximum analyticity for the four-point function (demonstrated to varying degrees of rigour using field theory). The scattering matrix possesses branch-point singularities at two-particle, three-particle, ... thresholds in all channels; the singularities lying along real and positive s-axis in the s-plane, along the real and positive t-axis (for the channel where t is the energy) and likewise for the third channel. Note the elegance, as well as the simplicity, of this conjectured singularity structure related as it so intimately is to physical particle spectrum.

D. DEDUCTIONS FROM CANONS OF RQM

So much for the basic principles. In this section we list some of the important deductions that have been made from these principles and which provide the practical working tools of hadron theory.

a) <u>Dispersion relations (consequences of the assumed analyticity of the S-matrix)</u>

If one knows the singularity structure of the scattering amplitudes, (in particular that the singularities lie along the real axes in complex s and t planes) one may write integral representations - the so-called dispersion relations - connecting real and imaginary parts of the scattering amplitudes. For example, from Cauchy's theorem, infer,

$$F(s,t) = \frac{1}{2\pi i} \int_C ds' \quad \frac{F(s',t)}{s'-s-i\epsilon}$$

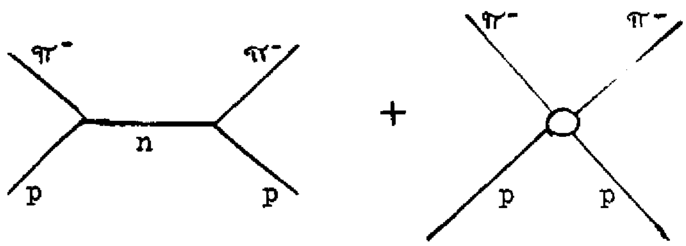
with the contour C as shown in Fig.4. To evaluate the integral along the large circle, we need to know the behaviour of F(s,t) as $|s| \to \infty$. If F(s,t) falls sufficiently fast (and we consider this in more detail in the next subsection), we may expect the following to hold:

$$F(s,t) = \frac{1}{\pi} \int \left(\frac{F(s',t)}{s'-s+i\epsilon} - \frac{F(s',t)}{s'-s-i\epsilon} \right) ds'$$

plus pole term contribution. Rewrite in the Hilbert form:

$$\mathrm{Re}\ F(s,t) = \frac{g^2}{s-s_0} + \frac{1}{\pi} \ P \int \frac{\mathrm{Im}\ F(s',t)}{s'-s} \quad ds'$$

This is the typical structure of a typical dispersion relation; one may exhibit the pole and the integral contributions diagrammatically (for πp scattering) thus:



Since Re F(s,t) and Im F(s,t) are both experimentally accessible quantities, the dispersion equation provides a determination of the pion-nucleon coupling parameter g^2.

b) High-energy behaviour of $F(s,t)$ (consequences of combined unitarity and analyticity)

Combining analyticity with unitarity, it has been possible to obtain restrictions governing the growth of physical amplitudes for large s or t. These again have no counterpart in non-relativistic theory. It is impossible to show how these results are derived either in field theory or in the S-matrix dispersion approach. We shall simply exemplify by stating one of the outstanding results.

Limitation growth of forward scattering amplitude

$$|F(s, t = 0)| < C \, s \log^2 (s/s_0) \quad .$$

Using the optical theorem, this implies that total cross-sections in physics can only grow less fast than *)

$$\sigma_T < C' \log^2 (s/s_0) \quad .$$

Experimentally, up to 30 BeV (and if cosmic ray data is believed up to 1000 or more BeV), total cross-sections appear to approach constant limits asymptotically. Contrast this with the theoretical prediction above; clearly we are still far from gleaning from theory the best possible bound. The fact, however, that such bounds exist at all (and are after all not outrageously weak) makes the physicist in this field feel truly arrogant at the power of RQM.

c) Regge trajectory exchanges

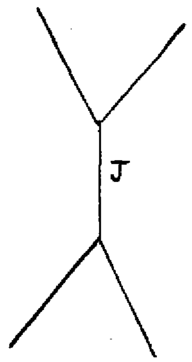
Perhaps the most powerful deduction from crossing and high-energy bounds has been the demonstration that hadrons must lie on Regge trajectories. The argument goes like this: (for more detail, see Professor Van Hove's lecture):

---

*) The constant C' can actually be evaluated from field theory; for example, one finds the theoretical estimate

$$C' \leqslant \frac{12}{m_\pi^2} \quad \text{for } \pi N \text{ scattering} \quad ,$$

Consider in a scattering problem the exchange of a spin J particle; the pole approximation to the amplitude gives:



$$F(s,t) \approx g^2 \frac{P_J(\cos\theta_t)}{t-m_J^2}$$

$$\rightarrow g^2 \frac{(\cos\theta_t)^J}{t-m_J^2} \approx g^2 \frac{s^J}{t-m_J^2}$$

for large s.

If $J > 1$, this will violate the bound derived above from RQM. Either there are no particles with spins > 1, or there is some physical mechanism which smears out the s^J contribution.

Remarkably, with the hadron spectrum known at present, one does not have far to seek for such a mechanism. As pointed out in Part I and specifically in Fig.2, hadrons appear to occur in Regge families with their masses increasing with spin. Each exchanged hadron of mass m_J will contribute a term of the above type; the total pole contribution to the amplitude equals

$$F(s,t) = \sum_J \frac{g_J^2 \, s^J}{t-m_J^2} .$$

This sum can be approximated to by an integral, if the number of particles involved is fairly large. Thus,

$$F(s,t) \approx \int g_J^2 \frac{s^J}{\sin\pi J} \frac{dJ}{t - m_J^2} .$$

To evaluate the integral, solve $t - m_J^2 = 0$ in the form,

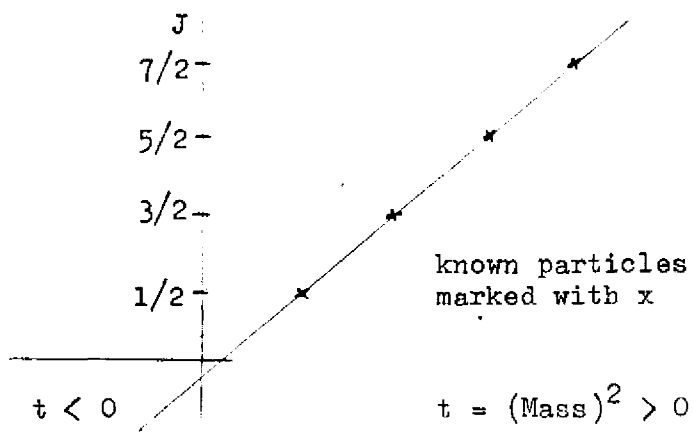
$$J = \alpha(t) .$$

Thus

$$F(s,t) \approx \int \frac{g_J^2 \, s^J}{\sin\pi J} \frac{dt}{t - m_J^2} \frac{dJ}{dt}$$

$$\sim \frac{\alpha'(t)}{\sin\pi\alpha(t)} g_{\alpha(t)}^2 \, s^{\alpha(t)} .$$

The high-energy behaviour of the amplitude is controlled not by the spin of any of the particles exchanged but by an effective spin $J = \alpha(t)$ dependent on momentum transfer t. And since in the scattering region $t \leqslant 0$, one must make a contination of J from the positive values of t (given by Rosenfeld's tables) to negative t.



Consider now, for this continuation, one specific trajectory; for example, the nucleon trajectory in Fig. 1. Extrapolate (as a plausible continuation) the known linear plot of the trajectory in the first quadrant, backwards to negative t-values. Clearly empirically $\alpha(t) \leqslant 1$ for the scattering region. No contradiction with high-energy theorems could thus possibly arise whenever the nucleon trajectory is exchanged in a scattering process (for example, in $\pi + M \rightarrow N + \pi$). Likewise for all known trajectories; the extrapolated values of $J = \alpha(t)$ always empirically satisfy the rule $\alpha(t) \leqslant 1$ ($T \leqslant 0$).

What is the moral of this for hadron spectrum - perhaps (1) all hadrons lie on Regge trajectories; *) (2) for all trajectories, $\alpha(t)$ must lie lower than unity when continued to negative t. There are fewer places in particle theory where the power of RQM has evidenced itself to greater effect than in this beautiful Regge synthesis of known hadron spectra ($t > 0$) with asymptotics of scattering amplitudes ($t < 0$).

---

*) We shall hear, in Professor M. Toller's lecture, that Regge trajectories (possibly) occur in tribes and families as a further consequence of some unexplored aspect of Lorentz symmetry and analyticity at $t = 0$.

E. HADRON SPECTRUM AND THE S-MATRIX PROGRAMME

We have now surveyed the structure of RQM; we can see that in dispersion theory and in the Regge trajectory exchange models of high-energy scattering there is tremendous amount of predictive power. The chief problem however remains. Where in all this is the analogue of the familiar non-relativistic Schrödinger theory, to yield the particle spectrum?

The closest in spirit to the traditional Schrödinger method are field theories of Class I. Except for the masses associated with the distinguished fields and their coupling parameters, the parameters of all bound states could, in principle, be read off from the scattering matrix, computed using the given field equations. In Part IV we shall see that it may yet be that aristocratic fields do exist and that the future of particle physics lies along this traditional path. The milieu of our age, however, is somehow against this.

What could substitute for field equations in field theories of Class II or in the S-matrix approach? Since for both approaches the basic assumption is that all particles are equally elementary or equally composite, there is but one way to attack the problem - rely on self-consistency of any assumed spectrum in satisfying the relations provided by the theory. Among those relations one has worked with are (1) relations in Class II field theories provided by equal-time commutation rules (one can test if they are self-consistently saturated by an assumed spectrum) or (2) the so-called "superconvergence relations". *)

---

*) These are dispersion relations for amplitudes which fall so fast in s that the relation reduces to $\int \mathrm{Im}\, A(s',t)\, ds' = 0$).

This happens, for example, when incoming and outgoing particles carry spin; one can show from RQM that, for a general amplitude,

(cont.)

(Footnote cont.)

$$\left| F_{\lambda_1, \lambda_2, \lambda_3, \lambda_4} (s,t) \right| < c\, s^{1-\max(|\lambda|,|\mu|)}$$

where $\lambda = \lambda_1 - \lambda_2$, $\mu = \lambda_3 - \lambda_4$ and the λ's are the spin-polarizations of the incoming and outgoing particles. This important dependence of high-energy behaviour on spin means large spin-flip amplitudes fall fast for **large s.** For such amplitudes (and for appropriate values of $|\lambda|$ and $|\mu|$), the normal dispersion relation

$$A(s,t) = \frac{1}{2\pi} \int \frac{\operatorname{Im} A(s',t)\, ds'}{s'-s+i\epsilon}$$

reduces to its superconvergent form $\int \operatorname{Im} A(s',t)\, ds' = 0$. To see how such relations work, take the example of $\pi-\rho$ scattering. Saturating the appropriate relation with ω and ϕ resonances only, one converts the integral $\int \operatorname{Im} A\, ds' = 0$ into an algebraic consistency formula which reads:

$$g^2_{\pi\rho\omega} \left(m^2_\omega - m^2_\rho - m^2_\pi\right) + g^2_{\pi\rho\phi} \left(m^2_\phi - m^2_\rho - m^2_\pi\right) + \dots = 0.$$

Empirically, we know that the masses of ω, ρ and π mesons satisfy

$$m^2_\omega - m^2_\rho - m^2_\pi = 0 \qquad \text{I}$$

while

$$g^2_{\pi\rho\phi} = 0 \qquad \text{II}$$

($\phi \rightarrow \rho + \pi$ decay is nearly suppressed). The relation is thus (miraculously) satisfied showing either (i) that RQM is a wonderful theory, or (ii) that the existence of ω (and the hypothesis of saturation of the superconvergence relation with a few resonances) implies that ϕ must also exist. One may take one's choice.

One cannot say that such bootstrapping self-consistency ideas have had more than marginal success in the past, though with the greater use of the superconvergence relations the situation may improve.

The boostrap idea is attractive. Part of its attraction lies in the possibility that the internal symmetries themselves may possibly arise as necessary preconditions for the existence of a particle theory satisfying the very stringent restrictions of crossing, analyticity and unitarity of RQM. We may yet find that we are living (with Voltaire) not only in the best of all possible worlds but indeed in the only possible world.

SUMMARY

DEDUCTIONS FROM GENERAL PRINCIPLES OF RQM

1) Examples of bounds from unitarity and analyticity

$$|F(s,t=0)| < C\, s \log^2 (s/s_0)$$

$$\sigma_{total} < C' \log^2 (s/s_0)$$

2) A typical dispersion relation

$$\text{Re } F(s,0) = \frac{g^2}{s-s_0} + \frac{1}{\pi} \int_C \frac{\text{Im } F(s',0)}{s'-s} \, ds'$$

3) Regge trajectory's contribution to a scattering amplitude

A trajectory contributes

$$\sum_J g_J^2 \frac{P_J(\cos\theta_t)}{t-m^2(J)} \approx \beta(t)\, s^{\alpha(t)}$$

for large s, where $\alpha(t) = J \Longleftrightarrow t - m^2(J) = 0$.

4) Some achievements of the S-matrix approach

1) Accurate determination from dispersion relations of $g_{\pi NN}$, $g_{KN\Lambda}$, etc.

2) Bootstrap generation of certain resonances in pion-nucleon sector.

3) Regge analysis of high-energy scattering data.

TABLE 9

PART III

THE DYNAMICAL ROLE OF CHARGES AND SHARING OF CURRENTS BETWEEN STRONG, WEAK AND E.M. INTERACTIONS

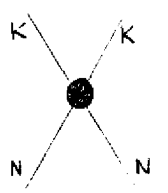
In Part II we spoke of abstracting from field theory the general canons of relativistic quantum mechanics. I now wish to speak of the second important idea, again an abstraction from field theory, but this time from one special theory - the quantized *) Maxwell-Dirac field theory of electrons interacting with photons. This is the notion of field-theoretic currents associated with the charges introduced in Part I, the dynamical role of charges and currents - particularly for low-energy phenomena and the sharing of the currents between strong, weak and E.M. interactions.

A. DYNAMICAL ROLE OF CHARGE

The dynamical role of electric charge is familiar from Coulomb force law, where the sign and magnitude of charge determines the long-range Coulomb force $\left(\approx \frac{e_1 e_2}{r^2}\right)$. Likewise for the gravitational force which is proportional to the product of gravitational charges (mass) $\left(\approx \frac{m_1 m_2}{r^2}\right)$.

Less familiar, but equally well established, is the role of hyper-, isotopic and other charges in the same context; the long-range parts of the relevant forces are proportional to the charge strengths; for example,

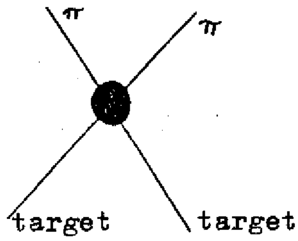
1. Hypercharge



*) We shall need familiarity with field-theoretic notation in this part of the lecture.

The long-range part of K^-N force is attractive, while for K^+N this force is repulsive. This corresponds to the sign of the product of hypercharges $Y_K Y_N$ which is -1 for the K^-N and $+1$ for the K^+N case.

2. Isotopic charge



The scattering length for pions (isotopic charge $\vec{I}_\pi$) interacting with a target of isotopic charge $\vec{I}_t$ is experimentally found proportional to the scalar product of the two isotopic charges

$$\vec{I}_\pi \cdot \vec{I}_t = \tfrac{1}{2} \left[(\vec{I}_\pi + \vec{I}_t)^2 - \vec{I}_\pi^2 - \vec{I}_t^2 \right] \quad .$$

The conserved charges thus do appear to play a dynamical role similar to the electric charge for low-frequency phenomena. For the E.M. case, one knows one can go further. In Part II we saw that the Maxwell-Dirac Hamiltonian uses currents $j_\mu(x)$ associated with the electric charge. Could this analogy be taken over for the other charges as well?

B. DYNAMICAL ROLE OF CURRENTS

Given a charge $Q(t)$, it is well known that one can construct a local four-vector current operator $J_\mu(x)$ associated with it,

$$Q(t) \Longrightarrow J_\mu(x)$$

where

$$Q(t) = \int_t J_0(\underline{x},t)\, d^3x \quad .$$

If the current is conserved, i.e., $\partial_\mu J_\mu = 0$, then the charge is a constant of motion $(\dot{Q}(t) = 0)$, and vice-versa.

For spin-half charged particles (electrons or muons, for example), one can go further; we may define left and right currents corresponding to left and right charges in the zero mass idealization.

The sums and differences of these are the vector and axial-vector currents:

$$V_\mu(x) \equiv J_\mu^{V}(x) = J_\mu^L(x) + J_\mu^R(x)$$

$$A_\mu(x) \equiv J_\mu^A(x) = -J_\mu^L(x) + J_\mu^R(x) \quad .$$

Likewise for right and left hypercharges, isotopic and unitary charges.

Now, one of the important experimental discoveries is that these currents - <u>and also their divergences</u> - if considered as fields, appear to possess <u>particles associated with them</u>. (Table 10)

Also, one of the recent insights of <u>local</u> field theory is that all operators representing a particle are equivalent so long as they possess the requisite quantum numbers. One may therefore express this association of particles and currents by writing a set of (approximate) field-current identities. Considering isotopic $SU_L(2) \times SU_R(2)$, for example, one may write:

$$\vec{J}_\mu^V = \frac{m_\rho^2}{g_\rho} \vec{\rho}_\mu + \ldots , \quad \vec{J}_\mu^A = \frac{m_A^2}{g_A} \vec{A}_\mu + \ldots$$

$$\frac{\partial \vec{J}_\mu^A}{\partial x_\mu} = c_\pi \vec{\pi} + \ldots$$

The last relation is called the PCAC (short for partially-conserved-axial-current hypothesis) relation.

Now where does dynamics come into this?

Ideally, one might hope that since for every current, a corresponding particle exists, by analogy with the two well-known classical Hamiltonians:

(a) Gravitational $g\,\theta_{\mu\nu} G_{\mu\nu}$

(b) E.M. $e\, J_\mu^{E.M.} A_\mu$,

which are simple products of currents x associated particle fields ($\theta_{\mu\nu}$ = stress tensor, $G_{\mu\nu}$ = graviton, A_μ = photon), the strong and weak Hamiltonians may also be written in the form, e.g.,*)

---

*) We have written H^{weak} in the form current x intermediate boson mediating weak interactions. No such intermediate boson has yet been experimentally discovered. Present experiments test only the effective interaction $H_{eff} \simeq J_\mu^{weak} J_\mu^{weak\dagger}$.

CURRENTS AND ASSOCIATED PARTICLES			
	Charge	Conserved status of current	Associated particle
	Gravitational	$\partial_\mu \theta_{\mu\nu} = 0$	$\theta_{\mu\nu} \Rightarrow 2^+$ (graviton?)
	Electromagnetic	$\partial_\mu J_\mu = 0$	$J_\mu \Longrightarrow 1^-(\gamma)$
Strong	Baryon	$\partial_\mu J_\mu = 0$	$J_\mu \Rightarrow 1^-(\omega)$
Strong	SU(3)	$\partial_\mu J_\mu^V = 0$ if neglect medium strong symmetry breaking	$J_\mu \Rightarrow 1^-$ octet $(\rho,\phi,K^*,\bar{K}^*)$
Chiral Strong	$SU_L(3) \times SU_R(3)$	$\partial_\mu J_\mu^A \neq 0$	$J_\mu^A \Rightarrow 1^+$ octet$(A_1,D,K_A,\bar{K}_A)$ $\partial_\mu J_\mu^A \Rightarrow 0^-$ octet$(\pi,\eta,K,\bar{K})$
Weak $\Delta Y=0$	Weak currents J_μ^V(weak), $\partial_\mu J_\mu^V(\text{weak}) = 0$ J_μ^A(weak); $\partial_\mu J_\mu^A(\text{weak}) \neq 0$		$J_\mu^{weak} \Longrightarrow$ Intermediate vector bosons (?)

Sharing of (hadronic) currents between strong, E.M. and weak forces.

$J_\mu^{E.M.} = J_\mu^3 + \frac{1}{\sqrt{3}} J_\mu^8$ (J_μ^3, J_μ^8 refer to SU(3) currents)

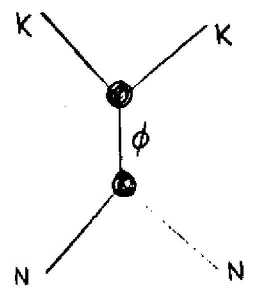
$J_\mu^{weak}(\Delta Y = 0) = J_{\mu\,L}^{1,2}$ ($J_\mu^1, J_\mu^2, J_\mu^3$ form isotopic SU(2), $J_\mu^1 \pm i J_\mu^2$ are isotopic lowering and raising currents.)

TABLE 10

$$H^{strong} = g_V J^i_\mu \; V^i_\mu + g_A J^{Ai}_\mu \; A^i_\mu \quad *)$$

$$H^{weak} \; = g^{weak} \; J_{\mu L} \; W^{weak}_\mu \;\; ,$$

where V^i and A^i are the strongly interacting 1^- and 1^+ octets of particles; W^{weak} are the weak intermediate vector mesons of Table 10. This is an attractive hypothesis.**) From this point of view, it would be easy to understand why, for example, the long-range part of $\left.\begin{matrix} K^- - N \\ K^+ - N \end{matrix}\right\}$ force is attractive or repulsive. One would expect the long-range part of the interaction to be (dominantly) produced by a Yukawa exchange



of the ϕ -meson just like the Coulomb case where the potential is a consequence of a single photon exchange.

---

*) For exact SU(3) x SU(3), $g_A = \pm g_V$. See Table 11 for the theoretical prediction when brokenness of the SU(3) x SU(3) symmetry is taken into account (expressed by $\frac{\partial J^A_\mu}{\partial x_\mu} \neq 0$ in contrast to $\frac{\partial J^V_\mu}{\partial x_\mu} = 0$).

**) One great virtue of such a strong Hamiltonian is that the dilemma presented by whether nature prefers field theories of Class II (democratic) or Class I (aristocratic) no longer arises. The currents $j_\mu(x)$ could be made up either of just the distinguished fields, or of all fields democratically. A slightly different version is the one very recently proposed

$$H^{strong} = \frac{g}{m^2} \left(J^i_\mu J^i_\mu + J^{Ai}_\mu J^{Ai}_\mu \right)$$

where even the aristocracy of the 1^- and 1^+ octet particles V^i_μ and A^i_μ is ended.

Unfortunately this argument is no more than suggestive, since, for a strong interaction theory, the largeness of the coupling parameter g makes it next to impossible, in practice, to decide on one "fundamental" Hamiltonian in contrast to another. (See Part IIC(i), footnote.) To show that the currents defined in Table 10 do play a role in strong dynamics, we use a different technique, as discussed in Part IIID. Before going over this, however, we must understand the second aspect of these currents; their property of being shared between disparate forces.

C. UNIVERSALITY OF CURRENTS

1. For the electric charge of hadrons we noted the remarkable relation

$$Q^{\text{hadron}} = I_3 + \tfrac{1}{2} Y = F^3 + \frac{1}{\sqrt{3}} F^8 \quad .$$

A similar relation would naturally exist for the corresponding hadron currents

$$J_\mu^{EM(\text{hadrons})} = J_\mu^3(x) + \frac{1}{\sqrt{3}} J_\mu^8(x) \ ,$$

where $J_\mu^3(x)$ is the (neutral) isotopic and $J_\mu^8(x)$ is the hypercharge current. This relation is remarkable enough. Even more remarkable is the postulate that the two remaining isotopic left currents $J^1_{\mu L}$ and $J^2_{\mu L}$ (or rather their combinations

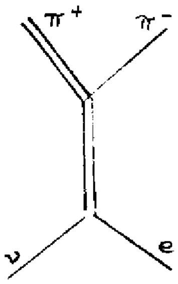
$$J^{\pm}_{\mu L}(x) = J^1_{\mu L} \pm i\, J^2_{\mu L}(x) \ )$$

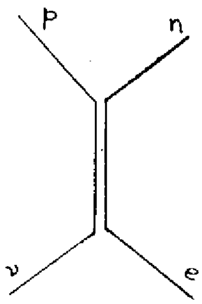
are precisely the weak hadron currents responsible for $\left.\begin{matrix} n \rightarrow p \\ p \rightarrow n \end{matrix}\right\}$ transitions in β-decay. In other words, nature believes in an economy of charges and currents; once the isotopic and hyper - or more generally, SU(3) x SU(3) - set of charges and currents were invented, it was decreed that they would serve not only as strong charges and strong currents but also as E.M. charges and E.M. currents (with a change of scale, of course) as well as for weak charges and weak currents (again with another change of scale) whenever hadron matter was involved.

2. The statement that the same currents make their appearance in strong, weak and E.M. interactions of hadrons, has been confirmed in numerous situations; to take just one representative example:

Consider the decay $\pi^+ \longrightarrow \pi^0 + (e^+ + \nu)$

compared with $n \longrightarrow p + (e^- + \bar{\nu})$





If the ideas outlined are correct, the ratio of the two hadronic transitions $\left.\begin{matrix} n \rightarrow p \\ \pi^+ \rightarrow \pi^0 \end{matrix}\right\}$ *) should be proportional to the isotopic charges of pions and nucleons. This indeed is the case experimentally. If the concept of isotopic charge for hadrons had not already emerged from <u>strong</u> interaction physics, it would surely have been invented from the <u>weak</u> hadronic phenomena alone.

3. Electric charge and weak charges are more universal in character than strong charges. Electric charge (as well as weak charges) is shared properties of leptons as well as hadrons. Thus the total electric charge equals

$$Q = Q^{\text{lepton}} + Q^{\text{hadron}}$$

and the equality $Q^{\text{hadron}} = I_3 + Y/2$ naturally holds only for its

---

*) More precisely, since $\pi^+ \rightarrow \pi^0 + (e + \nu)$ proceeds purely through the <u>vector</u> current, for comparison one must consider only the <u>vector</u> (the so-called Fermi) part of neutron β-decay.

hadronic part. Likewise for the weak charges and weak currents.*)

Now, one of the remarkable features (noted in Part I) of electric charge is the exact equality of electron and proton charges (in other words, Q^{leptons} and Q^{hadrons} - and therefore also $J_\mu^{\text{leptons}}(\text{E.M.})$ and $J_\mu^{\text{hadrons}}(\text{E.M.})$ - possess the same scale). Is this also true of $J_\mu^{\text{weak}}(\text{leptons})$ and $J_\mu^{\text{weak}}(\text{hadrons})$?

Before we can answer this question, recall that weak forces affect left-spinning matter only; more precisely, (primitive) weak currents are left-currents $J_{\mu L}$, which are equal mixtures of vector and axial currents $(J_{\mu L} = J_{\mu V} - J_{\mu A})$ both for leptons and hadrons. And of these two types only the vector ones are conserved, with the consequence that only the "vector" weak charges are time-independent.

We can now answer the question posed. The weak vector lepton charge does indeed equal in magnitude the weak vector hadron charge; one of the striking confirming pieces of evidence is the well-verified equality of the vector (Fermi) decay constant $n \to p + (e^- + \bar{\nu}_e)$ with the constant**) determining μ-decay $\mu^- \to \nu_\mu + (e^- + \bar{\nu}_e)$.

---

*) The full weak current is made up of three left-spinning parts

$$J^{\pm}(\text{weak}) = J^{\pm}_{\mu L}(\text{leptonic}) + J^{\pm}_{\mu L}(\text{hadronic} \quad \Delta Y = 0)$$

$$+ J^{\pm}_{\mu L}(\text{hadronic} \quad \Delta Y = \pm 1) .$$

The last piece $J^{\pm}_{\mu}(\Delta Y = \pm 1)$ is that part of the weak current which induces transitions of the type $\left.\begin{matrix}\Lambda \to n \\ n \to \Lambda\end{matrix}\right\}$ where hypercharge changes by one unit $(\Delta Y = \pm 1)$. The currents involved here are once again made up of $SU_L(3)$ currents. The fact that one is dealing with left currents only in weak interactions implies that weak forces are not left-right symmetric; that they do not preserve space-reflection (parity) symmetry. This fascinating aspect of weak interactions we must unfortunately omit.

**) The axial constants do not display this equality; this corresponds to the lack of J_μ^A conservation.

D. LOW-FREQUENCY DYNAMICS

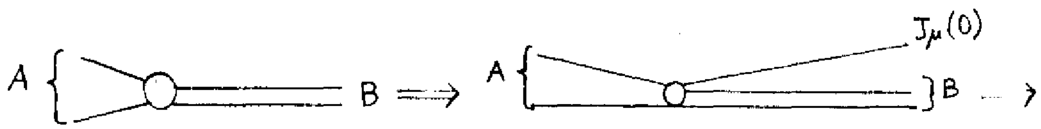
Turn back to strong interactions. We said, though the form of E.M. and weak Hamiltonian $eJ_\mu^{E.M.} \cdot A_\mu$ and $g_{weak}J_{\mu L}W_\mu$ could be tested – since the coupling constants e and g_{weak} are rather small – (and these have indeed been found to be in accord with experiment), no immediate tests of the form of the strong Hamiltonian are possible on account of the large coupling consant involved. How could one then be sure that the SU(3) x SU(3) currents do play a role in strong interaction physics? Fortunately a powerful set of tests are available. This is the set of low-energy theorems which exist whenever a symmetry does. And when, in addition, the currents have physical particles associated with them (field-particle identity of Table 10), the experimental verification of these low-energy theorems becomes not too difficult. The theorems are of the following variety:

Theorem I

Whenever a symmetry exists, broken or exact, and a corresponding charge and therefore current $J_\mu(p)$ exists ($J_\mu(p)$ is the Fourier transform of $J_\mu(x)$), one can derive relations connecting the processes

$$\begin{array}{l} A \longrightarrow B \\ \left.\begin{array}{l} A \longrightarrow B + J_\mu(0) \\ A \longrightarrow B + J_\mu(0) + J_\nu(0) \end{array}\right\} \text{soft } J_\mu\text{-emission} \\ \ldots\ldots \end{array}$$

$J_\mu(0)$ are the zero-frequency components of $J_\mu(p)$.



We shall specify the detailed structure of the relation in a few examples later.

Theorem II

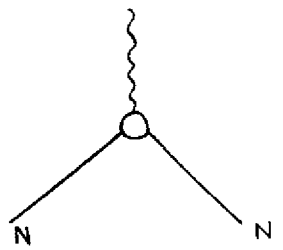
If the current is not conserved but $\partial_\mu J_\mu = \chi$, then similar relations hold for the sequences of processes

$$\begin{array}{l} A \longrightarrow B \\ \left.\begin{array}{l} A \longrightarrow B + \chi(0) \\ A \longrightarrow B + \chi(0) + \chi(0) \end{array}\right\} \text{soft } \chi\text{-emission} \\ \ldots\ldots \end{array}$$

Examples

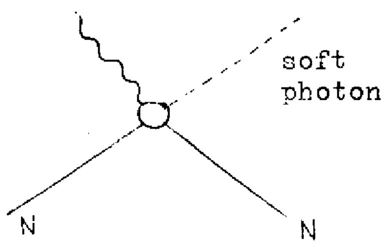
1. Electrodynamics

The theorem states that the vertex (a)



(a)

is related to the process (b) →



(b)

Write $F(\omega)$ for the "Compton" scattering of photons on nucleons represented in (b). Then, according to the theorem, the kinematic structure of $F(\omega)$ at low frequencies is specified completely by parameters of the vertex (a). In detail,

$$F(\omega) = F_1(0)\ \underset{\sim}{\epsilon}' \cdot \underset{\sim}{\epsilon} + \omega F_2(0)\ i\,\underset{\sim}{\sigma} \cdot (\underset{\sim}{\epsilon}' \times \underset{\sim}{\epsilon})$$

where

$$F_1(0) = \frac{\alpha}{m}\ Q^2 \qquad\qquad \alpha = e^2 = 1/137$$

$$F_2(0) = -\ \frac{\alpha}{2m}\quad \kappa^2 ,$$

are the two parameters relating to the process (a). Here $\underset{\sim}{\epsilon}$, $\underset{\sim}{\epsilon}'$ are the polarizations of the photons, $\underset{\sim}{\sigma}$ is the spin of the nucleon, eQ is its charge and Ke/2m its anomalous magnetic moment.*)

---

*) This low-energy theorem, a generalization of the well-known Thomson limit theorem in classical electrodynamics can be used as a boundary condition on a dispersion relation one may postulate for $F(\omega)$; thus

$$\frac{\alpha \kappa^2}{2m^2} = \frac{1}{4\pi^2} \int_0^\infty \frac{d\omega}{\omega}\ (\sigma_P(\omega) - \sigma_A(\omega)) .$$

Here σ_P and σ_A are the photon-nucleon cross-sections with photon polarization parallel and antiparallel to nucleon spin. The relation appears experimentally satisfied. This combination of a (postulated) dispersion relation and a low-energy theorem illustrates the theoretical methodology that has come to be employed more and more in particle physics.

2. Ratio of nucleon-vector and nucleon-axial-vector coupling constants from low-frequency techniques

One of the most celebrated low-energy relations is the one which connects the nucleon-J_{μ}^{V}-vertex to soft pion-nucleon scattering and computes thereby the physically important ratio of the (effective) coupling constants $g_V = g_{NNV}$ and $g_A = g_{NNA}$ (g_{NNV} and g_{NNA} are the zero-frequency limiting values of the appropriate vertex functions, see IIA2). The derivation of the relation is sketched in Table 11; its most important aspect is the purposeful use one must make of both strong and weak data to verify it - once again confirming: (a) that the same currents are operative in both interactions, so far as hadrons are concerned, (b) that these currents - or rather their charges - define the algebra of SU(3) x SU(3), (c) that the techniques of RQM - as formulated through field theories of Class II - and dispersion theory, are marvellously effective in capable hands.

3. Soft-pion processes

A host of low-frequency relations have been derived in strong, weak and E.M. physics, connecting amplitudes like $A \longrightarrow B$ with amplitudes for soft-pion emission $A \rightarrow B + \pi(0)$, $A \longrightarrow B + \pi(0) + \pi(0)$, ... I shall merely mention that these relations exist and are well verified: Quite recently, a new technique (using non-linear representations of the chiral group $SU_L(3) \times SU_R(3)$) has been perfected, which makes their derivation a relatively painless task. We shall be hearing more on this from Professors Weinberg and Zumino during the Conference.

RATIO OF VECTOR AND AXIAL-VECTOR COUPLING PARAMETERS
DERIVED USING LOW-ENERGY THEOREMS

(1) $\partial_\mu \vec{J}^A_\mu = m_\pi^2 \, c_\pi \, \vec{\pi}$ PCAC

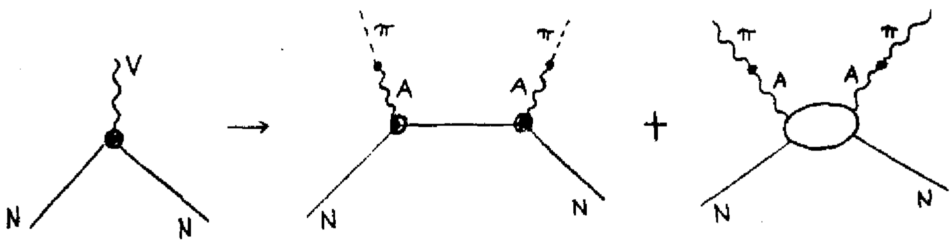
From weak interactions, estimate $(p|J^A_\mu|n)$, to obtain

$$c_\pi = \sqrt{2} \; \frac{g_A}{g_V} \; \frac{m_N}{g_{\pi NN}} \quad .$$

(2) Use the equal-time commutation relation

$$(N \,|\, V \,|\, N) = (N \,|\, [A \; , \; A] \,|\, N)$$

and techniques of field theories of Class II to relate



deriving thereby,

$$1 = c_\pi^2 \left[\frac{g^2_{\pi NN}}{2m_N^2} + \int \frac{d\omega}{\omega} \, (\sigma^+ - \sigma^-) \right] .$$

Use c_π above to predict

$$|\, g_A/g_V \,| = 1.19 \quad .$$

Experiment -1.18 .

TABLE 11

D. PARTICLE SPECTRUM AND THE SYMMETRY ALGEBRA PROGRAMME

The correlation of low-frequency data achieved by the use of the field theoretic method (the essential ingredient being the equal-time commutation rules of SU(3) x SU(3) algebra) was impressive enough that one felt tempted - just as the S-matrix theorist had been sorely tempted earlier with dispersion and superconvergence relations - to consider taking the SU(3) x SU(3) algebra as substitute for a complete dynamical theory. The hope was that one may possibly derive the hadron spectrum by attempting to saturate the identities provided by the SU(3) x SU(3) commutation rules (see Part IIB(c)). Like the S-matrix theorist's attempt to derive the full particle spectrum by bootstrap procedures, the symmetry algebraist's attempt has also met with scant success. The symmetry algebras play an important role in low-frequency dynamics; apparently they do not constitute the complete theory.

PART IV

THE STRUCTURE OF HADRONS,

STILL HIGHER DYNAMICAL SYMMETRIES

In Part III we have been concerned with the low-frequency dynamics which the existence of any given symmetry (isotopic, unitary, etc.) implies. Specifically, we saw that one can derive relations between processes involving bosons associated with the (symmetry) currents (0^-, 1^-, 1^+ octets). But these bosons by no means exhaust the full spectrum of hadrons. What about the multitude of other objects the hadron spectrum consists of, and in particular higher spin baryons? Is it conceivable that there are new dynamical symmetries still to be discovered for an elucidation of the rest of the particle spectrum, particularly as SU(3) x SU(3) appears to be undistinguished for classifying particles?

What exactly do we mean by a dynamical symmetry?

To answer this, consider the familiar case of atomic physics. The relativistic Maxwell-Dirac quantized electrodynamics may indeed be the fundamental theory of charged particle interactions, but to obtain the hydrogen spectrum, one still goes back to the non-relativistic Schrödinger equation with just the static Coulomb potential. And, as is well known, this equation possesses a completely unsuspected O(4) symmetry, first studied by Fock. The emergence of the O(4) is purely "accidental" in that it arises from the particular form of the (Coulomb) Schrödinger equation. However, it is this dynamical symmetry, rather than the fundamental Dirac-Maxwell form of the Hamiltonian, which dominates the hydrogen spectrum.*)

*) Table 12 shows how the symmetry arises and the classification of the hydrogen levels using O(4).

THE HYDROGEN ATOM

Hamiltonian $E = \frac{1}{2} p^2 - \frac{1}{r}$

Define $\underline{L} = \underline{r} \times \underline{p}$

$\underline{M} = \sqrt{-8E} \ (\underline{L} \times \underline{p} - \underline{p} \times \underline{L}) + \frac{\underline{r}}{r}$.

Operaters $\underline{I} = \frac{1}{2} (\underline{L} + \underline{M})$ ⎫ generate independent

and $\underline{K} = \frac{1}{2} (\underline{L} - \underline{M})$ ⎭ $0_3 \times 0_3 \sim 0_4$ rotations

Since $\underline{L} \cdot \underline{M} = 0$ so $|\underline{I}| = |\underline{K}|$.

Verify $-4E = [\underline{I}^2 + \underline{K}^2 + \frac{1}{2}]^{-1}$

$= [i(i+1) + k(k+1) + \frac{1}{2}]^{-1}$

Since $|I| = |\underline{K}| = 0, \frac{1}{2}, 1, \ldots$ (I,k generate 0_3)

obtain $E = -\frac{1}{2n^2}$,

where $n = (2i + 1) = (2k + 1)$.

A tower of levels, $(i,k) = (0,0)$; $(\frac{1}{2},\frac{1}{2})$: $(1,1), \ldots$ corresponds to a single representation of the non-compact group 0_{4+1}.

TABLE 12

The same thing seems to be happening in particle physics where we appear to find a dynamical symmetry SU(6) more successful than SU(3) x SU(3) for particle classification and for a description of vertex functions.

The development of ideas I shall now sketch started with Wigner and his postulated supermultiplet symmetry SU(4) of nuclear physics. Starting with the notion of spin and isotopic charge independence of nuclear forces, Wigner came upon the dynamical group SU(4) as a natural completion of the isotopic $SU_I(2)$ and the spin $SU_J(2)$ groups; $(SU(4) \subset SU_I(2) \times SU_J(2))$. For particle physics, replace isotopic by the unitary symmetry; the simplest completion group, which includes both the unitary $SU_F(3)$ and the spin $SU_J(2)$, is SU(6). Assume that SU(6) (like the O(4) for the hydrogen atom case) is the <u>dynamical symmetry</u> responsible for the hadron spectrum. Each representation of SU(6) would combine within it, particles of different spins and unitary charges.

The fundamental representation of SU(6) is $\underset{\sim}{6}$ dimensional. This would correspond to three quarks of spin up and three quarks of spin down. Other representations are

$\underset{\sim}{35}$, $\underset{\sim}{56}$, $\underset{\sim}{70}$, $\underset{\sim}{405}$, ... with content

$$SU(6) \rightarrow SU_F(3) \times SU_J(2)$$

$$\underset{\sim}{35} = \underset{\sim}{1} \times \underset{\sim}{3} + \underset{\sim}{8} \times \underset{\sim}{3} + \underset{\sim}{8} \times \underset{\sim}{1}$$

(i.e., a spin 1 nonet + a spin zero octet)

$$\underset{\sim}{56} = \underset{\sim}{10} \times \underset{\sim}{4} + \underset{\sim}{8} \times \underset{\sim}{2}$$

(i.e., a 3/2 decuplet + 1/2 octet) .

A glance at the Rosenfeld table, where the lowest mass boson entries are just the 1^- nonet + the 0^- octet (constituting together a $\underset{\sim}{35}$ of SU(6)) and the lowest mass baryons are precisely the $3/2^+$ decuplet + $1/2^+$ octet (together constituting the $\underset{\sim}{56}$ of SU(6)) convinces one that SU(6) makes very good sense.

I shall not describe here the extension of SU(6) to U(6) x U(6) symmetry which distinguishes the fundamental quark representation 6 from the antiquark $\overline{6}$ - nor its formulation $\widetilde{U}(12)$ needed to give correctly the relativistic kinematics of vertex functions. Professor F. Gürsey will be dealing with the subject in detail. In Tables 14-17 are given some of the large variety of predictions - all reasonably well substantiated. The most noteworthy are the predictions of the ratio of magnetic moments of protons and neutrons - obtained essentially as a kinematic prediction of the theory - and the immediate explanation of the scaling law of E.M. form factors mentioned in Part IIA.

Predictions from SU(6) , $\tilde{U}(12)$.

(1) $\frac{\mu_p}{\mu_n} = -3/2$ (see Table 16).

(2) Host of coupling oonstant relations (see Table 17).

(3) Scaling law for form factors (see Table 17).

(4) Mass formulae (see Tables 13 and 14)

$$M^2 = M_0^2 + a\,J(J+1) + b(\vec{I}^2 - \tfrac{1}{4}Y^2) + cY .$$

The problem which immediately arises with the undoubted successes of SU(6) and U(12) symmetries for the multiplet and vertex structures of the well-known mesons and baryons is how to reconcile this with, for example, SU(3) x SU(3). What is the nature of charges associated with SU(6)? Are they conserved? What, if any, are the currents? Does spin act as charge, and anyway what is the precise definition of spin used in SU(6)? Or is it that we are perhaps trying to force totally unrelated and distinct ideas into the same mould? A number of answers to these challenging problems have been advanced. Nobody, seems to know for sure. In the meanwhile, however, a different approach - the aristocratic approach to hadron dynamics - has emerged - an approach which frankly negates all one's notions about relativistic quantum mechanics - but one which is amazingly simple and fruitful. Treat known mesons and baryons as composites of basic quarks in an unashamed non-relativistic sense; the composites are listed below:

$$q \to \underset{\sim}{6}$$

$$q\bar{q} \to 6 \times \bar{6} = \underset{\sim}{35} + \underset{\sim}{1}$$

$$qqq \to \underset{\sim}{6} \times \underset{\sim}{6} \times \underset{\sim}{6} = \underset{\sim}{56} + \underset{\sim}{70} + \underset{\sim}{70} + \underset{\sim}{20} .$$

MASS FORMULAE

SU(3)

Mass formula for SU(3) multiplets:

$$M = a + bY + c \left[I\,(I + 1) - \tfrac{1}{4} Y^2 \right]$$

Baryon octet

Predict $\quad M_{\Sigma} + 3M_{\Lambda} = 2(M_N + M_{\Xi})$

Experiment $\quad 4539.7 \sim 4512.8$ MeV

Decuplet

Predict equal spacing rule

Theory $\quad M_{Y_1^*} - M_{N^*} = M_{\Xi^*} - M_{Y_1^*} = M_{\Omega^-} - M_{Y_1^*}$

Experiment $\quad 149 \sim 145 \sim 142$ MeV

Electromagnetic mass differences

Theory $\quad \Sigma^- - \Sigma^+ - n + p = \Xi^- - \Xi^0$

Experiment $\quad 6.6 \pm 0.1$ MeV $= 6.5 \pm 1.0$ MeV

TABLE 13

MASS FORMULAE

SU(6)

Baryon $\underset{\sim}{56}$

Theory $M = M_{00} + M_1\ J(J+1) + M_2 Y + M_3 \left[I\ (I+1) - \frac{1}{4}\ Y^2 \right].$

An average value of $M_{00} \sim 1065$ MeV gives the masses of SU(3) octet and decuplet correctly

Meson $\underset{\sim}{35}$

SU(6) reproduces SU(3) result

$$4K^2 - \pi^2 = 3\eta^2$$

and gives in addition

$$\rho^2 - \pi^2 = K^{*2} - K^2$$

Experiment 0.571 MeV $\sim$ 0.553 MeV

Electromagnetic mass differences

Theory $n - p = 1/3\ (\Delta^- - \Delta^{++})$

Expt. 1.3 MeV $\sim$ 2.7 $\pm$ 1.3 MeV

TABLE 14

ELECTROMAGNETIC MASS DIFFERENCE OF HADRONS IN STATIC SU(6) MODEL USING QUARKS

		THEORY	EXPERIMENT
Baryons		MeV	MeV
	$\Xi^- - \Xi^0$	6.3 ± 0.3	6.5 ± 1
	$\Xi^{*-} - \Xi^{*0}$	3.1 ± 0.3	6 ± 3
	$N^{*0} - N^{*++}$	0.7 ± 0.6	0.4 ± 0.8
	$N^{*-} - N^{*++}$	3.6 ± 0.6	0.6 ± 5
	$Y_1^{*-} - Y_1^{*+}$	4.4 ± 0.5	4.3 ± 2
Bosons			
	$\pi^+ - \pi^-$	4.2 ± 0.4	4.6 ± 0.007
	$K^0 - K^+$	-0.9 ± 0.2	4.2 ± 0.5

TABLE 15

MAGNETIC MOMENTS

		THEORY	EXPERIMENT
SU(3)			
μ_{Λ}		-0.96	$-0.73 \pm 0.16 \left(\frac{e\hbar}{2m_p c}\right)$
μ_{Σ^+}		2.78	2.5 ± 0.7
SU(6)			
$\frac{\mu_p}{\mu_n}$	$=$	$-\frac{3}{2}$	-1.46
$\mu_{N^* \to N+\gamma}$	$=$	$\frac{2\sqrt{2}}{3}\mu_p$	$(1.28 \pm 0.02)\frac{2\sqrt{2}}{3}\mu_p$

TABLE 16

COUPLING CONSTANT RELATIONS FROM $\tilde{U}(12)$

1) $F^c(q^2)/F^M(q^2) \sim 1 + \frac{m}{2M}$

$F^{c,M}$ are Sachs EM form factors.

2) $\mu_p = 1 + 2M/m, \quad \mu_n = -\frac{2}{3}(1 + 2M/m)$

m is mean mass of 1^- multiplet and M nucleon mass.

3) Meson baryon vertex

(V = 1^- nonet : D = $\frac{3}{2}^+$ decuplet):

Predict

$$g_{PNN} : g^{ch}_{VNN} : g^{Mag}_{VND} : g_{PND} : g_{VND} : g_{PDD} : g^{ch}_{VDD}$$

$$= \left(1 + \frac{2M}{m}\right)_{3D+2F} : \left(1 - \frac{m^2}{4M^2}\right) : \left(1 + \frac{2M}{m}\right)_{3D+2F} : \left(1 + \frac{2M}{m}\right) : \left(1 + \frac{2M}{m}\right) : \left(1 - \frac{m^2}{4M^2}\right)$$

Also predict

$$g_{N^*N\pi} = \frac{3}{5}\left(1 + \frac{2M}{m}\right) g_{NN\pi} \, .$$

Taking $\langle m \rangle = 700$ MeV $\langle M \rangle = 1300$ MeV,

Predict $\Gamma_{N^*N\pi} \sim 110$ MeV

Experiment ~ 100 MeV

TABLE 17

Assume a soft (oscillator) potential for the constituent quarks. The baryon multiplet $\underset{\sim}{56}$ represents a three-body symmetric wave function, the meson $\underset{\sim}{35}$ is a two-body quark-antiquark composite. The model goes on to describe scattering and production phenomena in meson-baryon scattering. The fundamental assumption made is that there are just two independent amplitudes for q-q and q-$\bar{q}$ scattering, all amplitudes for composite particles being obtained by simple additivity. The only ingredient of RQM the quark model needs, is the superposition principle.

As I said earlier, the model seems to negate the S-matrix notion that hadrons are made up of each other. Since within its lights it succeeds, it poses one of the many mysteries of our subject. If physical quarks were discovered (and so far the search has not been successful - their massiveness presumably makes their production with present accelerator energies impossible), one would really have to come to grips with the new (dynamical) problem of reconciling the aristocratic with the democratic approach.*)

---

*) To take an example of the type of problem posed, consider Regge trajectories, treated characteristically differently in S-matrix theory, in the higher symmetry schemes and in the quark model. The quark model explanation is the simplest; the particles on a boson Regge trajectory, for example, are orbital angular momentum ℓ-excitations of a bound q$\bar{q}$ system. The higher symmetry schemes employ a description - as an abstraction from the dynamics - associated with infinite-dimensional unitary representations of non-compact groups like U(6,6) or U(6,6) x O(3). For analogy, one may draw once again on the hydrogen spectrum, where the hydrogen levels (in O(4) language) are given by the representations,

$$(0,0),\ (\tfrac{1}{2},\tfrac{1}{2}),\ (1,1),\ \dots$$

This set constitutes <u>one single unitary irreducible</u> representation of the non-compact group O(4,1). The exploitation of these notions together with infinite-component field equations, first discussed by Majorana, will be the subject of later lectures (particularly of Professor C. Fronsdal).

SUMMARY

This concludes our brief survey of what has been achieved in theoretical understanding in particle physics. To recapitulate, up to the energies for which systematic experimentation has been possible, the highly restrictive and tight structure of RQM seems to hold. We believe we know what some of the internal symmetries of physical particles are; we know that hadronic matter exhibits the same internal symmetries whether acting strongly, weakly or electromagnetically, and we know that these symmetries not only serve to classify particle, but also dominate low-frequency dynamics.

So much for the ideas and principles that have reasonably succeeded. We have been less successful in understanding the architecture of matter itself. The mathematically intractable, though aesthetically attractive, S-matrix bootstrap idea which considers all hadrons as composites of each other, appears at present irreconcilable with the simpler additive aristocratic quark model. The attractive (dynamical) higher symmetries of hadrons like SU(6), U(6) x U(6), $\widetilde{U}(12)$, ... which bypass these difficulties (essentially treating quarks as mathematical entities and giving quark-model-like results without quarks) still need to be reconciled with symmetries like $SU_L(3) \times SU_R(3)$.

Notwithstanding some notable successes in weak interaction theory (the demonstration of the two-component nature of the neutrino, the sharing of $SU_L(3)$ (left, V-A) currents between hadronic strong and weak forces, the well-verified postulate of a suppression of $\Delta Y = \pm 1$ weak effects relative to weak $\Delta Y = 0$ - important topics which I have had no time to discuss) there is much that is dark; the recently discovered superweak CP violation and the possible existence of exotic types of yet undiscovered matter (A-matter of T.D. Lee, introduced specifically to explain CP-violation, magnetic monopoles of Dirac to explain why electric charge must be quantized), the behaviour of weak forces at higher energies; all these are question marks needing much experiment and deep thought. And then, As I said in the very beginning of my lecture, the energies to which our systematic experimentation extends are painfully low on the cosmic scale. There could be nothing more pretentious than the (unqualified) title of this lecture -"A fundamental theory of matter"- but such, fortunately indeed, is the encompassing conceit of the human mind!

Then there are the really deep problems: the origin of different types of forces - strong, weak, E.M. gravitational - their outrageously different couplings, the selective division they impose on matter into leptonic and hadronic, into charged and uncharged, into left and right-spinning varieties. Considering what we have already achieved, one feels proud of the work of our generation. Considering what still remains to comprehend, one feels truly humble.

SOME UNRESOLVED PROBLEMS

(1) Origin and reconciling of dynamical symmetries like SU(6) with $SU_L(3) \times SU_R(3)$.

(2) Nature of internal symmetries; do the postulates of RQM, together with hadron democracy idea, imply the existence of these symmetries?

(3) Theory of CP violation; behaviour of weak interactions at high energies. Origin of the suppression of $\Delta Y = \pm 1$ weak forces relative to $\Delta Y = 0$ forces.

(4) The large numbers like $\frac{m_\mu}{m_e}$, α^{-1}, $(g^2_{weak})^{-1}$, ...

(5) Exotic forms of matter; quarks, magnetic monopoles.

I should like to end with a quotation from J.R. Oppenheimer, who helped in the planning of this Symposium and whose warmth, whose insight and inspiration I personally miss so deeply to-day: "We are so engulfed by the changes in the current scene in physics: by their ferocity, their brashness, their virtuosity, their diffusion, that we don't understand them very well, and it may not be possible for us to understand them. ..... The future will be only more radical and not less, only more strange and not more familiar, and it will have its own new insights for the inquiring mind."

ACKNOWLEDGMENTS

This paper was prepared with the collaborative assistence of Dr. Harun-ar-Rashid, to whom I owe much for critical comments and for preparation of the tables.

Table I: STABLE PARTICLES, January, 1968.

From Data on Particles and Resonant States, UCRL-8030. Printed at LRL and CERN

A. H. Rosenfeld, N. Barash-Schmidt, A. Barbaro-Galtieri, L. R. Price, Matts Roos, Paul Söding, W. J. Willis, C. G. Wohl

(Quantities in italics have changed by more than one standard deviation since January, 1967.)

	$I^G(J^P)C$	Mass (MeV)	Mass difference (MeV)	Mean life (sec) cτ(cm)	Mass² (GeV)²	Decays: Partial mode	Fraction	Q(MeV)	p or p_{max}† (MeV/c)	Partial Rates (sec^{-1})
γ	$0,1(1^-)^-$	0		stable	0	stable				
ν (ν_e, ν_μ)	$J=\frac{1}{2}$	0(<0.2 keV) 0(<2.1 MeV)		stable	0	stable				
e	$J=\frac{1}{2}$	0.511006 ±.000002		stable (>2×10^{21}y)	0.000	stable	μ_e = 1.001159596 ±.000000023 $\frac{e\hbar}{2m_ec}$			
μ	$J=\frac{1}{2}$	105.659 ±.002	−33.920 ±.014 (μ−π±)	2.1983×10^{-6} ±.0008, cτ = 6.592×10^4	0.011	$e\nu\bar{\nu}$	100	105	53	
	μ_μ = 1.0011666 ±.0000005 $\frac{e\hbar}{2m_\mu c}$					eγγ	(<1.6)10^{-5}	105	53	
						3e	(<1.3)10^{-7}	104	53	
						eγ	(<6)10^{-9}	105	53	
$\pi^\pm$	$1(0^-)$	139.579 ±.014	4.6041 ±.0037 ($\pi^\pm-\pi^0$)	2.604×10^{-8} ±.007, S=2.3*, cτ = 781, $(\tau^+-\tau^-)/\bar{\tau}$=(.4±.2)% (test of CPT)	0.019	μν	100 %	34	30	
						eν	(1.24±0.03)10^{-4}	139	70	
						μνγ	(1.24±0.25)10^{-4}	34	30	
						$\pi^0e\nu$	(1.03±0.07)10^{-8}	4	5	
						eνγ	(3.0 ±0.5)10^{-8}	139	70	
π^0	$1^-(0^-)^+$	134.975 ±.014		d 0.89×10^{-16} ±.18, S=1.6*, cτ = 2.67×10^{-6}	0.018	γγ	(98.83±0.04)%	135	67	
						γe^+e^-	(1.17±0.04)%	134	67	
						γγγ	(<5)10^{-6}	135	67	
						$e^+e^-e^+e^-$	b (3.47)10^{-5}	133	67	
$K^\pm$	$\frac{1}{2}(0^-)$	493.83 ±0.11	−3.93 ±0.17 ($K^\pm-K^0$)	1.235×10^{-8} ±.005 S=2.1*, cτ = 370, $(\tau^+-\tau^-)/\bar{\tau}$=(.09±.12)% (test of CPT) S=1.3*	0.244	μν	c (63.58±0.29)%	388	236	c (51.49 ±0.27)10^6
						$\pi\pi^0$	(20.94±0.28)% S=1.1*	219	205	(6.96 ±0.23)10^6 S=1.1*
						$\pi\pi^-\pi^+$	(5.57±0.04)%	75	126	(4.51 ±0.03)10^6
						$\pi\pi^0\pi^0$	(1.70±0.05)%	84	133	(1.38 ±0.04)10^6
						$\mu\pi^0\nu$	(3.38±0.17)% S=1.6*	253	215	(2.74 ±0.14)10^6 S=1.6*
						$e\pi^0\nu$	(4.83±0.12)% S=1.4*	358	229	(3.91 ±0.10)10^6 S=1.4*
						$\pi\pi^\mp e^\pm\nu$	(3.8 ±0.8)10^{-5}	214	204	
						$\pi\pi^\pm e^\mp\nu$	(<2)10^{-6}	214	204	
						$\pi\pi^\mp\mu^\pm\nu$	(≤1.4)10^{-5}	109	151	
						$\pi\pi^\pm\mu^\mp\nu$	(<3)10^{-6}	109	151	
						eν	(1.24±0.40)10^{-5}	493	247	
						$\pi\pi^0\gamma$	(2.2 ±0.7)10^{-4}	219	205	
						$\pi\pi^+\pi^-\gamma$	(10 ±4)10^{-5}	75	126	
						πeνγ	(6 ±4)10^{-4}	354	227	
						πe^+e^-	(<1.1)10^{-6}	353	227	
						$\pi\mu^+\mu^-$	(<2.4)10^{-6}	143	172	
						πγγ	(<1.1)10^{-4}	354	227	
K^0	$\frac{1}{2}(0^-)$	497.75 ±0.16		50% K_{short}, 50% K_{long}						
K^0_S	$\frac{1}{2}(0^-)$		$-0.48\times\frac{1}{\tau_S}$ ±0.02 (K_S-K_L)	0.874×10^{-10} ±.011 S=1.3*, cτ = 2.61	0.248	$\pi^+\pi^-$	(68.4 ±1.0)%	219	206	(0.783±0.015)10^{10}
						$\pi^0\pi^0$	(31.6 ±1.0)%	228	209	(0.362±0.012)10^{10}
K^0_L	$\frac{1}{2}(0^-)$			5.30×10^{-8} ±.13, cτ = 1593	0.248	$\pi^0\pi^0\pi^0$	c (25.5 ±1.9)%	93	139	c (4.81 ±0.41)10^6
						$\pi^+\pi^-\pi^0$	(12.1 ±0.4)%	84	133	(2.29 ±0.09)10^6
						πμν	(27.3 ±1.3)%	253	216	(5.15 ±0.26)10^6
						πeν	(35.1 ±1.4)%	358	229	(6.62 ±0.29)10^6
						$\pi^+\pi^-$	(0.149±0.006)%	219	206	(0.028±0.001)10^6
						d) $\pi^0\pi^0$	still uncertain	218	209	
						$\pi^+\pi^-\gamma$	(<0.3)%	219	206	
						d) γγ	still uncertain	498	249	
						eμ	(<0.8)10^{-5}	392	238	
						$\mu^+\mu^-$	(<1.5)10^{-6}	287	225	
						e^+e^-	(<1.7)10^{-5}	497	249	

	$I^G(J^P)C$	Mass (MeV)	Mass difference (MeV)	Mean life (sec) cτ(cm)	Mass² (GeV)²	Partial mode	Fraction	Q(MeV)	p or p_{max}† (MeV/c)	Magnetic Moment ($e\hbar/2m_pc$)	Decay Parameters†: Measured α	Measured Φ(degree)	Derived γ	Derived Δ(degree)
η	$0^+(0^-)^+$	548.8 ±0.6		Γ=(2.3±0.5)keV		Neutral decays 71.0%: γγ	(42 ± 3.4)% (d)	549	274					
						$\pi^0\gamma\gamma$	(1 ± 1.9)% (d)	414	258					
						$3\pi^0$	(28 ± 1.8)% (d)	144	179					
						Charged decays 29.0%: $\pi^+\pi^-\pi^0$	(23.4 ± 1.1)%	135	174					
						$\pi^+\pi^-\gamma$	(5.5 ±0.5)%	269	236					
						$\pi^0e^+e^-$	(<0.04)%	413	258					
						$\pi^+\pi^-e^+e^-$	(0.1 ±0.1)%	268	236					
p	$\frac{1}{2}(\frac{1}{2}^+)$	938.256 ±0.005	−1.2933 ±.0001 (p−n)	stable (>6×10^{27}y)	0.880					2.792763 ±.000030				
n	$\frac{1}{2}(\frac{1}{2}^+)$	939.550 ±0.005		(1.01±0.03)10^3, cτ = 3.03×10^{13}	0.882	$pe^-\nu$	100 %	1	1	−1.913148 ±.000066	$\frac{g_A}{g_V}$ = −1.198 ± .022(e),†			
Λ	$0(\frac{1}{2}^+)$	1115.50 ±0.08		2.52×10^{-10} ±.04 S=1.4*, cτ = 7.61	1.245	$p\pi^-$	(65.3 ±1.2)%	38	100	−0.73 ±.16 S=1.2*	0.647±0.016	(−6±7)°	0.75	(7±8)°
						$n\pi^0$	(34.7 ±1.2)%	41	104		0.71 ±0.18	S=1.2*		
						peν	(0.88±0.15)10^{-3}	177	163		0.06 ±0.19			
						pμν	(1.35±0.60)10^{-4}	72	131		For Λ → peν, $\frac{g_A}{g_V}$ = $-1.14^{+0.23}_{-0.33}$ †			
Σ^+	$1(\frac{1}{2}^+)$	1189.47 ±0.08	−7.95 ±.11 ($\Sigma^+-\Sigma^-$)	0.810×10^{-10} ±.013, cτ = 2.43	1.412	$p\pi^0$	(52.8 ±1.5)%	116	189	2.5 ±.7	−.955±0.070 S=1.1*			
						$n\pi^+$	(47.2 ±1.5)%	110	185		+.017±0.037	(180±30)°	−0.99	(0±85)°
						pγ	(1.9 ±0.4)10^{-3}	251	225					
						$n\pi^+\gamma$	(≈1.)10^{-3}	110	185					
						$\Lambda e^+\nu$	(2.2 ±0.7)10^{-5}	73	72					
						$n\mu^+\nu$	(<0.7)10^{-4}	144	202					
						$ne^+\nu$	(<0.3)10^{-4}	249	224					
				$\frac{\Gamma(\Sigma^+\to\ell^+n\nu)}{\Gamma(\Sigma^-\to\ell^-n\nu)}=.04^{+.04}_{-.02}$										
Σ^0	$1(\frac{1}{2}^+)$	1192.54 ±0.10	−4.88 ±.06 ($\Sigma^0-\Sigma^-$)	$<1.0\times10^{-14}$, cτ$<3\times10^{-4}$	1.422	Λγ	100 %	77	75					
						Λe^+e^-	b (5.45)10^{-3}							
Σ^-	$1(\frac{1}{2}^+)$	1197.41 ±0.09		1.66×10^{-10} ±.03 S=1.3*, cτ = 4.95	1.434	$n\pi^-$	100 %	118	193		−.06 ±0.05	(22±30)°	0.90	
						$ne^-\nu$	(1.25±0.17)10^{-3}	257	230					
						$n\mu^-\nu$	(0.62±0.12)10^{-3}	152	210		S=1.6*			
						$\Lambda e^-\nu$	(0.66±0.11)10^{-4}	81	79		For $\Sigma^-\to\Lambda e\nu$, $\frac{g_A}{g_V}$ = 0.3 ± 0.3†			
						$n\pi^-\gamma$	(≈1)10^{-3}	118	193					
Ξ^0	$\frac{1}{2}(\frac{1}{2}^+)$	1314.9 ±0.8	−6.3 ±.7 ($\Xi^0-\Xi^-$)	2.9×10^{-10} ±.4 S=1.2*, cτ=8.85	1.728	$\Lambda\pi^0$	100 %	64	135		−.33 ±0.10			
						$p\pi^-$	(<0.5)%	237	299					
						$pe^-\nu$	(<0.6)%	376	323					
						$\Sigma^+e^-\nu$	(<0.7)%	125	119					
						$\Sigma^-e^+\nu$	(<0.6)%	117	112					
						$\Sigma^+\mu^-\nu$	(<0.7)%	20	64					
						$\Sigma^-\mu^+\nu$	(<0.6)%	12	49					
						$p\mu^-\nu$	(<0.6)%	271	309					
Ξ^-	$\frac{1}{2}(\frac{1}{2}^+)$	1321.3 ±0.2		1.73×10^{-10} ±.05, cτ=5.20	1.746	$\Lambda\pi^-$	100 %	66	139		−.402±.051 S=1.4*	(6±8)° S=1.3*		
						$\Lambda e^-\nu$	($0.90^{+0.71}_{-0.43}$)10^{-3}	205	190					
						$n\pi$	(<5)10^{-3}	242	303					
						$\Lambda\mu^-\nu$	(<1.2)%	100	163					
						$\Sigma^0e^-\nu$	(<0.3)%	128	122					
						$\Sigma^0\mu^-\nu$	(<0.5)%	23	70					
						$n e^-\nu$	(<1.0)%	381	327					
Ω^-	$0(\frac{3}{2}^+?)$	1672±1		$1.1^{+0.6}_{-0.5}\times10^{-10}$, cτ=3.3	2.795	$\Xi^0\pi^-$		217	293					
						$\Xi^-\pi^0$		216	289					
						ΛK^-		63	210					

General Atomic and Nuclear Constants [a]

N	$=6.02252\times10^{23}$ mole^{-1} (based on $A_{C^{12}}=12$)
c	$=2.997925\times10^{10}$ cm sec^{-1}
e	$=4.80298\times10^{-10}$ esu = 1.60210×10^{-19} coulomb
1 MeV	$=1.60210\times10^{-6}$ erg
ħ	$=6.5819\times10^{-22}$ MeV sec $=1.05449\times10^{-27}$ erg sec
ħc	$=1.9732\times10^{-11}$ MeV cm = 197.32 MeV fermi
k	$=8.6171\times10^{-11}$ MeV deg^{-1} (Boltzmann const.)
α	$=e^2/\hbar c$ = 1/137.0388
m_e	$=0.511006$ MeV/c^2 = 1/1836.10 m_p
m_p	$=938.256$ MeV/c^2 = 1836.10 m_e = 6.721 $m_{\pi^\pm}$ $=1.00727663\ m_1$ (where m_1 = 1 amu = $\frac{1}{12}m_{C^{12}}$ = 931.478 MeV/c^2)
r_e	$=e^2/m_ec^2$ = 2.81777 fermi (1 fermi = 10^{-13}cm)
λ_e	$=\hbar/m_ec=r_e\alpha^{-1}=3.86144\times10^{-11}$ cm
a_∞ Bohr	$=\hbar^2/m_ee^2=r_e\alpha^{-2}$ = 0.529167 A (1 A = 10^{-8} cm)
$\sigma_{Thomson}$	$=\frac{8}{3}\pi r_e^2=0.66516\times10^{-24}$ cm^2 = 0.66516 barn
μ_{Bohr}	$=e\hbar/2m_ec=0.578817\times10^{-14}$ MeV gauss^{-1}
μ_{nucl}	$=e\hbar/2m_pc=3.1524\times10^{-18}$ MeV gauss^{-1}

$\frac{1}{2}\omega_{cyclotron}$ (rad sec^{-1} gauss^{-1})
$=e/2m_ec=8.79404\times10^6$
$=e/2m_pc=4.7895\times10^3$

(continued on the other side)

[a] Based mainly on E. R. Cohen and J. W. M. DuMond, Rev. Mod. Phys. 37, 537 (1965).

Note that there are some disagreements with the results of recent experiments.

CP violation parameters

$$\eta_{+-\atop(00)}=\frac{A(K_L\to\pi^+\pi^-)}{A(K_S\to\pi^+\pi^-)}=|\eta_{+-\atop(00)}|e^{i\phi_{+-\atop(00)}}$$

$|\eta_{+-}|$ = (1.89±0.09)$\times10^{-3}$

ϕ_{+-} = (65±11)° S=1.1*

$|\eta_{00}|$ still uncertain, see data listings

†The definition of these quantities is as follows:

$$\alpha=\frac{2\,\mathrm{Re}(S^*P)}{|S|^2+|P|^2};\quad \beta=\frac{2\,\mathrm{Im}(S^*P)}{|S|^2+|P|^2};\quad \gamma=\frac{|S|^2-|P|^2}{|S|^2+|P|^2}$$

$\tan\Phi=\beta/\gamma$ $\qquad$ $\tan\Delta=-\beta/\alpha$

g_A/g_V defined by $M\propto\langle B_f|\gamma_\lambda(g_V-g_A\gamma_5)|B_i\rangle$

* S = Scale factor = $\sqrt{\chi^2/(N-1)}$ where N ≈ number of experiments. S should be ≈ 1. If S > 1, we have enlarged the error of the mean, δx, i.e., δx→S δx. This new convention is still inadequate, since if S > 1, the real uncertainty is probably even greater than Sδx. See text of January 1967 edition.

† In decays with more than two bodies, P_{max} is the maximum momentum that any particle can have.

b. Theoretical value, see also data card listings.

c. For comparison with predictions of the ΔI=½ rule, see Appendix I.

d. See note in data card listings.

e. This value, not listed in the data cards, was obtained by G. Conforto (Acta Phys. Acad. Hungar. 22, 11 (1967).)

STABLES & TABLES

Quantities in italics have changed by more than one standard deviation since January, 1967.

Name	Symbol (I^G)	$I^G(J^P)C_n$ ⟼ estab. ? = guess	Mass M (MeV)	Width Γ (MeV)	M^2 ±ΓM[(a)] (GeV)^2	Partial decay modes: Mode	Fraction %	Q (MeV)	p or p_{max}[(b)] (MeV/c)
$\pi^\pm(140)$ $\pi^0(135)$	$\pi(0^-)$	$1^-(0^-)+$	139.58 / 134.98		0.019 / 0.018	See Table S			
$\eta(549)$	$\eta(0^-)$	$0^+(0^-)+$	548.8 ±0.6	2.3 keV ±.5 keV	0.301 ±.000	all neutral	71	See Table S	
						$\pi^+\pi^-\pi^0+\pi^+\pi^-\gamma$	29		
$\rho^\pm(765)$	$\rho(1^-)$	$1^+(1^-)-$	755-775 (h)	110-140 (h)	0.585 ±.095	$\pi\pi$	≈ 100	491	353
						$\pi^\pm\pi^+\pi^-\pi^0$	< 0.2	212	247
						$\pi^\pm\pi^-\pi^+\pi^-$	< 0.15	212	247
						$\pi^\pm\gamma$	< 0.4	630	372
$\rho^0(765)$			760-780 (h)	90-150 (h)	0.590 ±.090	$\eta\pi^\pm$	< 0.8	82	146
						e^+e^-	.0055±.0007 (k)	769	385
						$\mu^+\mu^-$	.0058±.0010	559	370
$\omega(783)$	$\phi'(1^-)$	$0^-(1^-)-$	783.3 ±0.7 S=1.9*	12.2 ±1.3	0.614 ±.009	$\pi^+\pi^-\pi^0$	≈ 90	369	328
						$\pi^+\pi^-$	seen (c)	504	366
						$\pi^0\gamma$	9.5±0.9	648	380
						η+ neutral	< 1.5	234	199
						$\pi^+\pi^-\gamma$	< 5	504	366
						$\pi^0\pi^0\gamma$	< 1	513	368
						e^+e^-	$.0051^{+.0015}_{-.0020}$ (j)	782	392
						$\mu^+\mu^-$	< 0.10	572	377
$\eta'(958)$ or X^0 See note (u).	$\eta'(0^-)$	$0^+(0^-)+$	958.3 ±0.8	< 4	0.918 ±.004	$\eta\pi\pi$	67 ± 10	131	232
						$\pi^+\pi^-\gamma$(incl. $\rho^0\gamma$)	22 ± 3	679	458
						neutrals (excl. $\eta\pi\pi$) 11	± 9		
						for upper limits see footnote (f)			
$\delta(962)$ Seen in only one experiment	?( )	≥1 ( )	962 ±5	< 5	0.927 ±.005	$\delta^\pm$ → 1 charged+neutral(s) ≈ 60			
						$\delta^\pm$ → ≥ 3 charged+neutral(s)≈40			
H(990) existence in doubt (see note (i))	φ(A) ?	$0^-(1^+)-$??	≈990	≤ 80	0.98	3π	only mode seen (see note (n))	575	440
$\eta_V(1016)$ → K$\bar{K}$	η(0+)	$0^+(0^+)+$	1016 ±10 (if resonance)	25 -5	1.032 ±.025	$K^0\bar{K}^0$	only mode seen	24	110
						ηπ	< 70	328	342
Resonance, virtual bound state, or antibound state, still not distinguished.									
φ(1019)	φ(1−)	$0^-(1^-)-$	1019.7 ±0.6 S=1.5*	3.4 ±0.8	1.039 ±.004	K^+K^-	47.5±3.2	32	126
						$K_L K_S$	38.9±3.1	24	109
						$\pi^+\pi^-\pi^0$ (incl. ρπ)	13.8±4.3	605	462
						e^+e^- indic. seen,	< 0.2	1018	510
						$\mu^+\mu^-$ seen,	< 0.5	808	499
						for other upper limits see footnote (g)			
$\eta_V(1070)$ → K_SK_S	η(0+)	$0^+(0^+)+$	1069 ±10 S=2.3*	~ 80 see note (l)	1.14 ±.09	ππ	< 70	793	516
(Some data still favor large scattering length)						K$\bar{K}$	> 30	76	198
A1(1070) Existence in doubt (see note (d))	π(1+)	$1^-(1^+)+$ (J^P=2− not yet excluded)	1070 ±20	80 ±35	1.12	3π (see note (n))	≈ 100	651	488
						K$\bar{K}$	< 0.25, G=(−1)$^{J+1}$ forbids this		
B(1220)	ρ(A) ?	$1^+(1^+)-$?? (J^P=2+ not excluded)	1220 ±11	129 ±14 S=1.3*	1.46 ±.16	ωπ	≈ 100	297	339
						for upper limits see footnote (m)			
f(1260)	η'(2+)	$0^+(2^+)+$	1263 ±10	151 +13 S=2.9*	1.57 ±.15	ππ	large	984	616
						$2\pi^+2\pi^-$	< 4	705	552
						K$\bar{K}$ indic. seen,	< 2.5	267	388
D(1285)	η'(A)	$0^+(1^+)+$ (J^P= 0−, 2− not excluded)	1285 ±4	32 ±8	1.65 ±.04	K$\bar{K}$π (mainly η_V(1016)π)	only mode seen	154	304
						$K^*\bar{K}+\bar{K}^*K$		-100	
						ππρ	not seen	256	356
A2(1300) Some evidence for > 1 meson (see data listings).	π(2+)	$1^-(2^+)+$	1305 ±10 S=2.2*	90 ±10 S=1.2*	1.70 ±.12	ρπ (see note (n))	85.7±2.9	395	410
						K$\bar{K}$	4.0±1.1 S=1.1*	313	424
						ηπ	10.5±2.1	617	526
						η'π	< 10	207	275
						$\pi^+\pi^-\pi^0$(excl. ρπ)	< 17	891	615
E(1420)	η(A)	$0^+(0^-)+$ (J^P=1+ not yet excluded)	1424 ±6	71 ±10 S=1.2*	2.03 ±.10	$K^*\bar{K}+\bar{K}^*K$	50 ±10	38	157
						η_V(1016)π	50 ±10	268	328
						ππη	< 88	596	569
						ππρ	not seen	384	455
f'(1514)	η(2+)	$0^+(2^+)+$	1514 ±5	73 ±23 S=1.8*	2.29 ±.11	K$\bar{K}$	72 ± 12 (e)	518	570
						$K^*\bar{K}+\bar{K}^*K$	10 ± 10 (e)	128	294
						ππ	< 14	1235	744
						ηππ	18 ± 10	686	624
						ηη	< 40	417	522
π_A(1640) → 3π not yet a completely established resonance	π(A) ?	$1^-(A)+$?	1654 ±12 S=1.6*	109 ±30	2.73 ±.18	3π	appears dominant	1245	797
						[ρπ†]	< 40]	745	637
						[fπ†]	appears dominant]	252	320
						K$\bar{K}$	< 40	654	658
ρ_N(1650) g → 2π	ρ(V)	$1^+(V)-$ (J^P= 1−, 3−, ... with 3− favored)	1660 ±17 S=1.4*	169 ±30	2.76 ±.28	2π	appears dominant	1385	820
						4π	probably observed	1106	773
						ρππ		615	617
						K$\bar{K}$ indication seen, < 10		668	667
						ωπ indication seen		745	638
→The remaining data, --e. g., R, S, T, U mesons--in this mass region are too confused to tabulate. →See sketch at upper right.									
$K^\pm(494)$ $K^0(498)$	K(0−)	$1/2(0^-)$	493.83 / 497.75		0.244 / 0.248	See Table S			
$K^*(890)$ m_0-m_+ = 6.3±4.1	K(1−)	$1/2(1^-)$	892.7 ±0.5 S=1.2*	49.2 ±1.0	0.797 ±.044	Kπ	≈ 100	259	289
						Kππ	< 0.2	120	216
K_A(1230) or C	K(A)	$1/2(1^+)$ Not yet completely established. See footnote (p). J^P=2− not yet ruled out.	1230	60	1.51 ±.07	See footnote (p)			
K_A(1320)	K(A)	$1/2(1^+)$ See footnote (p). J^P=2− not yet completely ruled out.	1320	60	1.74 ±.08	See footnote (p)			
K_N(1420)	K(2+)	$1/2(2^+)$	1418.6 ±3.2 S=1.3*	89.1 ±5.1 S=1.1*	2.012 ±.126	Kπ	51 ± 3 } S=1.5*	787	615
						K*π	33 ± 3	348	413
						Kρ	12 ± 3	156	318
						Kω	2.4± 1	143	304
						Kη	2.1± 1	378	482
K_A(1800) or L	K(A)	$1/2(A)$?	1781 ±14 S=1.7*	74 ±24 S=1.3*	3.17 ±.13	Kπ	< 2.3	1167	825
						K*π	24 ± 8	772	670
						K_N(1420)π	16 ± 8	245	317
						Kρ	10 ± 6	532	632
						Remaining K*ππ	45 ± 15	1032	807
						Kω	4.8± 2	519	624
						Kη	< 1	754	732

→ The following bumps, excluded above, are listed among the data cards: φ(1410); ε(730), $A2_{L,2}(1320)\to\rho^0\pi^-$, ρρ(1410), K_SK_S(1440); η(1600) → 4π; ρ(1700) → 4π, $K^*\bar{K}$; $R_{I=0}$(1630-1830); η(1830) → 4π, φ(1830) → ωππ; S(1930); T(2200); $N\bar{N}$ 0(2380), U(2380); κ(725); K_V(1080); $K_A(I=3/2)(1175)$; $K_A(I=3/2)(1265)$; $K_A(I=1/2)(1280)$, $K_V(I=1/2)(1660)$.

* Quoted error includes scale factor $S = \sqrt{\chi^2/(N-1)}$. See footnote to Table S.

† Square brackets indicate a subreaction of the previous (unbracketed) decay mode.

(a) ΓM is the half-width of the resonance when plotted against M^2.

(b) For decay modes into ≥ 3 particles p_{max} is the maximum momentum that any of the particles in the final state can have. The momenta have been calculated using the averaged central mass values, without taking into account the widths of the resonances.

(c) Reported values range between 1% and 15%, and depend on assumptions on ρ-ω interference.

Footnotes continued at right.

Particle or resonance	I(J^P) —— = estab.	Beam π, K (BeV) (BeV/c)	Mass (MeV)	Γ (MeV)	M²±ΓM (BeV²)	Mode	Fraction (%)	p or p†max (MeV/c)	4πλ² (mb)
N									
p n	1/2(1/2⁺)		938.1 939.6		0.880 0.883	See Table S			
N′(1470)	1/2(1/2⁺) P11	T=0.53πp p=0.66	1470	210	2.16 ±0.31	Nπ Nππ {Nσ}[a]	65 35 [domin]	420	27.8
N(1518)	1/2(3/2⁻) D13	T=0.62 p=0.75	1525	115	2.33 ±0.18	Nπ Nππ [Δ(1236)π][a] Nη	55 45 [domin] ~0.5	460 414 229 161	23.2
N(1550)	1/2(1/2⁻) S11	T=0.66 p=0.79	1550	130	2.40 ±0.20	Nπ Nη Nππ	30 70 small	477 210 434	21.5
N(1680)	1/2(5/2⁻) D15	T=0.88 p=1.02	1680	170	2.82 ±0.29	Nπ Nππ [Δ(1236)π][a] ΛK Nη	40 dom. inel. [?] <1.6 <2.5	567 533 365 218 379	15.2
N(1688)	1/2(5/2⁺) F15	T=0.90 p=1.03	1690	130	2.86 ±0.22	Nπ Nππ [Δ(1236)π][a] ΛK Nη	65 dom. inel. [?] <.13 <1.5	574 540 374 234 390	14.9
N′(1710)	1/2(1/2⁻) S11	T=0.94 p=1.07	1710	300	2.92 ±0.51	Nπ	80	587	14.2
N(2190)	1/2(7/2⁻) G17	T=1.96 p=2.10	2200	250	4.84 ±0.55	Nπ	30	894	6.13
N(2650)	1/2(?⁻)	T=3.12 p=3.26	2650	360	7.02 ±0.95	Nπ	(J+1/2)x=0.45[b]	1154	3.67
N(3030)	1/2(?)	T=4.26 p=4.40	3030	400	9.18 ±1.21	Nπ	(J+1/2)x=0.05[b]	1377	2.62
Δ									
Δ(1236)	3/2(3/2⁺) P33	T=0.195 p=0.304	(++) 1236.0 ±0.6	120 ±2	1.53 ±0.15	Nπ Nπ⁺π⁻	100 0	231 89	91.9
	$m_0 - m_{++}$ = 0.45±0.85		$m_- - m_{++}$ = 7.9±6.8						
Δ(1640)	3/2(1/2⁻) S31	T=0.81 p=0.94	1640	180	2.69 ±0.30	Nπ Nππ	30 dom. inel.	540	16.8
Δ(1920)	3/2(7/2⁺) F37	T=1.41 p=1.54	1950	220	3.80 ±0.43	Nπ ΣK	40 seen	741 453	8.91
Δ(2420)	3/2(11/2⁺)	T=2.50 p=2.64	2420	310	5.86 ±0.75	Nπ	11	1024	4.67
Δ(2850)	3/2(?⁺)	T=3.71 p=3.85	2850	400	8.12 ±1.14	Nπ	(J+1/2)x=0.25[b]	1266	3.05
Δ(3230)	3/2(?)	T=4.94 p=5.08	3230	440	10.4 ±1.4	Nπ	(J+1/2)x=0.05[b]	1475	2.24
Z									
Z_0(1865)	0(?)	p=1.15 K⁺p	1865	180	3.47 ±0.34	NK	(J+1/2)x=0.35[b]	579	14.6
	Resonance interpretation not established.								
Λ									
Λ	0(1/2⁺)		1115.5		1.24	See Table S			
Λ(1405)	0(1/2⁻) S01	p<0 K⁻p	1405	50	1.97 ±0.07	Σπ	100	140	
Λ(1520)	0(3/2⁻) D03	p=0.392	1518.8 ±1.5	16 ±2	2.31 ±0.02	NK̄ Σπ Λππ	45±4 }S=1.8* 45±4 10±1	235 258 251	83.6
Λ′(1670)	0(1/2⁻) S01	p=0.74	1670	18	2.79 ±0.03	NK̄ Λη	K⁻p→Λη seen	410 66	28.5
Λ′(1690)	0(3/2⁻) D03	p=0.78	1690	45	2.86 ±0.08	NK̄ Σπ	20 58	429 403	26.1
Λ(1815)	0(5/2⁺) F05	p=1.05	1816 ±2 S=1.3*	74 ±5	3.30 ±0.14	NK̄ Σπ Σ(1385)π Λη	63 11 11 ~1	538 500 359 346	16.7
Λ(1830)	0(5/2⁻) D05	p=1.08	1827	76	3.34 ±0.14	NK̄ Σπ	8 42	547 508	16.0
Λ(2100)	0(7/2⁻) G07	p=1.68	2100	140	4.41 ±0.29	NK̄ Σπ Λη ΞK Λω	33 4 <3 ~1 <10	748 699 617 483 443	8.68
Λ(2350)	0(?)	p=2.29	2350	210	5.52 ±0.49	NK̄	(J+1/2)x=0.7[b]	913	5.85
	Seen in total c.s.								
Σ									
Σ	1(1/2⁺)		(+)1189.5 (0)1192.5 (-)1197.4		1.41 1.42 1.43	See Table S			
Σ(1385)	1(3/2⁺) P13	p<0 K⁻p	(+)1382.2±0.8 S=1.6*	(+)37±3 S=2.1*	1.92 ±0.05	Λπ Σπ	91±3 9±3 S=1.4*	208 117	
		S=4.8* ↑ ←(-)	1388.0±3.0	(-)38±8, S=3.7* ↑					
Σ(1660)	1(3/2⁻) D13	p=0.72	1660	50	2.76 ±0.08	Λ(1405)π NK̄	large small for both	197 400	29.9
	Decay modes of these two states not separated yet.								
Σ(1690)	1 (? ?)	p=0.80	1690	120	2.89 ±0.19	Λπ Σπ	large not disentangled	455 395	25.1
Σ(1770)	1(5/2⁻) D15	p=0.95	1767 ±4 S=1.5*	95 ±12 S=2.3*	3.13 ±0.16	NK̄ Λπ Λ(1520)π Σ(1385)π Ση Σπ	46 15 15 14 0.5 ~1	497 519 190 317 140 463	19.4
Σ(1910)	1(5/2⁺) F15	p=1.25	1910	60	3.65 ±0.11	NK̄ Λπ Σπ	8 10 3	612 619 568	12.9
	A small effect. Not established beyond all question.								
Σ(2030)	1(7/2⁺) F17	p=1.52	2030	120	4.12 ±0.24	NK̄ Λπ Σπ ΞK	11 36 9 <2	700 700 652 412	9.92
Σ(2250)	1(?)	p=2.04	2250	200	5.06 ±0.45	NK̄	(J+1/2)x=0.3[b]	849	6.76
	Seen in total c.s.								
Σ(2455)	1(?)	p=2.57	2455	~140	6.03 ±0.34	NK̄	(J+1/2)x=0.26[b]	979	5.08
	Seen in total c.s.								
Σ(2595)	1(?)	p=2.95	2595	~140	6.73 ±0.36	NK̄	(J+1/2)x=0.26[b]	1064	4.30
	Seen in total c.s.								
Ξ									
Ξ	1/2(1/2⁺)		(0)1314.7 (-)1321.2		1.73 1.75	See Table S			
Ξ(1530)	1/2(3/2⁺)		(0)1528.9±1.1 (-)1533.8±1.9	7.3 ±1.7	2.34 ±0.01	Ξπ	100	145	
Ξ(1815)	1/2(?) p-wave		1815 ±3	16 ±8 S=2.2*	3.29 ±0.03	ΛK̄ Ξπ Ξππ [Ξ(1530)π][a] ΣK̄	~65 ~10 ~25 [~20] <2	391 409 351 220 299	
	Branching ratios poorly known.								
Ξ(1930)	1/2(?)		1930	140	3.72 ±0.27	Ξπ ΛK̄	seen	501 504	
	Needs confirmation.								
Ω									
Ω⁻	0(3/2⁺)		1672		2.80	See Table S			

See footnote c.

APPENDIX B

Take two machines as typical: (a) CERN PS (strong focusing)
(b) Nimrod (weak focusing)

CERN PS

Maximum energy	28.5 GeV
Protons per pulse	$\sim 10^{12}$ internal
Pulse repetition rate	12 per minute

Remarks: There is an agreed improvement programme which will double the repetition rate and increase the circulating beam to $\sim 10^{13}$ protons per pulse.

Secondary beams are available both from internal targets and external targets. It is usually possible to have several experiments running on the machine at any one time.

Nimrod

Maximum energy	7 GeV
Protons per pulse	1.5×10^{12} internal
Pulse repetition rate	23 per minute

Typical beams

The number of useable particles can vary enormously because usually it is necessary to make many compromises to fit in all the experimental teams. Here are some actual Nimrod beam figures. Yields are obtained at the end of the beam line so decay has been taken into account.

Particle	Beam	Yield per pulse with 1.5×10^{12} protons circulating with $\Delta P/P = 1\%$ FWHH	Momentum GeV/c
π^-	2 tank separated beam	1.5×10^6	1.5
π^+		2.5×10^6	1.5
K^-		2.5×10^3	1.5
K^+		1.5×10^4	1.5

The following table gives some similar CERN figures:

Particle	Beam	Yield per 10^{12} protons with $\Delta P/P = 1\%$ FWHH	Momentum GeV/c
π^-	Unseparated	$\sim 2 \times 10^6$	1.5
π^+		$\sim 3 \times 10^6$	1.5
π^+		$\sim 3 \times 10^6$	2.5
π^-		$\sim 2 \times 10^6$	2.5
K^+		1.2×10^5	1.5
K^-		5×10^4	1.5
$\bar{p}$		10^4 ($\Delta P/P$ unknown)	3.0
K^-		1.5×10^3	7.5
$\bar{p}$		10^2	12.7

These are typical beams as actually used in counter experiments. Bubble chamber beams usually only require $\sim$10-100 particles per pulse, but have a high degree of separation and so may use 10 - 50% of the circulating beam depending on the energy. Obviously the higher the energy the greater the fraction of beam required. Neutrino experiments, of course, need the full beam and, generally speaking, no other experiment can be run at the same time. The following table gives some absolute yield figures for the CPS.

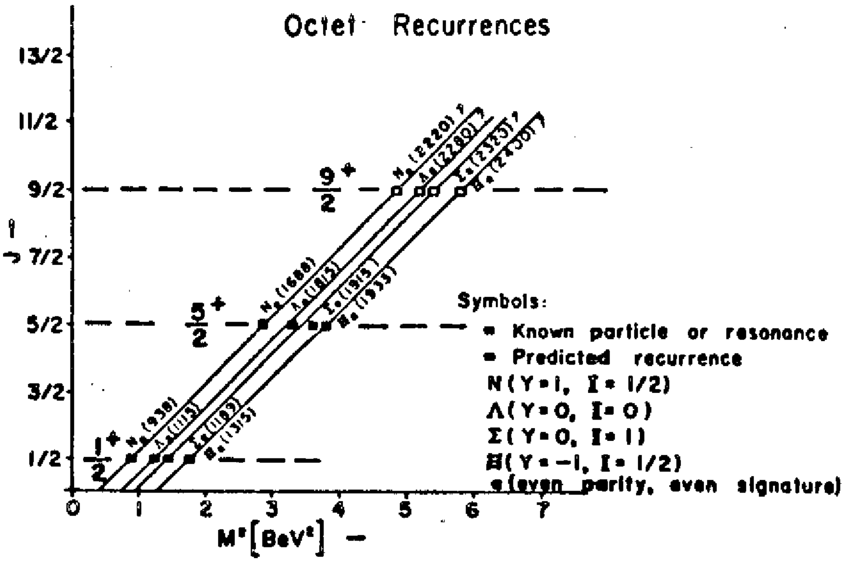
CERN yields from one interaction length of lead at 18.8 GeV/c per 10^{12} protons per pulse and 20 pulses per minute (Ster^{-1} GeV/c^{-1}).

Momentum GeV/c	1	2	3	4	6	8	10	12
π^+ ($\times 10^{11}$)	3.9	3.7	4.0	3.2	3.2	2	0.7	0.4
π^- ($\times 10^{11}$)	3.8	3.2	2.8	1.9	1.2	0.6	0.3	0.1
K^+ ($\times 10^9$)	46	45	48	40	27	22	-	6
K^- ($\times 10^9$)	45	34	27	16	7	3	0.6	0.2

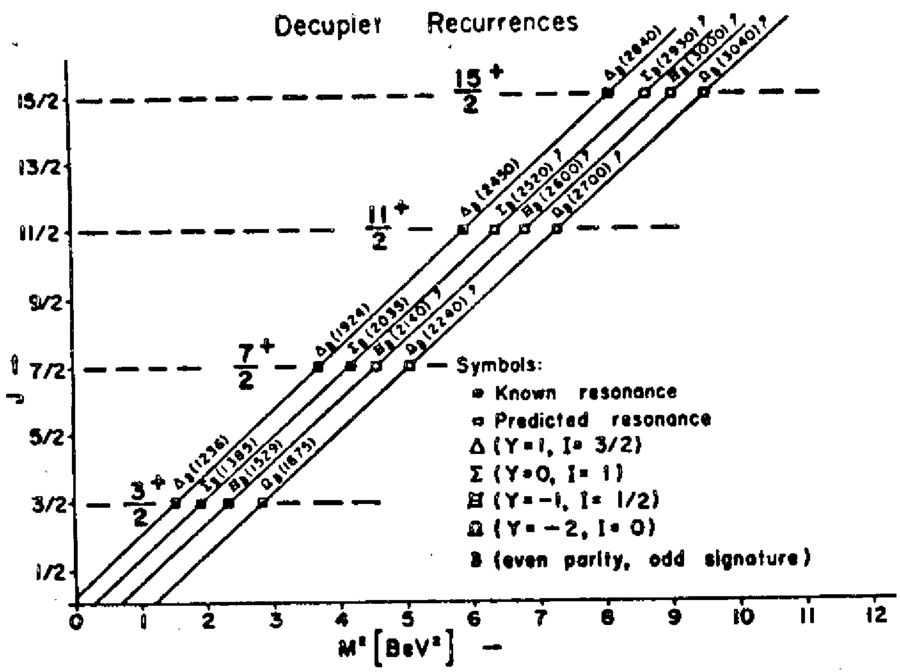
G.H. Stafford

REGGE TRAJECTORY ASSIGNMENTS OF KNOWN PARTICLES

IN $\underset{\sim}{8}$ and $\underset{\sim}{10}$'s OF SU(3)



$$J(M^2) = 0.15 + 0.90\ M^2$$



$$J(M^2) = -0.39 + 1.01\ M^2$$

Fig. 1

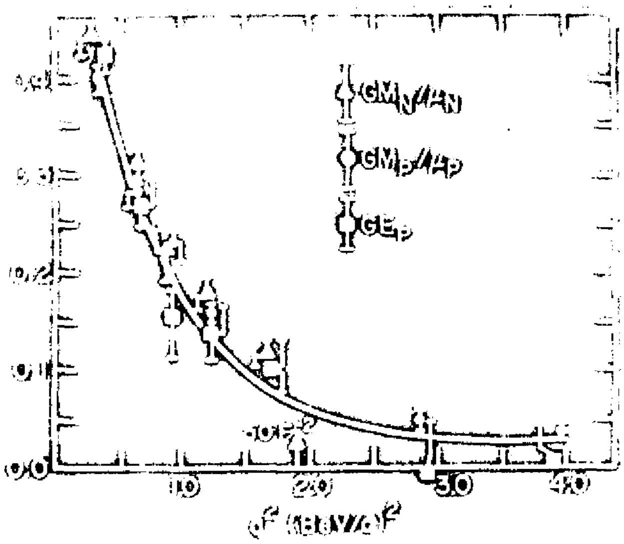


Fig.2

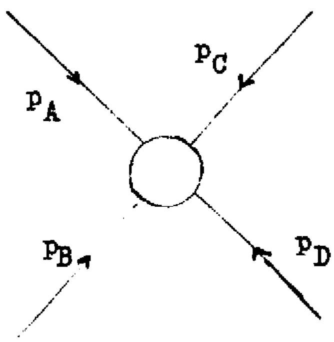


Fig.3

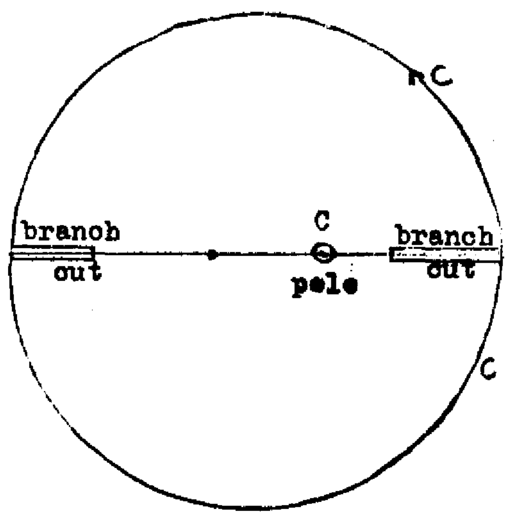


Fig.4