THE $K^4$ FORM FACTORS IN CURRENT ALGEBRA WITH HARD PIONS AND KAON

A. Q. SARKER

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THE K\textsubscript{2} FORM FACTORS IN CURRENT ALGEBRA WITH HARD PIONS AND KAON *

A.Q. Sarker **

International Centre for Theoretical Physics, Trieste, Italy, and
Institute of Physics, University of Islamabad, Rawalpindi, Pakistan

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**Visiting scientist at ICTP, Trieste, June-August 1968
The axial form factors in $K_{l4}$ decays are calculated using the technique of current algebra, hard kaon and pions, the assumption of PCAC and the spectral function sum rules. The assumption of meson-dominance of certain vertex functions and inverse propagators as given by Schnitzer and Weinberg is inherent in such calculations. Taking the $\xi$-parameter from the experimental $K_{l3}$ decays and using the predicted axial form factors at their zero arguments, the decay rates for $K^+ \rightarrow \pi^+ \pi^- e^+ \nu$ and $K^+ \rightarrow \pi^+ \pi^- \mu^+ \nu$ are then calculated. On comparison with their recent observed rates, it is found that the value $\xi_{br} = 0.6$ is strongly favoured as opposed to $\xi_{pol} = -1.0$. The values of the form factors for other $K_{l4}$ decays are also given.
I. INTRODUCTION

One of the early successes of the technique of current algebra, the soft pion limits and the assumption of the partial conservation of the axial-vector current (PCAC) is the calculation of the axial vector form factors in $K_{l4}$ decay by Weinberg $^1$. However, Berends, Donnachie and Oades $^2$ analysed the recent experimental data of the $K_{l4}$ and $K_{l3}$ decays and observed that the results of the current algebra predict for the total decay rates of $K^+ \rightarrow \pi^+ \pi^- e^+ \nu$ and $K^+ \rightarrow \pi^+ \pi^- \mu^+ \nu$ to be too small compared to the experimentally observed rates. One is therefore tempted to conclude that the soft pion calculations are not very reliable. Apart from this, the work of Weinberg admits the following criticism:

1) the kaon and the pions were not treated symmetrically,
2) certain terms were evaluated by taking into account only the kaon pole and in the soft pion limits,
3) extensive use of the Callan-Treiman $^3$ relation was made with the implicit assumption of the constant behaviour of the $K_{l3}$ form factors in their analytic continuation.

Using the technique of hard kaon and pion in context of the algebra of currents, recently given by Schnitzer and Weinberg $^4$, $^5$, we are now able to calculate the $K_{l4}$ form factors in a rigorous way. Along with PCAC, we have also assumed the meson-dominance of certain "proper" vertex functions and inverse propagators which states precisely that these functions and inverse propagators are smooth functions of relevant momenta. By taking advantage of the various Ward-type identities for the vertex and four-point functions, we can calculate these three- and four-point functions rigorously and in better approximations. The Schwinger terms are automatically taken care of through the Weinberg $^6$ first sum rules. Further, in the present technique we obtain also the $s,t,u$ dependence of the $K_{l4}$ form factors. Using these form factors at their zero momentum transfer values, we then calculate the total decay rates of the various $K_{l4}$ decays. Comparing these predictions with the recent experimentally observed rates $^2$, $^6$ of $K^+ \rightarrow \pi^+ \pi^- e^+ \nu$ and $K^+ \rightarrow \pi^+ \pi^- \mu^+ \nu$ we find that the parameter $\zeta_{br} = 0.6$ of $K_{l3}$ decay, obtained from the
branching ratio, $K^y/K^z$, measurements, is strongly favoured. On the other hand, the value $\xi_{pol} = -1.0$, obtained from the polarization measurements in $K^-\mu^3$ decay, predicts rates which are too small for both $K^+ \rightarrow \pi^+\pi^-e^+\nu$ and $K^+ \rightarrow \pi^+\pi^-\mu^+\nu$ rates. We also predict the values of the axial form factors for the $K^+ \rightarrow \pi^0\pi^0\ell^+\nu$ decays.

A few remarks about the vector form factor in $K_{\ell 4}$ decay are in order. MAHAMT and MARSHAK used the current algebraic technique to relate the vector form factor to the known $\pi^0 \rightarrow 2\gamma$ decay and found that the vector contribution to the $K_{\ell 4}$ decay rates is indeed small, contrary to the assertion of Berends, Donnachie and Oades, who tried to fit the observed decay rates of $K_{\ell 4}$ with a large value of the vector form factor in addition to the axial form factors, as given by Weinberg. It is heartening to observe that we have been able to predict the observed $K^+ \rightarrow \pi^+\pi^-e^+\nu$ and $K^+ \rightarrow \pi^+\pi^-\mu^+\nu$ decay rates (within errors) by using the axial form factors predicted in this paper and taking the value of the $\xi$ parameter to be $\xi_{br} = 0.6$ and without any large value of the vector form factor. In the following paper, we shall calculate the vector form factors using the present technique and relevant remarks concerning the magnitude of the vector form factor contributions in $K_{\ell 4}$ decays will be made there.

In the present calculations we shall neglect the $\sigma$-type terms to simplify the algebra to a considerable extent. We shall come to this point again in the conclusion and shall try to justify neglecting the $\sigma$-terms a posteriori. The kappa-term is quite small as discussed by RIAZUDDIN and SARKER and we shall neglect it without any further comment.

We briefly mention the various other approaches for the calculations of the $K_{\ell 4}$ form factors. CLAVELLI essentially showed that all the current algebraic results of the $K_{\ell 4}$ form factors in the zero mass limits of the pions can be reproduced from the assumption of vector-dominance of the form factors with gauge invariance at the vertices. In the present work we observe that all terms which should correspond to those of the vector-dominance method are cancelled out in the final expressions of the $K_{\ell 4}$ form factors. So the present work seems to rule out any relevance of the assumption of the vector-dominance hypothesis to the $K_{\ell 4}$ form factors. In a recent paper BERMAN and ROY criticised the
calculations of Weinberg and argued that the procedure of taking simultaneously both the pions in the soft limits is inconsistent with the non-vanishing of the contribution of the \( \sigma \)-type term. Their criticism may be a valid one in the soft pion limits, but in the present hard kaon and pion calculations we do find explicit kaon pole contributions to the form factor \( F_3 \) (which is the relevant quantity in the present context) as found by Weinberg in the soft pion technique and it seems to have no apparent connection with the vanishing or non-vanishing of the contribution of the \( \sigma \)-type term. On the possibility of extracting the \( \Pi-\Pi \) phase shifts from the \( K_{14} \) decay (and the earlier references) the reader is referred to a recent paper by Pais and Treiman 11).

The plan of the paper is as follows: in the next section we write down the definitions we are going to use throughout the present work. The main body of the calculations are presented in Secs.III,IV and V. In Sec.VI we give the results for the \( K_{14} \) form factors. In Sec.VII we briefly show how to recover the results of the soft pion calculations from our general method. A few more remarks are then included in the conclusion.

II. DEFINITIONS

a) Kinematics and \( \Delta I = \frac{1}{2} \) Rules

We consider the processes

\[
\begin{align*}
(A) \quad K^+(k) & \rightarrow \pi^- (p) + \pi^0 (q) + l^+ (p_L) + \gamma (p_\gamma) \\
(B) \quad K^+(k) & \rightarrow \pi^0 (p) + \pi^0 (q) + l^+ + \gamma \\
(C) \quad K^0 (k) & \rightarrow \pi^- (p) + \pi^0 (q) + l^+ + \gamma
\end{align*}
\]

where \( l^+ \) denotes a positron or muon. The four-momenta of the kaon, the first and the second pions are denoted by \( k, p \) and \( q \), respectively, and those of the leptons by \( p_L \) and \( p_\gamma \). We introduce the following notations for suitable linear combinations of these momenta:
\[ l = - (K + P + q) = (P_X + P_Y) \]  

\[ l_1 = p + q, \quad l_2 = K + P, \quad l_3 = K + q, \]  

\[ s = - (l_1)^2 = - (p + q)^2 \]  

\[ t = - (l_2)^2 = - (K + P)^2 \]  

\[ u = - (l_3)^2 = - (K + q)^2. \]

From (2.2) and the total energy-momentum conservation we have

\[ s + t + u = m_K^2 + 2m_P^2 - \mathcal{L}^2. \]

It is sometimes useful to introduce the following variable:

\[ \eta = \frac{1}{2} (t - u) = \mathcal{L} (p - q). \]

The set, s, t and u form a complete set of independent variables for describing the form factors of the processes (A), (B) and (C). It is sometimes also convenient to use the set, s, t and \( \mathcal{L}^2 \). It is to be noted that the kaon momentum has been considered in (2.1) and (2.2) with a negative sign, so that the kaon is outgoing in the processes, (A), (B) and (C).

We can write down the matrix element for the process (A), assuming a local V-A coupling for the leptons and to first order in weak interaction as

\[
(2\pi)^4 \, \delta^4 (K + P + q + l) \, \frac{G}{\sqrt{2}} \left[ \sum_{\alpha} \, \bar{V}_{\alpha} (1 + \gamma_5) \gamma_\alpha \right] \, \left< K^+(K), \pi^+(P), \pi^+(q) \mid V_{\alpha}^d(0) + A_{\alpha}^d(0) \mid 0 \right>
\]

where \( G \) is the universal Fermi coupling and \( V_{\alpha}^d(0) \) and \( A_{\alpha}^d(0) \) are the
$\Delta Y = -1$ changing weak vector and axial-vector currents. In the present paper we shall be concerned with only the axial-vector part $A^d_{\tau}$ of the full current. On invariance grounds the $K_{\ell 4}$ form factors are defined by

$$T_{\tau} = i (2\pi)^{3/2} (8 R_0 p_{\ell} q_{\rho})^{1/2} \langle K^+(p), \pi^-(p), \pi^+(q_\rho) | A^d_{\tau}(0) | 0 \rangle$$

where the form factors $F_i$ are functions of $q_{\rho}$ only. Similarly, for the processes (B) and (C) the form factors will be denoted by $F_i^{(B)}$ and $F_i^{(C)}$, respectively. From the $\Delta I = \frac{1}{2}$ rule we have one relation between $F_i^{(A)}$, $F_i^{(B)}$ and $F_i^{(C)}$:

$$F_i^{(A)} = F_i^{(B)} + \frac{1}{\sqrt{2}} F_i^{(C)}$$

Using then simple isotopic spin rotation one gets:

$$\langle 2\pi \rangle^3 (4 p \cdot q_\rho)^{1/2} \langle \pi^-(p), \pi^+(q_\rho) | \nu^{\pi}_{\lambda}(0) | 0 \rangle$$

$$= \int_1 p^\pi (s) (q_\rho - p) \lambda$$

Using then simple isotopic spin rotation one gets:
As for the $K^0$ form factors, we have the following normalizations:

$$ (2\pi)^3 (4 k \cdot p) \nu \langle K^+(p), \pi^0(q) | V^{\nu_{1\nu}}_\lambda | 0 \rangle $$

$$ = \sqrt{2} f_+^\nu (s) (p - q)_\lambda $$

As for the $K_L^*$ form factors, we have the following normalizations:

$$ (2\pi)^3 (4 k \cdot p) \nu \langle K^0(p), \pi^0(q) | V^{\nu_{1\nu}}_\lambda | 0 \rangle $$

$$ = \frac{i}{\sqrt{2}} \left[ f_+ (t) (p - k)_\lambda - f_- (t) (p + k)_\lambda \right] $$

$$ (2\pi)^3 (4 k \cdot p) \nu \langle K^0(p), \pi^0(q) | V^{\nu_{1\nu}}_\lambda | 0 \rangle $$

$$ = \left[ f_+ (u) (q - k)_\lambda - f_- (u) (q + k)_\lambda \right] $$

The form factors $f_+^\nu (0)$ and $f_-^\nu (0)$ are normalized to unity, while $f_-^\nu (0)$ is zero in the exact SU(3) limit. The form factor $f_+^\nu (0)$ is also unity up to the first-order SU(3)-violating effect due to the ADEMOLLO-GATTO theorem.

b) "Proper" three- and four-point functions

We shall use the assumption of the partial conservation of the axial-vector current in the form

$$ \partial_y A_y^a (x) = \frac{f_a}{\sqrt{2}} m_a^2 \phi_a (x), \quad (\text{no summation over } a) $$

where $f_a$ is the decay constant of the pseudoscalar meson. We use the latin superscript for the SU(3) indices and the greek subscripts for the space-time vector indices. We then have

$$ \left\langle 0 \right| T \left\{ \partial_y A_y^a (x), A_y^b (0) \right\} \right| 0 \rangle $$

$$ = - \frac{\delta^{ab}}{2} \frac{f_a^2 m_a^2}{(p^2 + m_a^2)} $$

$$ = \frac{\delta^{ab}}{2} \frac{f_a^2 m_a^2}{(p^2 + m_a^2)} $$

$$ (2.14) $$
We also define the covariant spin-1 part of the unrenormalized vector and axial vector propagators

\[ \Delta_{\mu
u}^V(p) = \int d^4\mu^2 \frac{g_{\nu\nu}(\mu^2)}{(p^2 + \mu^2)^2} \times \left[ \delta_{\mu\nu} + \frac{p_\mu p_\nu}{\mu^2} \right], \]  
\[ (2.16) \]

\[ \langle V_{\mu}^a(x), V_{\nu}^b(0) \rangle_0 = \frac{\delta^{ab}}{2} (2\pi)^{-3} \int d^4p \theta(p) \times \]  
\[ e^{-ip\cdot x} p_{\nu}(p^2) \left[ \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right], \]  
\[ (2.17) \]

\[ \Delta_{\mu
u}^A(p) = \int d^4\mu^2 \frac{g_{\nu\nu}(\mu^2)}{(p^2 + \mu^2)^2} \times \left[ \delta_{\mu\nu} + \frac{p_\mu p_\nu}{\mu^2} \right], \]  
\[ (2.18) \]

\[ \langle A_{\mu}^a(x), A_{\nu}^b(0) \rangle_0 = \frac{\delta^{ab}}{2} (2\pi)^{-3} \int d^4p \theta(p) \times \]  
\[ e^{-ip\cdot x} p_{\nu}(p^2) \left[ \delta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right]. \]  
\[ (2.19) \]

We then have 4)

\[ \int d^4x e^{-ip\cdot x} \langle T \{ V_{\mu}^a(x), V_{\nu}^b(0) \} \rangle_0 = -\frac{i\delta^{ab}}{2} \Delta_{\mu\nu}^V(p) \]  
\[ = -i \frac{\delta^{ab}}{2} \left[ \Delta_{\mu\nu}^V(p) - C_{\mu}^a \eta_\mu \eta_\nu \right], \]  
\[ (2.20) \]
\[ \int d^{4}x \left\langle T\{ A_{\mu}^{a}(x), A_{\nu}^{b}(0) \} \right\rangle = -i \frac{\delta^{ab}}{2} \Delta_{\mu \nu}(p) = -i \frac{\delta^{ab}}{2} \left[ \Delta_{\mu \nu}(p) \right. \\
+ \frac{p_{\mu}p_{\nu}f_{a}}{(p^{2} + m_{a}^{2})} - \eta_{\mu} \eta_{\nu} (C_{\mu}^{a} + f_{a}^{2}) \right], \] (2.21a)

where
\[ C_{\nu} = \int d\kappa^{2} \mu^{-2} \rho_{\nu}(\kappa^{2}), \] (2.21b)
\[ C_{\lambda} = \int d\kappa^{2} \mu^{-2} \rho_{\lambda}(\kappa^{2}), \] (2.21c)

and
\[ \eta_{\mu} = \{ 0, 0, 0, 1 \}. \] (2.21d)

We also have
\[ p_{\mu} \Delta_{\mu \nu}(p) = C_{\nu, \lambda} p_{\lambda}. \] (2.22)

We now define certain "proper" vertex and four-point functions of vector and axial-vector currents by subtracting out the $0^-$ pole contributions in all channels from the usual vacuum expectation values of the time-ordered products of these currents. The procedure is as follows: whenever there is an axial-vector current $A_{\lambda}^{a}$ in the time-ordered product, we define the "proper" vertex and four-point functions by replacing $A_{\lambda}^{a}$ by $A_{\lambda}^{a} - \eta_{\mu} m_{a}^{-2} q_{\mu} \partial_{\lambda} \Gamma_{\lambda}^{a}$. We illustrate it by the following examples which in turn also explain the other notations to be used later on.

\[ N_{\lambda}^{abc} = \int d^{4}x d^{4}y e^{-i(k \cdot x + p \cdot y)} \left\langle T\{ \partial_{\mu} A_{\mu}^{a}(x), \partial_{\nu} A_{\nu}^{b}(y), V_{\lambda}^{c}(0) \} \right\rangle, \]
\[ = \frac{i f_{a} f_{b} m_{a}^{2} m_{b}^{2}}{2(k^{2} + m_{a}^{2})(p^{2} + m_{b}^{2})} \frac{g_{V}^{-1}}{\sqrt{2}} \Delta_{\lambda \nu}(l_{2}) \Gamma_{\nu}^{a} \Gamma_{\lambda}^{b} \Gamma_{\nu}^{c} (k, p). \] (2.23)
\[ N_{\gamma \lambda} = \int d^4x d^4y e^{-i(k \cdot x + \mu \cdot y)} \left\langle T \{ \partial_{\mu} A_{\mu}^a(x), A_{\nu}^b(y), V_\lambda^c(0) \} \right\rangle = \]
\[ \frac{f_\alpha f_\beta f_\gamma}{2 \sqrt{2} (k^2 + m_\alpha^2)} g_\gamma^{-1} g_\lambda^{-1} \Delta_{\nu \beta}^{(\gamma)}(p) \Delta^{(\lambda)}(l_2) \Gamma_{\gamma \lambda}^{abc}(R, p) \]
\[ + \frac{f_\alpha f_\beta f_\gamma f_\delta}{2 \sqrt{2} (k^2 + m_\alpha^2)(p^2 + m_\delta^2)} g_\gamma^{-1} \Delta^{(\lambda)}(l_2) \Gamma^{abc}(R, p). \]

(2.24)

where \( g_\lambda \) and \( g_\nu \) are the coefficients of \( \delta^2(\mu^2 - m_\lambda^2) \) and \( \delta^2(\mu^2 - m_\nu^2) \)
in \( \rho_\lambda(\mu^2) \) and \( \rho_\nu(\mu^2) \) defined by (2.21c) and (2.21b), respectively. The corresponding four-point functions of \( K_L^4 \) decay will be denoted by \( M^{ab, cd}_T \), \( \Pi^{ab, cd}_T \), etc.

\[ M^{ab, cd}_T = \int d^4x d^4y d^4z e^{-i(k \cdot x + p \cdot y + q \cdot z)} \]
\[ \times \left\langle T \{ \partial_{\mu} A_{\mu}^a(x), \partial_{\nu} A_{\nu}^b(y), \partial_{\lambda} A_{\lambda}^c(z), A_{\alpha}^{d}(0) \} \right\rangle = \]
\[ \frac{f_\alpha f_\beta f_\gamma f_\delta}{4 (k^2 + m_\alpha^2)(p^2 + m_\beta^2)(q^2 + m_\delta^2)} g_\alpha^{-1} \Delta^{(\alpha \beta \gamma \delta)}(l) \Pi^{abcd}_T \eta^{(R, p, q)}, \]

(2.25)

\[ M^{ab, cd}_\lambda = \int d^4x d^4y d^4z e^{-i(k \cdot x + p \cdot y + q \cdot z)} \]
\[ \times \left\langle T \{ \partial_{\mu} A_{\mu}^a(x), \partial_{\nu} A_{\nu}^b(y), A_{\lambda}^c(z), A_{\alpha}^{d}(0) \} \right\rangle \]
One can now write down easily the similar definitions for $\Pi_{\epsilon \eta}^{ab\delta}$, etc., $\Pi_{\epsilon \eta}^{ab\delta} (k,p,q)$ etc., and $\Pi_{\epsilon \eta}^{ab\delta} (k,p,q)$. Since some of these expressions are quite lengthy, we shall not write them down explicitly, although we shall have opportunities to use them.

III. FORMALISM

We begin with the following identity for the time-ordered product of four (axial-vector) currents:

$$
\begin{align*}
&= \frac{f_a f_b f_c m_a^2 m_b^2}{4 (k^2 + m_a^2) (p^2 + m_b^2)} g_{A A}^{-1} \Delta_{\lambda\gamma}(q) \Delta_{\epsilon\eta}(u) \Pi_{\epsilon \eta}^{ab\delta} (k,p,q) \\
&\quad + \frac{f_a f_b f_c m_a^2 m_b^2}{4 (k^2 + m_a^2) (p^2 + m_b^2)} g_{A A}^{-1} \Delta_{\lambda\gamma}(q) \Delta_{\epsilon\eta}(u) \Pi_{\epsilon \eta}^{ab\delta} (k,p,q).
\end{align*}

(2.26)

One can now write down easily the similar definitions for $\Pi_{ab\delta}^{ab\delta}$, etc., $\Pi_{ab\delta}^{ab\delta} (k,p,q)$ etc., and $\Pi_{ab\delta}^{ab\delta} (k,p,q)$. Since some of these expressions are quite lengthy, we shall not write them down explicitly, although we shall have opportunities to use them.

$$
\begin{align*}
&= \frac{\partial}{\partial x_{\lambda}} \frac{\partial}{\partial y_{\gamma}} \frac{\partial}{\partial y_{\delta}} \{ A_{a}^{\mu}(x), A_{b}^{\nu}(y), A_{c}^{\rho}(z), A_{R}^{0}(0) \} \\
&\quad - [\delta(x_{0}) \{ A_{a}^{\mu}(x), A_{R}^{0}(0) \}, \partial_{\gamma} A_{b}^{\nu}(y), \partial_{\delta} A_{c}^{\rho}(z)] \\
&\quad + \left\{ \frac{1}{2} \delta(y_{0}) \delta(z_{0}) \{ A_{a}^{\mu}(x), A_{b}^{\mu}(y), A_{c}^{\nu}(z), A_{R}^{0}(0) \}, \partial_{\lambda} A_{a}^{\mu}(x) \right\}
\end{align*}

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The $\sigma$-terms, which are grouped together at the end of the identity (3.1), will be neglected throughout the rest of the calculations.

For the process (A), we take $a = 4 - i5$, $b = 1 + i2$, $c = 1 - i2$ and $d = 4 + i5$. The notation is such that $V_\Delta^{4+15}(0)$ denotes $V_\Delta(0) + iV_\Delta(0)$, and similarly for others. We then contract the kaon and two pions in (2.6) and make use of the PCAC in the form (2.13) and the identity (3.1) and then, using the well-known equal-time current commutation relations as given by Gell-Mann, obtain $(e = b + i7$, $h = b - 17)$
\[
\frac{f_{\pi} f_{\pi}^2 m_{\pi}^2 m_{\pi}^4}{(k^2 + m_{\pi}^2)(q^2 + m_{\pi}^2)} \ T^{(\Lambda)}
\]

\[= i K_{\mu} P_{\nu} \gamma_{\lambda} \Gamma_{\mu \nu \lambda \tau}^{abcde} (k, p, q) \]

\[- K_{\mu} \left\{ \frac{1}{6} \tilde{\Delta}^{A(k)}_{\mu \tau} (r) + \frac{1}{3} \tilde{\Delta}^{A(k)}_{\mu \tau} (l) - \frac{1}{2} \tilde{\Delta}^{A(k)}_{\mu \tau} (l_2) \right\} \]

\[- \gamma_{\lambda} \left\{ \frac{1}{6} \tilde{\Delta}^{A(\pi)}_{\lambda \tau} (q) + \frac{1}{3} \tilde{\Delta}^{A(k)}_{\lambda \tau} (l) - \frac{1}{2} \tilde{\Delta}^{A(\pi)}_{\lambda \tau} (l_1) \right\} \]

\[+ P_{\nu} \left\{ \frac{1}{3} \tilde{\Delta}^{A(\pi)}_{\tau \nu} (p) + \frac{2}{3} \tilde{\Delta}^{A(k)}_{\tau \nu} (l) \right. \]

\[- \frac{1}{2} \tilde{\Delta}^{A(\pi)}_{\tau \nu} (l_1) - \frac{1}{2} \tilde{\Delta}^{A(k)}_{\tau \nu} (l_2) \left\} \right. \]

\[- \frac{1}{3} \left[ \frac{2 f_{\pi}^2 m_{\pi}^2}{(p^2 + m_{\pi}^2)} P_{\tau} - \frac{f_{\pi}^2 m_{\pi}^2}{(q^2 + m_{\pi}^2)} q_{\tau} - \frac{f_{k}^2 m_{k}^2}{(k^2 + m_{k}^2)} K_{\tau} \right] \]

\[+ \frac{i f_{\pi}^2 m_{\pi}^4}{\sqrt{2} (p^2 + m_{\pi}^2)(q^2 + m_{\pi}^2)} \ g_p^{-1} \Delta^{(\pi)}_{\tau \eta} (l_1) \Gamma_{\eta}^{bcde} (p, q) \]

\[- \frac{i f_{k} f_{\pi} m_{k}^2 m_{\pi}^2}{(k^2 + m_{k}^2)(p^2 + m_{\pi}^2)} \ g_{k}^{-1} \Delta^{(k)}_{\tau \eta} (l_2) \Gamma_{\eta}^{abce} (k, p) \]

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where we have defined

\[
\int d^4x \, d^4y \, e^{\frac{i}{\hbar} (k \cdot x + l \cdot y)} \langle T \{ \partial_\mu A^a_\mu (x), A^d_\tau (0), V^3_\nu (y) \} \rangle_0
\]

\[
= \frac{1}{\sqrt{2}} \Delta_{\nu \beta} (l_1) \frac{f_{K} m_{K}^2}{(k^2 + m_{K}^2)} \left[ f^{a d} \Delta_{\tau \eta}^{A(k)} (l_1) \right]
\]

\[
\Gamma^{(0)}_{\beta \eta} (k, l_1) = \frac{f_{K} l_{c}}{l_{c}^2 + m_{K}^2} \Gamma^{a d}_{\beta \eta} (k, l_1)
\]
The various terms in (3.2) have singularities with the kaon and the pions on the mass shell.

Substituting the definition of $M_{\mu \nu \lambda \kappa} (k, p, q)$ in terms of the "proper" four-point functions and of $\Delta^A_{\mu \nu}$ and $\Delta^A_{\kappa \eta}$, etc., we equate the coefficients of each order of these singularities from both sides of (3.2). This gives us a set of eight equations and after eliminating the terms $\prod_{\delta \eta}$, $\prod_{\beta \eta}$, etc., from them, we obtain

\[ T^{(A)} = -i \oint_{k_A} \Delta^A_{\kappa \eta} (l) \prod_{\eta}^{abcd} (k, p, q) \]

\[ - i f_k f_{\pi} g_{k_A}^{-1} \Delta^A_{\eta \eta} (l) \prod_{\eta}^{abcd} (k, p, q) = \]

\[ i g_{A_1}^{-2} g_{k_A}^{-2} C_A \pi C_A \pi C_A \pi k_{\kappa} p_{\eta} q_{\lambda} \Delta^A_{\kappa \eta} (l) \prod_{\mu \nu \lambda \eta} (k, p, q) \]

\[ + \frac{1}{3} \left[ f_k f_{\pi} k_{\kappa} + f_{\pi} q_{\eta} - 2 f_{\pi} p_{\kappa} \right] \]
- \kappa x \left[ \frac{1}{6} \Delta^{A(k)}_{\mu \tau}(k) + \frac{1}{3} \Delta^{A(k)}_{\mu \tau}(l) \\
- \frac{1}{2} \Delta^{V(\pi)}_{\mu \tau}(l_2) + \frac{1}{6} \eta_{\mu} \eta_{\tau} \left( C^k_A + f^2_k \right) \right] \\
- q_{\lambda} \left[ \frac{1}{6} \Delta^{A(\pi)}_{\lambda \tau}(q) + \frac{1}{3} \Delta^{A(k)}_{\lambda \tau}(l) \\
- \frac{1}{2} \Delta^{V(\pi)}_{\lambda \tau}(l_1) + \frac{1}{6} \eta_{\lambda} \eta_{\tau} \left( C^\pi_A + f^2_\pi \right) \right] \\
+ p_{\nu} \left[ \frac{1}{3} \Delta^{A(\pi)}_{\nu \tau}(p) + \frac{2}{3} \Delta^{A(k)}_{\nu \tau}(l) - \frac{1}{2} \Delta^{V(\pi)}_{\nu \tau}(l_1) \\
- \frac{1}{2} \Delta^{V(k)}_{\nu \tau}(l_2) + \frac{1}{3} \eta_{\nu} \eta_{\tau} \left( C^\pi_A + f^2_\pi \right) \right] \\
+ \frac{i}{\sqrt{2}} f_{\pi} g^{-1}_{\pi} \Delta_{\tau \eta}(l_1) \Gamma_{\eta}^{bc 3}(p, q) \\
- i f_{\pi} f_k g^{-1}_k \Delta_{\tau \eta}(l_2) \Gamma_{\eta}^{ab e}(k, p) \\
+ \frac{f_k}{\sqrt{2}} g^{-1}_{\pi} g^{-1}_A (q - p) \Delta_{\nu \mu}^{V(\pi)}(l_1) \Delta^{A(k)}_{\nu \eta}(l) \Gamma_{\eta}^{(1)}_{\mu \eta}(k, l_1) \\
+ f_{\pi} g^{-1}_k g^{-1}_A (k - p \mu) \Delta_{\eta \mu}^{V(k)}(l_2) \Delta^{A(k)}_{\eta \eta}(l) \Gamma_{\eta}^{(2)}_{\eta}(q, l_2) \\
+ i \frac{f_{\pi} g^{-1}_{\pi}}{\sqrt{2} (k^2 + m_k^2)} (q - p) \Delta_{\nu \mu}^{V(\pi)}(l_1) \Delta_{\nu \eta}^{A(k)}(l) \Gamma_{\eta}^{ad 3}_{\mu \eta}(k, l_1) \\
-16-
\[-i \frac{f_{\pi} f_K g_{K^*}}{2(x^2 + m_K^2)} (r - p)_\mu \Delta^{\nu(k)}_{\mu \lambda}(l_2) \lambda \tau \Gamma^\mu_{\alpha\beta}(q, l_2).\]

(3.5)

For the process (B) we obtain similarly

\[\frac{1}{2} f_K f_{\pi}^2 T_{(\pi)}^{(B)} = \]

\[\begin{align*}
&+ \frac{i}{2} g_{A_1} g_{K^*} C_A^\pi C_A^\nu C_A^\mu \mathcal{K}_{\mu \nu \lambda \eta}(K, P, q) \Delta^{A(k)}_{\nu \lambda \eta}(l) \Pi_{\alpha \beta \gamma \delta}^{(k)}(K, P, q) \\
&+ \frac{1}{6} \left[ f_K^2 r_{\nu \lambda} - \frac{1}{2} f_{\pi}^2 r_{\nu \lambda} - \frac{i}{2} f_{\pi}^2 q_{\nu \lambda} \right] \\
&- k_{\mu} \left[ \frac{1}{12} \Delta^{A(k)}_{\mu \nu \lambda \eta}(K) + \frac{1}{6} \Delta^{A(k)}_{\nu \lambda \eta \tau}(l) - \frac{1}{8} \Delta^{V(k)}_{\nu \lambda \eta \tau}(l_2) \\
&- \frac{1}{8} \Delta^{V(k)}_{\nu \lambda \eta \tau}(l_3) + \frac{1}{12} \eta_{\mu \nu \lambda \eta \tau}(C_A + f_{\pi}^2) \right] \\
&+ q_{\nu \lambda} \left[ \frac{1}{24} \Delta^{A(\pi)}_{\lambda \nu \eta \tau}(q) + \frac{1}{12} \Delta^{A(k)}_{\lambda \nu \eta \tau}(l) \\
&- \frac{1}{8} \Delta^{V(k)}_{\lambda \nu \eta \tau}(l_3) + \frac{1}{12} \eta_{\lambda \nu \eta \tau}(C_A + f_{\pi}^2) \right]
\]
\[ + P_Y \left[ \frac{1}{2\pi} \Delta_A^{(\pi)} (\lambda) + \frac{1}{2} \Delta_A^{(\kappa)} (\lambda) \right. \\
\left. - \frac{1}{8} \Delta^{V(\kappa)} (\lambda_2) + \frac{1}{24} \eta_y \eta_\tau (C_A^\pi + f_\pi^2) \right] \]

\[ - \frac{i}{2\sqrt{2}} f_\pi f_k g_k^{-1} \left[ \Delta^{V(\kappa)} (\lambda_2) \Gamma_{\eta}^{a_3 d} (\kappa, \rho) \\
+ \Delta^{V(\kappa)} (\lambda_3) \Gamma_{\eta}^{a_3 d} (\kappa, \sigma) \right] \]

\[ - \frac{i f_\pi f_k g_k^{-1} \eta_\tau}{4\sqrt{2} (\lambda^2 + m_k^2)} \left[ (R - P)_\mu \Delta^{V(\kappa)} (\lambda_2) \Gamma_\alpha^{3d a} (R, \lambda_2) \\
+ (R - q)_\nu \Delta^{V(\kappa)} (\lambda_3) \Gamma_\beta^{3d a} (R, \lambda_3) \right] \]

\[ + \frac{1}{4\sqrt{2}} f_\pi g_k^{-1} g_k^{-1} \Delta^{A(\kappa)} (\lambda) \left[ (R - P)_\mu \Delta^{V(\kappa)} (\lambda_2) \\
\Gamma^{(3)}_{\alpha \eta} (q, \lambda_2) + (R - q)_\nu \Delta^{V(\kappa)} (\lambda_3) \Gamma^{(4)}_{\lambda \eta} (P, \lambda_3) \right], \]

where we have defined
and a similar expression with $p$ and $q$ interchanged.

The "proper" three-point functions $\gamma^3_{\mu}$, are related to the $\pi_{l3}$ and $K_{l3}$ form factors. These and the functions $T_{\tilde{\eta}n}$ which satisfy vector constraint equations from the Ward identities will be evaluated in the next section.

If we use the current algebraic result

\[
\frac{\partial^2}{\partial q^2} = \frac{g^2}{m^2_p}, \quad (3.8)
\]

we observe that the Schwinger terms, which are the coefficients of $\gamma^3_{\mu} \gamma^2_v$ in $\gamma^A_{\mu \nu}$, etc., occurring in (3.5) (and similarly also those in (3.6)), are completely cancelled out due to the WEIBERG first sum rules

\[
C^\pi_V = C^\pi_A + f^2, \quad (3.9a)
\]

\[
C^K_V = C^K_A + f^2, \quad (3.9b)
\]

We shall also be using very often the following results from the WEINBERG second sum rules

\[
g_A = g^A_v, \quad g_A^* = g_{K_A}, \quad (3.10)
\]

and the KAWARABAYASHI-SUZUKI-RIAZUDDIN-FAYYAZUDDIN(KSRF) relation

\[
g_A^2 = 2 f^2 \pi m^2_p. \quad (3.11)
\]
IV. THREE-POINT FUNCTIONS

We first evaluate the three-point functions \( \Gamma_{\eta}^{abc}(p, q) \), \( \Gamma_{\eta}^{ab}(k, p) \), \( \Gamma_{\chi}(k, l) \) and \( \Gamma_{\alpha}(q, l) \) and express them in terms of the \( \Pi_{\alpha} \) and \( K_{\alpha} \) form factors.

Using the definitions (2.9)-(2.12) and the Weinberg identity, one can easily find \( \Gamma_{\eta}^{abc}(p, q) \) (with \( \epsilon = \gamma + i\eta, \gamma = 6 - i7 \))

\[
- \frac{i}{\sqrt{2}} \eta p^{-1} \Delta^{v(\eta)}_{\alpha} (l_1) \Gamma^{bc3}_{\eta} (p, q) = f^{\pi}_{+} (s) (q - p)_{\alpha},
\]

\[
- i q^{-1}_{k} \Delta^{v(k)}_{\alpha} (l_2) \Gamma^{abe}_{\eta} (k, p) = \left[ f^{\pi}_{+} (t) (p - k)_{\alpha} - f^{\pi}_{-} (t) (p + k)_{\alpha} \right],
\]

\[
- \frac{i}{\sqrt{2}} q^{-1}_{s} \Delta^{v(\eta)}_{\chi} (l_1) \Gamma^{ad3}_{\chi} (k, l) = - \frac{1}{2} f^{\pi}_{+} (s) (-2 k - p - q)_{\lambda},
\]

\[
- i q^{-1}_{k} \Delta^{v(k)}_{\alpha} (l_2) \Gamma^{cdk}_{\chi} (q, l) = \left[ f^{\pi}_{+} (t) (-2 q - k - p)_{\mu} - f^{\pi}_{-} (t) (-k - q)_{\mu} \right]
\]
We then have from (4.3) and (4.4)

\[
\frac{i f_k^2}{\sqrt{2}} \frac{g_p^{-1}}{(l^2 + m_k^2)} (q - p) \Delta_{\Lambda Y}^{(n)}(l) \ell_{\tau} \Gamma_{\gamma}^{a d 3} (K, l)
\]

\[
= - \frac{f_k^2}{2} \frac{f_{\pi}^0(s)}{(l^2 + m_k^2)} \frac{2 l \cdot (q - p) \ell_{\tau}}{2 (q - p) \ell_{\tau}},
\]

(4.5)

\[
\frac{i f_{\pi} f_k}{2} \frac{g_{K*}^{-1}}{(l^2 + m_k^2)} (K - p) \Delta_{\mu, \lambda}^{(K)}(l, 2) \ell_{\tau} \Gamma_{\lambda}^{C d h} (q, l)
\]

\[
= + \frac{f_{\pi} f_k}{2} \frac{2 l \cdot (2 q, (K - p) + (K^2 - p^2)) f_+(t)}{(l^2 + m_k^2)} \left[ - (K^2 - p^2) f_-(t) \right]
\]

(4.6)

The s, t, dependence of the form factors f_+ and f_- are known and we have in the linear approximations \((4.7, 8)\)

\[
f_+^\Pi(s) = (1 + 0.020 \frac{s}{m_{\Pi}^2})
\]

(4.7)

and

\[
f_+(t) = (1 + 0.017 \frac{t}{m_{\Pi}^2}) \]

(4.8)

\[
f_-(t) = (\xi - 0.005 \frac{t}{m_{\Pi}^2})
\]

(4.9)

One gets from the technique of current algebra with hard pions and kaons \((5)\) \(\xi = 0.05\). The \(K^0_3/3\mu_3\) branching ratio measurements \((19)\) give \(\xi_{BR} = 0.6 \pm 0.3\), while from the polarization experiments one obtains \(\xi_{POL} = -1.0 \pm 0.2\).
In order to determine the vertex functions, \( \Gamma_{\beta \eta}(K, p+q) \) and \( \Gamma_{\alpha \eta}(q, k+p) \), we note the following Ward-type identities for them:

\[
\frac{f_{\pi}}{\sqrt{2}} \Gamma_{\beta \eta}^{(1)}(K; l_1) = - \frac{1}{\sqrt{2}} g_{\pi}^{-1} K^K \, C_\alpha \, \Gamma_{\beta \eta}^{(1)}(K, l) \\
+ \frac{1}{2} g_{\pi} g_{K^\ast} [\Delta_{\beta \eta}^{(1)}(l_1)^{-1} - \Delta_{\beta \eta}^{A(K)}(l)^{-1}] 
\]

(4.10)

\[
\frac{f_{\pi}}{\sqrt{2}} \Gamma_{\alpha \eta}^{(2)}(q, l_2) = - g_{A, 0}^{-1} q_{\alpha \gamma} C_\alpha \, \Gamma_{\alpha \gamma \eta}^{(2)}(q, l) \\
+ \frac{1}{2} g_{K^\ast} g_{K^\ast} [\Delta_{\alpha \eta}^{(2)}(l_2)^{-1} - \Delta_{\alpha \eta}^{A(K)}(l)^{-1}] 
\]

(4.11)

where the vertex functions \( \Gamma_{\alpha \eta} \) and \( \Gamma_{\alpha \gamma \eta} \) satisfy the following vector constraint equations

\[
\sqrt{2} g_{\pi}^{-1} l_{1\gamma} \Delta_{\gamma \beta}^{(1)}(l_1) \, \Gamma_{\beta \eta}^{(1)}(K, l) \\
= g_{K^\ast}^{-1} \frac{1}{2} C_\alpha \, \Gamma_{\beta \eta}^{A(K)}(l) - \Delta_{\beta \eta}^{A(K)}(l)^{-1} 
\]

(4.12)

\[
2 g_{K^\ast}^{-1} l_{2\beta} \Delta_{K \gamma}^{(2)}(l_2) \, \Gamma_{\gamma \eta}^{(2)}(q, l) \\
= g_{A, 0} g_{K^\ast} \, \Gamma_{\gamma \eta}^{A(K)}(l) - \Delta_{\gamma \eta}^{A(K)}(q)^{-1} 
\]

(4.13)

The assumption of meson dominance states \(^4\) that these vertex functions be smooth functions of the momenta, subject to the requirement of the Ward identities (4.12) and (4.13). This means that \( \Gamma_{\alpha \gamma \eta}^{(2)}(q, l) \)
and $T^{(1)}_{\alpha \beta}(k, l)$ are linear in momenta and hence the inverse vector and axial propagators are quadratic in momenta. The forms of the inverse vector and axial vector propagators, subject to the conditions (2.22) can be easily found out:

$$\Delta^{-1}_{\mu \nu} (p) = g^{-2}_{V,A} \{ (m_{V,A}^2 + p^2) \delta_{\mu \nu} - p_\mu p_\nu \} . \quad (4.14)$$

Substituting (4.14) in (4.12) and (4.13), we obtain

$$\Gamma^{(1)}_{\alpha \beta \eta \eta} (k, l) = - \frac{g_p}{\sqrt{2} c_{\nu}} \left[ \delta_{\alpha \eta} (l - k) \delta_{\beta \eta} + \delta_{\beta \eta} k_\eta - \delta_{\beta \eta} l_\eta + \{ \delta_{\beta \eta} l_\eta \delta_{\alpha \eta} - \delta_{\beta \eta} \delta_{\alpha \eta} \} (2 + \delta_{\lambda \lambda}) \right], \quad (4.15a)$$

$$\Gamma^{(2)}_{\alpha \beta \eta \eta} (q, l) = \frac{1}{\alpha} \frac{g_k^*}{c_{\nu}} \left( \frac{g_{A_1}}{\partial_{A_1}} + \frac{g_{K_A}}{\partial_{K_A}} \right) \left[ \delta_{\alpha \eta} (l - q) \right] + \delta_{\alpha \eta} q_\eta - \delta_{\alpha \eta} l_\eta + \{ \delta_{\alpha \eta} l_\eta - \delta_{\alpha \eta} l_\eta \} (2 + \delta_{\lambda \lambda}), \quad (4.15b)$$

where $\delta_A$ and $\delta_K$ are the parameters which occur in the $\rho \rightarrow \pi \pi$, $A_1 \rightarrow \rho \pi$ and $K^* \rightarrow K \pi$, $K_A \rightarrow K^* \pi$ systems respectively. If one now uses the expressions (4.15a) and (4.15b) in (4.10) and (4.11), respectively, one easily obtains

$$f_k (q - p) \lambda \frac{1}{\sqrt{2}} g^{-1}_p \Delta^{V(\pi)}_{\lambda \lambda} (l, l) \Delta^{-1}_{A(k)} (l) \Gamma^{(1)}_{\alpha \beta \eta \eta} (k, l)$$

$$= + \frac{1}{2} (q - p) \lambda \left[ \Delta^{A(k)}_{\lambda \lambda} (l) - \Delta^{V(\pi)}_{\lambda \lambda} (l) \right]$$

$$- \frac{1}{\sqrt{2}} g^{-2}_p g^{-1}_k c_{A_1} \left( q - p \right) \lambda \Delta^{V(\pi)}_{\lambda \lambda} (l) \Delta^{A(k)}_{\lambda \lambda} \Gamma^{(1)}_{\alpha \beta \eta \eta} (k, l)$$

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\[ \frac{1}{2} (q - p)_\lambda \left[ \Delta_{\lambda \tau}^{A(k)}(\ell) - \Delta_{\lambda \tau}^{V(\pi)}(\ell_1) \right] \]

\[ + \frac{1}{2} g_\rho^2 C_A^k (C_\pi^\rho)^{-1} \kappa_{\lambda \gamma} (q - p)_\lambda \left( \delta_{\lambda \gamma} + \ell_{\lambda \gamma} \ell_{1 \gamma} / m_\rho^2 \right) \]

\[ (\delta_{\tau \eta} + \ell_{\tau \eta} / m_{K_A}^2) (\ell_{1 \tau}^2 + m_\rho^2)^{-1} (\ell_{1 \eta}^2 + m_\rho^2)^{-1} \]

\[ \left[ \delta_{\lambda \eta} (\ell - k)_\gamma + \delta_{\lambda \eta} \kappa_{\eta} - \delta_{\tau \eta} \ell_\lambda - \{ \delta_{\tau \eta} \ell_{1 \alpha} \right. \]

\[ - \delta_{\lambda \eta} \ell_{1 \eta} \right] (2 + \delta_A) \]  

(4.16)

\[ \int (R - p)_\mu g_{K^*}^{-1} \Delta_{\mu \alpha}^{V(k)}(\ell_2) g_{K_A}^{-1} \Delta_{\lambda \tau}^{A(k)}(\ell) \Gamma^{(2)}_{\alpha \lambda}(q, \ell) \]

\[ = \frac{1}{2} (R - p)_\mu \left[ \Delta_{\lambda \tau}^{A(k)}(\ell) - \Delta_{\lambda \tau}^{V(k)}(\ell_2) \right] \]

\[ - q_{\lambda \gamma} (R - p)_\mu g_{A_1}^{-1} g_{K^*}^{-1} g_{K_A}^{-1} C_\pi^A \Delta_{\mu \alpha}^{V(k)}(\ell_2) \]

\[ \Delta_{\lambda \tau}^{A(k)}(\ell) \Gamma^{(2)}_{\alpha \lambda \eta}(q, \ell) \]
\[ \begin{align*}
&= \frac{1}{2} (R - p)_\mu \left[ \Delta^{A(k)}_{\mu \tau} (l_2) - \Delta^{V(k)}_{\mu \tau} (l_2) \right] \\
&\quad + \frac{1}{4} g^{-1}_{k^*} g_{kA} g^{2 v} \left( \frac{g_{A_1}}{g_{kA}} + \frac{g_{kA}}{g_{A_1}} \right) C^{v}_{A} (C^{v})^{-1} \phi_{\tau} \\
&\quad (R - p)_\mu \left( \delta_{\mu \alpha} + l_2 \delta_{2 \alpha/m^2_{k^*}} \right) \left( \delta_{\gamma \eta} + l_2 \delta_{\eta/m^2_{kA}} \right) \\
&\quad (l_2^2 + m^2_{k^*})^{-1} (l^2 + m^2_{kA})^{-1} \left[ \delta_{\gamma \eta} (l - q)_\alpha \\
&\quad + \delta_{\alpha \gamma} \phi_{\eta} - \delta_{\alpha \eta} \phi_{\gamma} - \left\{ \delta_{\alpha \gamma} \phi_{l_2 \gamma} - \delta_{\alpha \gamma} \phi_{l_2 \eta} \right\} \right] \\
&\quad \times (2 + \delta_{k}) \right].
\end{align*} \]

(4.17)

For the process (B), we obtain similarly

\[-i g^{-1}_{k^*} \Delta^{V(k)}_{\tau \eta} (l_2) \Gamma^{3 \alpha}_{\tau \eta} (k, p) \]

\[\begin{align*}
&= \frac{1}{\sqrt{2}} \left[ f_+ (t) (p - k)_\tau - f_- (t) (p + k)_\tau \right] \\
&\quad - i g^{-1}_{k^*} \Delta^{V(k)}_{\mu \alpha} (l_2) \Gamma^{3 \alpha}_{\tau \eta} (q, l_2) \\
&= \frac{1}{\sqrt{2}} \left[ f_+ (t) (-2q - k - p)_\tau = f_- (t) (-k - p)_\tau \right].
\end{align*} \]

(4.18)

(4.19)
\[
\begin{align*}
&\frac{f_\pi}{g_\pi} (k-p)_\mu g_{K*}^{-1} \Delta^{V(k)}_{\mu \alpha} (l_2) g_{K^*}^{-1} \Delta^{A(k)}_{\alpha \eta} (l) \Gamma^{(3)}_{\rho \eta} (q, l_2) \\
&= -\frac{1}{\sqrt{2}} (k-p)_\mu \left[ \Delta^{V(k)}_{\mu \alpha} (l_2) - \Delta^{A(k)}_{\alpha \eta} (l) \right] \\
&+ \frac{f_\pi^2}{2\sqrt{2}} \frac{m_p^2 + m_{K*}^2}{m_p^2} \frac{m_{K*}^2}{l_2^2 + m_{K*}^2} \frac{1}{l_2^2 + m_{K^*}^2} \left[ l \cdot (k-p) q_\rho ight] \\
&- l \cdot q_\rho (k-p)_\tau - 2 \left\{ (k-q)_\rho R_{\rho \eta} - (p-q)_\rho R_{\rho \eta} \right\} (2+\delta K) \\
&+ \frac{k^2 - p^2}{m_{K*}^2} \left\{ (k+p) \cdot q_\rho - (q, p) \cdot q_\rho \right\} \\
&+ \frac{l_\tau}{m_{K^*}^2} \left\{ (k^2 - p^2) (q \cdot l_2) - q_\mu (k-p) l_2^2 \right\} (2+\delta K) \\
\end{align*}
\]

(4.20)

and expressions similar to (4.18)-(4.20) having p and q interchanged.

Before we end this section we make the following observation. The expressions (4.5)-(4.6) are the kaon pole term taken into account by Weinberg in the soft pion limits as well. We now show that the sum of the pole term \(\Gamma^{p \eta}_\beta\) and \(\Gamma^{p \eta}_\beta\) given by (4.16), satisfies a simpler vector constraint equation. It is easy to see that the "proper" vertex function \(\Gamma^{p \eta}_\beta\) also satisfies the following identity:

\[
\frac{1}{2} \frac{g_{K^*}^{-1}}{g_{K^*}} \Delta^{A(k)}_{\eta \eta} (l) \Gamma^{(3)}_{\rho \eta} (k, l_2) = -\frac{1}{2} f_\pi^2 \Gamma^{(3)}_{\rho \eta} (4.21)
\]

One can show (after some algebra) that the solution of \(\Gamma^{p \eta}_\beta (k, l_2)\) given by (4.16) does satisfy (4.21) due to the Weinberg sum rules (3.9) and (3.10). Multiplying (4.3) by \((p+q)_\beta\) and adding it to (4.21) we obtain
\[
\frac{1}{\sqrt{2}} \partial_\mu \partial_\nu \Delta(H) \Delta^{(\mu)}_{\sigma}(l) \gamma^{(\nu)}_{\gamma}(k, l) + \frac{i \ell \tau f_\pi}{\ell^2 + m_\pi^2}
\]

\[
\Delta^{\gamma \beta}_{\gamma \beta}(l) \gamma^{(\alpha \delta \beta)}_{\gamma \beta}(k, l) = \frac{1}{2} f_\pi k \Delta_{\gamma \beta}
\]

(4.22)

which is the same vector constraint equation (eq. (14) of Ref. 1), required to be satisfied by the full vertex function, \( N^{a b c d}_{\gamma \gamma}(k, l) \).

V. THE SURFACE TERMS

The usual technique of calculating the surface terms \( \Pi_{\mu \nu \lambda \gamma}^{abcd}(k, p, q) \) is to make use of the conserved vector currents and the assumption of meson dominance 5,8). The surface terms \( \Pi_{\mu \nu \lambda \gamma}^{abcd} \) occurring in (3.5) and (3.6) consist of only axial-vector currents and therefore cannot be determined without further assumptions. We then assume in this section that the \( SU(3) \times SU(3) \) symmetry holds exactly, so that the current \( A_{\lambda}^{4+15}(0) \) is conserved and we will neglect the term \( 0 \left[ A_{\mu}^a(x) A_{\nu}^b(y) A_{\lambda}^c(z) \partial_\gamma A_{\xi}^{4+15}(0) \right] 0 \). We will also take the vector and axial-vector meson masses to be degenerate and their relevant coupling constants equal. We then have the following constraint equation for the surface term \( \Pi_{\mu \nu \lambda \gamma}^{abcd} \)

\( a = 4+15, b = 1+i2, c = 1-i2 \) and \( d = 4+15, e = 6+17, f = 6-i7 \)

\[
\partial_\lambda \Delta^{A(K)}_{\alpha}(l) \Pi^{abcd}_{\mu \nu \lambda \gamma}(k, p, q)
\]

\[
= \frac{1}{\sqrt{2}} \Delta^{A(K)}_{\alpha}(l) \Delta^{V(\gamma)}_{\beta}(\mu, k, l) \gamma^{(\nu \lambda \gamma)}_{\gamma}(\mu, \gamma)
\]

\[
- \Delta^{A(\pi)}_{\lambda}(q) \Delta^{V(K)}_{\gamma \gamma^'}(l, q) \gamma^{(\nu \lambda \gamma)}_{\gamma}(l, q)
\]

(5.1)

We now define certain contact terms, denoted by \( \Pi_{\mu \nu \lambda \gamma}^{(ct)} \), by subtracting out the 1" pole structures of the function \( \Pi^{abcd}_{\mu \nu \lambda \gamma} \).
As far as the current algebra is concerned, the contact terms \( \Pi_{\mu \nu \lambda \eta}^{(ct)} \) do not have any poles, so they can be approximated by smooth functions of the momenta in accordance with the hypothesis of meson-dominance. We then define (with our normalization)

\[
\Pi^{abcd}_{\mu \nu \lambda \eta}(k, p, q) = \Pi^{(ct)}_{\mu \nu \lambda \eta}(k, p, q)
\]

\[
- i g_{K^*}^{-2} \Gamma^{a b e}_{\mu \nu \sigma}(k, p) \Delta_{\sigma \sigma'}(l + q) \Gamma^{c d h}_{\eta \sigma'}(q, l)
\]

\[
- i g_{p}^{-2} \Gamma^{b c d}_{\nu \lambda \sigma}(p, q) \Delta_{\sigma \sigma'}(l + k) \Gamma^{a d h}_{\eta \sigma'}(k, l).
\]

(5.2)

We note

\[
- i g_{K_A}^{-1} l_\eta \Delta^{A(k)}_{\eta \beta}(l) \Gamma^{a d h}_{\mu \sigma \beta}(k, l)
\]

\[
= \frac{g_p g_{K_A}}{\sqrt{2}} \left[ \Delta_{\mu \sigma}^{V(\pi)}(k + l)^{-1} - \Delta_{\mu \sigma}^{A(k)}(k)^{-1} \right]
\]

(5.3)

\[
- i g_{K_A}^{-1} l_\eta \Delta^{A(k)}_{\eta \beta}(l) \Gamma^{c d h}_{\eta \beta \sigma}(q, l)
\]

\[
= g_{K_A} g_{K^*} \left[ \Delta_{\gamma \sigma}^{A(\pi)}(q)^{-1} - \Delta_{\gamma \sigma}^{V(k)}(q + l)^{-1} \right]
\]

(5.4)

From (5.1)-(5.4) one can obtain

\[
g_{K_A}^{-1} l_\tau \Delta^{A(k)}_{\tau \eta}(l) \Pi^{(ct)}_{\mu \nu \lambda \eta}(k, p, q)
\]

\[
= \frac{1}{\sqrt{2}} \Gamma^{b c d}_{\nu \lambda \mu}(p, q) - \Gamma^{a b e}_{\nu \mu \lambda}(p, q).
\]

(5.5)
The three-point functions, \( \Pi_{\mu \nu \lambda}^{bc3} \) and \( \Pi_{\mu \nu \gamma}^{ab6+17} \), satisfy the following vector constraint equations:

\[
\frac{i}{\sqrt{2}} \mathcal{L}_{\mathcal{H} \mathcal{J}} \, g_{F}^{-1} \Delta^{(n)}_{\mathcal{M} \mathcal{N}} \gamma_{\mu \lambda}^{\mathcal{J}} \gamma_{\nu \lambda}^{\mathcal{J}} (p) = \Delta^{(n)}_{\mathcal{M} \mathcal{N}} \gamma_{\mu \lambda}^{\mathcal{J}} \gamma_{\nu \lambda}^{\mathcal{J}} (p)
\]

\[
= g_{A}^{2} \left[ \Delta^{(n)}_{\mathcal{M} \mathcal{N}} \gamma_{\mu \lambda}^{\mathcal{J}} \gamma_{\nu \lambda}^{\mathcal{J}} (p) - \Delta^{(n)}_{\mathcal{M} \mathcal{N}} \gamma_{\mu \lambda}^{\mathcal{J}} \gamma_{\nu \lambda}^{\mathcal{J}} (p) \right],
\]

\[
(5.6)
\]

\[
- i \mathcal{L}_{2 \Omega} \, g_{K}^{-1} \Delta^{(n)}_{\mathcal{M} \mathcal{N}} \gamma_{\mu \lambda}^{\mathcal{J}} \gamma_{\nu \lambda}^{\mathcal{J}} (K) = \Delta^{(n)}_{\mathcal{M} \mathcal{N}} \gamma_{\mu \lambda}^{\mathcal{J}} \gamma_{\nu \lambda}^{\mathcal{J}} (K)
\]

\[
= g_{K}^{2} \left[ \Delta^{(n)}_{\mathcal{M} \mathcal{N}} \gamma_{\mu \lambda}^{\mathcal{J}} \gamma_{\nu \lambda}^{\mathcal{J}} (K) - \Delta^{(n)}_{\mathcal{M} \mathcal{N}} \gamma_{\mu \lambda}^{\mathcal{J}} \gamma_{\nu \lambda}^{\mathcal{J}} (K) \right],
\]

\[
(5.7)
\]

which admit the following solutions:

\[
\gamma_{\mu \lambda \gamma}^{bc3} \gamma_{\nu \lambda}^{(p, q)} = \frac{i \sqrt{2} g_{F} g_{A}^{2}}{\mathcal{C} \mathcal{V}} \left[ \delta_{\mu \lambda}^{(p, q)} + \delta_{\mu \lambda} \gamma_{\nu \lambda} \right]
\]

\[
- \delta_{\mu \lambda} \mathcal{L}_{\mathcal{H}} + \{ \delta_{\mu \lambda} \mathcal{L}_{1 \mathcal{H}} - \delta_{\mu \lambda} \mathcal{L}_{1 \mathcal{H}} \} (2 + \delta_{A})
\]

\[
(5.8)
\]

\[
\gamma_{\mu \lambda \nu}^{(\mu \nu)} (K, p) = - i \frac{g_{A}^{2}}{\mathcal{C} \mathcal{V}} \left[ \delta_{\mu \lambda}^{(K, p)} + \delta_{\mu \lambda} \mathcal{L}_{1 \mathcal{H}} \right]
\]

\[
- \delta_{\mu \lambda} \mathcal{L}_{\mathcal{H}} + \{ \delta_{\mu \lambda} \mathcal{L}_{2 \mathcal{H}} - \delta_{\mu \lambda} \mathcal{L}_{2 \mathcal{H}} \} (2 + \delta_{K})
\]

\[
(5.9)
\]

Using (5.8) and (5.9) in (5.5) we obtain the solution of \( \Pi^{(ct)}_{\mu \nu \lambda \eta} \) in the form (the coefficients of the terms \( \delta_{A} \) and \( \delta_{K} \) vanish)

\[
\Pi^{(ct)}_{\mu \nu \lambda \tau} = - \frac{i g_{F} g_{A}^{3}}{\mathcal{C} \mathcal{V}} \left\{ - 2 \delta_{\mu \lambda} \delta_{\tau \gamma} + \delta_{\mu \nu} \delta_{\tau \lambda} + \delta_{\nu \lambda} \delta_{\tau \mu} \right\}
\]

\[
(5.10)
\]
For the process \((B)\) we find similarly the solution of the contact term \(\Pi'_{\mu \nu \lambda \tau}(\xi)\) as

\[
\Pi'_{\mu \nu \lambda \tau}(\xi) = -i \frac{g_{\sigma} g_{\sigma'}}{C_A C_V} \left\{ -2 \delta_{\mu \lambda} \delta_{\tau \tau'} + \delta_{\mu \nu} \delta_{\tau \lambda} + \delta_{\mu \lambda} \delta_{\tau \nu} \right\}
\]

(5.11)

The vector meson pole (non-contact) terms in (5.2) can easily be evaluated. Their contributions are of the order of \(f(s^2, t^2, u^2)/m^2_K(\xi)\). Since these contributions are small we shall not include them here.

VI. RESULTS

We are now in a position to write down the final results for the \(K_{L4}\) form factors. Before we do so, let us discuss certain terms in (3.5) in the way they are written down in the final expressions, e.g.,

\[
-\frac{1}{6} \left[ \frac{1}{2} \Delta^{A(K)}_{\mu \tau}(l_1) + \frac{1}{3} \Delta^{A(K)}_{\mu \tau}(l_2) - \frac{1}{2} \Delta^{V(K)}_{\mu \tau}(l_2) + \frac{1}{3} \frac{\lambda_{\mu \tau} f_{\xi}^2}{l^2 + m_{\xi}^2} \right],
\]

(6.1)

where we have already taken care of the Schwinger terms. To the expression (6.1) we add the first two terms on the right-hand side of (4.22) with coefficient \(k_{\mu}\):

\[
-\frac{1}{2} R_{\mu} \left[ \Delta^{V(K)}_{\mu \tau}(l_2) - \Delta^{A(K)}_{\mu \tau}(l_2) \right],
\]

(6.2)

The vector propagators \(\Delta^{V(K)}_{\mu \tau}(l_2)\) are then cancelled out from the addition of (6.1) and (6.2). Making use of the Weinberg first sum rule (3.9), we can rewrite (6.1) and (6.2) in the following form:

\[
-\frac{R_{\mu}}{6} \left[ -\frac{f_{\xi}^2}{l^2 + m_{\xi}^2} C^K_{\nu \lambda} \delta_{\mu \tau} - \Delta^{A(K)}_{\mu \tau}(l_1) - \frac{\lambda_{\mu \tau} f_{\xi}^2}{l^2 + m_{\xi}^2} \right],
\]

(6.3)

Since \(C^K_{\nu \lambda}\) and \(\Delta^{A(K)}_{\mu \tau}(l_1)\) are known, we can evaluate (6.3) and express it now in simpler forms. Similarly, we can simplify the corresponding coefficients of \(\rho_{\nu}\) and \(\sigma_{\lambda}\) which occurs in (3.5). Now substituting expressions (3.7), (4.1), (4.2), (4.5), (4.6) and (5.10) in (3.5)

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and equating the coefficients of \((p+q)_\pi\), \((p-q)_\pi\) and \(-(k-p+q)_\pi\)

we obtain the following expressions for the \(K_{l4}\) form factors, \(F_{i}(A)\)
(keeping only linear terms in \(s, t,\) and \(u\) in the numerators):

\[
\frac{f_\pi}{m_K} F_1^{(A)}(s, t, u) = -\frac{1}{2} \frac{f_\pi}{f_K} + \frac{1}{2} \left[ f_+(t) + f_-(t) \right] - \frac{1}{4} \frac{f_\pi}{f_K} \\
+ \frac{1}{4} C_1 + C_2 \left\{ R \cdot (q-p)(3+\delta_A) \right\} \\
+ C_3 \left\{ -\frac{1}{2} \cdot l \cdot (R+3q-p) + q \cdot (R+2p)(2+\delta_K) \right\} \\
+ C' \left\{ \frac{1}{2} \cdot R \cdot p = q \cdot l_2 \right\}, \\
(6.4)
\]

\[
\frac{f_\pi}{m_K} F_2^{(A)}(s, t, u) = -\frac{3}{4} \frac{f_\pi}{f_K} + \frac{f_\pi}{f_K} f_+(s) \\
+ \frac{1}{2} \left[ f_+(t) - f_-(t) \right] + \frac{1}{4} C_1 + C_2 \left\{ l \cdot R \right\} \\
+ l_1 \cdot R \left( 2+\delta_A \right) + C_3 \left\{ -\frac{1}{2} \cdot l \cdot (R+p-q) \right\} \\
+(q \cdot R)(2+\delta_K) \right\} - C' \left\{ R \cdot (q + \frac{1}{2} p) \right\}, \\
(6.5)
\]

\[
\frac{f_\pi}{m_K} F_3^{(A)}(s, t, u) = -\frac{1}{2} \frac{f_\pi}{f_K} + \left[ f_+(t) + f_-(t) \right] \\
+ \frac{1}{6} C_1 + \frac{1}{3} \frac{f_\pi}{f_K} \left( \frac{m_{K^*}^2}{m_{K^*}^2} \right) \frac{1}{\lambda^2 + m_{K^*}^2} \left( \lambda_3 - 2p \right) \cdot l \\
- \frac{f_\pi}{f_K} \frac{R \cdot (q-p)}{\lambda^2 + m_k^2} \left( f_+(s) + \frac{1}{6} \right)
\]
\[- \frac{1}{\ell^2 + m^2_k} \left[ \{ q \cdot (R - p) + (p^2 - p^2) \} \{ f_+ (t) + f_- (t) \} \right] + \frac{1}{6} \left( \frac{f_+}{f_\pi} \right) - \frac{1}{2} \left( R^2 - p^2 \right) \{ f_+ (t) + f_- (t) \}\right] \]

\[+ C_2 \cdot \left( q \cdot p \right) + C_3 \left\{ 1 \cdot q + 2 (p \cdot q) (2 + \delta_k) \right\} \]

\[- C' (p \cdot q), \]

(6.6)

where

\[C_1 = \frac{2 f_\pi}{f_K} \frac{\ell^2 + m_{K_A^0}^2 - m_{K^*}^2}{\ell^2 + m_{K_A^0}^2}, \]

(6.7)

\[C_2 = \frac{f_\pi}{f_K} \left( \frac{m_{K^*}^2}{m_{K_A^0}^2} \right) \frac{m_p^2}{\ell^2 + m_p^2} \frac{1}{\ell^2 + m_{K_A^0}^2}, \]

(6.8a)

\[C_3 = \frac{f_\pi}{4 f_K} \left( \frac{m_{K^*}^2 + m_p^2}{m_p^2} \right) \frac{m_{K^*}^2}{\ell^2 + m_p^2} \frac{1}{\ell^2 + m_{K_A^0}^2}, \]

(6.8b)

\[C' = \frac{1}{2 f_\pi f_K} \left( \frac{q_v^2}{m_v^2} + \frac{q_A^2}{m_A^2} \right) \frac{1}{\ell^2 + m_{K_A^0}^2}, \]

(6.9)

Similarly, for the process (B), we obtain from (3.6), using the expressions (4.18), (4.19), (4.20) and similar ones with p and q interchanged and (5.11),
\[
\frac{f_\pi}{m_K} F_1^{(B)}(s,t,u) = -\frac{1}{4} \left[ \frac{f_\pi}{f_K} + 2 \frac{f_K}{f_\pi} \right] \\
+ \frac{1}{4} \left[ 2 f_+(t) + f_-(t) \right] + \frac{1}{4} \left[ 2 f_+(u) + f_-(u) \right] \\
+ \frac{1}{4} C_1 + \frac{1}{2} C_3(K,p) \left\{ \epsilon \cdot (k-q \cdot l_i) + q \cdot l_2 (2+\delta_k) \right\} \\
+ p \leftrightarrow q \right\} - C' \left\{ (p,q_i) + \frac{1}{2} (k,l) \right\} ,
\]

(6.10)

\[
\frac{f_\pi}{m_K} F_2^{(B)}(s,t,u) = +\frac{1}{4} \left[ f_+(t) - f_-(t) \right] \\
- \frac{1}{4} \left[ f_+(u) - f_-(u) \right] - \frac{1}{2} C_3(K,p) \left\{ \epsilon \cdot (k-l_i) \\
+ (q \cdot k) (2+\delta_k) \right\} + p \leftrightarrow q \right\} - \frac{1}{2} C' k \cdot (p-q) 
\]

(6.11)

\[
\frac{f_\pi}{m_K} F_3^{(B)}(s,t,u) = -\frac{1}{2} \frac{f_K}{f_\pi} + \frac{1}{2} \left[ f_+(t) - f_-(t) \right] \\
+ \frac{1}{2} \left[ f_+(u) - f_-(u) \right] + \frac{1}{6} C_1 - \frac{1}{12} \frac{f_\pi}{f_K} \frac{m_K^2 (2k \cdot l_i) \cdot \epsilon}{m_A (k^2 + m_A^2)} \\
- \frac{1}{2 \epsilon^2 + m_K^2} \left[ \left\{ (q \cdot (k-p)) + (k^2 - p^2) \right\} \left[ f_+(t) + \frac{1}{6} \frac{f_K}{f_\pi} \right] \\
- \frac{1}{2} (k^2 - p^2) \left( f_+(t) + f_-(t) \right) \right\} + \left\{ p \leftrightarrow q \right\} \\
+ \frac{1}{2} C_3(K,p) \left\{ \epsilon \cdot q - 2 p \cdot q \cdot (2+\delta_k) \right\} + p \leftrightarrow q \right\} \\
- 2 C' (p,q) .
\]

(6.12)
We finally give the numerical results and the predictions from our calculations. Using the well-known values \( f_K/f_\pi = 1.28 \) and \( \sin^2 \theta_A = 0.265 \) and taking the value of the \( \xi^- \) parameter from experiments, we get for \( \xi_{\text{b-f}} = 0.6 \)

\[
\begin{align*}
F_1^{(A)}(0) &= 1.291, & F_3^{(A)}(0) &= 1.175 \\
F_2^{(A)}(0) &= 0.718
\end{align*}
\] (6.13)

while for \( \xi_{\text{pol}} = -1.0 \)

\[
\begin{align*}
F_3^{(A)}(0) &= -0.432, & F_2^{(A)}(0) &= 1.522 \\
F_1^{(A)}(0) &= 0.487
\end{align*}
\] (6.14)

Using the set of kinematic factors given by BEREUDS, DONNCUIE, and OADES, we have the predictions from (6.13) for the decay rates

\[
\begin{align*}
\Gamma(K^+ \rightarrow \pi^- \pi^+ e^+ \nu) &= 2.81 \times 10^3 \text{ sec}^{-1} \quad (6.15a) \\
\Gamma'(K^+ \rightarrow \pi^- \pi^+ \mu^+ \nu) &= 0.45 \times 10^3 \text{ sec}^{-1} \quad (6.15b)
\end{align*}
\]

which are to be compared with the experimentally observed values

\[
\begin{align*}
\Gamma_{\text{exp}}(K^+ \rightarrow \pi^- \pi^+ e^+ \nu) &= (2.9 \pm 0.6) \times 10^3 \text{ sec}^{-1} \quad (6.16a) \\
\Gamma_{\text{exp}}'(K^+ \rightarrow \pi^- \pi^+ \mu^+ \nu) &= (1.1 \pm 0.7) \times 10^3 \text{ sec}^{-1} \quad (6.16b)
\end{align*}
\]

On the other hand, if one uses the set of values of \( F_i \) 's given in (6.14) for \( \xi_{\text{pol}} = -1.0 \), one has the prediction

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\[ \Gamma'(K^+ \rightarrow \pi^- \pi^+ \mu^+ \nu) = 0.12 \times 10^3 \text{ sec}^{-1}. \]  

Thus we observe that the experimental values \((6.16)\) favour strongly the value \(\xi_{br} = 0.6\), opposed to \(\xi_{pol} = -1.0\). If one would use the full expressions \((6.4)-(6.6)\) to calculate the decay rates, one may expect the predictions \((6.15)\) to be about 10-15\% larger, in better agreement with experiments. It would be interesting to see how the expressions \((6.4)-(6.6)\) fit the decay distribution in \(K^\pm\) decays.

If we use \(\xi = 0.05\) obtained from the technique of current algebra, hard pions and kaon, we obtain \(\Gamma(K^+ \rightarrow \pi^+ \pi^- e^+ \nu) = 1.67 \times 10^3 \text{ sec}^{-1}\) and \(\Gamma'(K^+ \rightarrow \pi^+ \pi^- \mu^+ \nu) = 0.26 \times 10^3 \text{ sec}^{-1}\). The corresponding predictions using the Weinberg results are quoted to be \(\Gamma(K^+ \rightarrow \pi^+ \pi^- e^+ \nu) = 1.79 \times 10^3 \text{ sec}^{-1}\) and \(\Gamma'(K^+ \rightarrow \pi^+ \pi^- \mu^+ \nu) = 0.33 \times 10^3 \text{ sec}^{-1}\). For the decay process \(K^+ \rightarrow \pi^0 \pi^0 \ell^+ \nu\), we obtain from \((6.10)-(6.12)\) for \(\xi = 0.6\):

\[
\begin{align*}
P_1^{(B)}(0) &= 1.291, & P_2^{(B)}(0) &= 0, \\
P_3^{(B)}(0) &= 1.175. 
\end{align*}
\]  

while for \(\xi_{pol} = -1.0\):

\[
\begin{align*}
P_1^{(B)}(0) &= 0.487, & P_2^{(B)}(0) &= 0, \\
P_3^{(B)}(0) &= -0.432. 
\end{align*}
\]  

From \((6.16)\) and \((6.19)\) we have the predictions for \(\xi = 0.6\):

\[ \Gamma'(K^+ \rightarrow \pi^- \pi^+ \ell^+ \nu) = 1.33 \times 10^3 \text{ sec}^{-1}. \]  

\[ (6.20a) \]
\[ \Gamma' \left( K^+ \rightarrow \pi^0 \pi^0 K^+ \nu \right) = 0.22 \times 10^3 \text{ sec}^{-1}, \quad (6.20b) \]

while for \( \xi_{\text{pol}} = -1.0 \)

\[ \Gamma' \left( K^+ \rightarrow \pi^0 \pi^0 e^+ \nu \right) = 0.19 \times 10^3 \text{ sec}^{-1}, \quad (6.21a) \]

\[ \Gamma' \left( K^+ \rightarrow \pi^0 \pi^0 \nu \right) = 0.03 \times 10^3 \text{ sec}^{-1}. \quad (6.21b) \]

As for the process \( K^0 \rightarrow \pi^- \pi^0 \pi^+ \nu \), one can use the 
\[ \Delta I = \frac{1}{2} \] relation (2.7). The experimental decay rates for \( K^+ \rightarrow \pi^0 \pi^0 \pi^+ \nu \) and \( K^0 \rightarrow \pi^- \pi^0 \pi^+ \nu \) are expected to be available in the near future and on comparison between them and our predictions (6.21), (6.22) one may again be able to draw conclusions about the value of the \( \xi_1 \) parameter in \( K_{\pi}^3 \) decays.

VII. THE SOFT PION RESULTS

In this section we first write down the results for the \( K_{\pi^4} \) form factors obtained in the soft pion and kaon limits (following the work of Weinberg) and then give the procedure for deriving them from our general method. In doing so we obtain consistency conditions which lead to the soft pion and kaon results for the \( K_{\pi^3} \) and \( \pi_{\pi^3} \) form factors. One can easily get the following results using the Weinberg identity:

\[ \lim \ p, q \rightarrow 0 \]

\[ F_1^{(A)} = \frac{m_K}{f_{\pi}} f_+ \]

\[ F_2^{(A)} = \frac{1}{2} \frac{m_K}{f_{\pi}} f_K \left[ 1 + \frac{R \cdot (p-q)}{R \cdot (p+q)} \right] \]

\[ (7.1a) \]

\[ F_1^{(B)} = \frac{m_K}{f_{\pi}} f_+ \]

\[ F_2^{(B)} = 0 \]

\[ F_3^{(B)} = \frac{1}{2} \frac{m_K}{f_{\pi}} f_K ; \quad (7.1b) \]
\[ \lim_{k,p \to 0} \]

\[ F_1^{(A)} = F_2^{(A)} = \frac{m_k}{f_{\pi}} \left( \frac{f_{\pi}}{f_k} \right), \quad F_3^{(A)} = \frac{1}{2} \frac{m_k}{f_{\pi}} \left( \frac{f_{\pi}}{f_k} \right) \left[ 1 + \frac{q_+ (p-k)}{q_+ (k+p)} \right], \]

(7.2a)

\[ F_1^{(B)} = \frac{1}{2} \frac{m_k}{f_{\pi}} \left[ \frac{f_{\pi}}{f_k} + \frac{f_k}{f_{\pi}} \right], \quad F_2^{(B)} = 0, \]

\[ F_3^{(B)} = \frac{1}{2} \frac{m_k}{f_{\pi}} \left[ \frac{f_k}{f_{\pi}} + \frac{1}{2} \frac{f_{\pi}}{f_k} \left \{ 1 + \frac{q_+ (p-k)}{q_+ (k+p)} \right \} \right]. \]

(7.2b)

\[ \lim_{k,q \to 0} \]

\[ F_1^{(A)} = \frac{m_k}{f_{\pi}} \left( \frac{f_{\pi}}{f_k} \right), \quad F_2^{(A)} = \frac{m_k}{f_{\pi}} \left( \frac{f_{\pi}}{f_k} \right), \quad F_3^{(A)} = \frac{m_k}{f_{\pi}} \left( \frac{f_{\pi}}{f_k} \right), \]

(7.3a)

\[ F_1^{(B)} = F_1^{(B)} (k,p \to 0), \quad F_2^{(B)} = 0, \quad F_3^{(B)} = F_3^{(B)} (k,p \to 0, p,q \to q). \]

(7.3b)

One notices that in three different limits, \( p,q \to 0 \), \( k,p \to 0 \) and \( k,q \to 0 \), different results follow. To rederive the above results from our general method we rewrite the identity \( (3.1) \) in a form which would exhibit the symmetry between only two SU(3) indices, say \( b \) and \( c \):

\[ T \left \{ \partial_\mu A_\mu^a (x), \partial_\nu A_\nu^b (y), \partial_\lambda A_\lambda^c (z), A_\tau^d (0) \right \} \]

\[ = \frac{\partial}{\partial z_\lambda} \frac{\partial}{\partial y_\nu} \frac{\partial}{\partial x_\mu} T \left \{ A_\mu^a (x), A_\nu^b (y), A_\lambda^c (z), A_\tau^d (0) \right \} \]

\[ - \left [ \delta (y_0) T \left \{ [A_0^b (y), A_\tau^d (0)], \partial_\mu A_\mu^a (x), \partial_\lambda A_\lambda^c (z) \right \} \right . \]
\[ + \delta(z_0) \mathcal{T} \left\{ [A^c_0(z), A^d_\alpha(0)], \partial_\lambda A^a_\mu(x), \partial_\nu A^b_\gamma(y) \right\} \]
\[ + \frac{1}{2} \delta(y_0) \delta(z_0) \mathcal{T} \left\{ [A^c_0(z), [A^b_\alpha(y), A^d_\alpha(0)], \partial_\mu A^a_\lambda(x) \right\} \]
\[ + \frac{1}{2} \delta(y_0) \delta(z_0) \mathcal{T} \left\{ [A^b_\alpha(y), [A^c_0(z), A^d_\alpha(0)], \partial_\mu A^a_\lambda(x) \right\} \]
\[ + \frac{1}{2} \left( \frac{\partial}{\partial z_\lambda} - \frac{\partial}{\partial y_\lambda} \right) \delta(y_0 - z_0) \mathcal{T} \left\{ [A^b_\alpha(y), A^c_\lambda(z)], \partial_\mu A^a_\lambda(x), A^d_\alpha(0) \right\} \]
\[ - \left[ \delta(z_0) \mathcal{T} \left\{ [A^a_\alpha(0), A^d_\alpha(0)], \partial_\nu A^b_\gamma(y), \partial_\lambda A^c_\lambda(z) \right\} \right. \]
\[ + \delta(x_0) \delta(z_0) \mathcal{T} \left\{ [A^c_0(z), [A^a_\alpha(x), A^d_\alpha(0)], \partial_\nu A^b_\gamma(y) \right\} \]
\[ + \delta(x_0) \delta(y_0) \mathcal{T} \left\{ [A^b_\alpha, [A^a_\alpha(x), A^d_\alpha(0)], \partial_\lambda A^c_\lambda(z) \right\} \]
\[ + \frac{1}{2} \delta(x_0) \delta(y_0) \delta(z_0) \left\{ [A^c_0(z), [A^b_\alpha(y), [A^a_\alpha(x), A^d_\alpha(0)]]] \right. \]
\[ + [A^b_\alpha(y), [A^c_0(z), [A^a_\alpha(x), A^d_\alpha(0)]]] \right\} \]
\[ + \frac{1}{2} \left( \frac{\partial}{\partial z_\lambda} - \frac{\partial}{\partial y_\lambda} \right) \delta(y_0 - z_0) \delta(x_0) \mathcal{T} \left\{ [A^b_\alpha(y), A^c_\lambda(z)], \right. \]
\[ \left. [A^a_\alpha(x), A^d_\alpha(0)] \right\} \]
where we have dropped the $\sigma$-terms. If one now uses the identity (7.4) to the $K^+_{14}$ decay amplitudes and takes the limit $p^2 = -m^2_a$, and $p^a p^b \to 0$, one notices that on the right-hand side of (7.4) the first term vanishes, the first group of terms within the large square bracket and the term on the left-hand side have singularity at $p^2 = -m^2_a$ while the remaining terms within the second large square bracket are not singular at the above limits. Equating the coefficients of the singularity and putting appropriate values of $a, b, c$ and $d$, one gets immediately the values of the $K^+_{14}$ form factors as given in (7.1)-(7.3) in the appropriate limits. The second group of terms yield in the limits $p^b, p^c \to 0$, the expressions for the $K^+_{13}$ (and the corresponding $\pi^+\pi^0$) form factors as given by RIAZUDDIN, SARKER and FAYYAZUDDIN 22.

VIII. CONCLUSION

In the calculations of the $K^+_{14}$ form factors from the identity (3.1) we have neglected the $\sigma$-type terms. Since the predicted results for the decay rates of $K^+ \to \pi^+\pi^-\mu^+\nu$ and $K^+ \to \pi^+\pi^-\mu^+\nu$ agree quite well with the experimentally observed rates, we are probably justified in neglecting the $\sigma$-type terms in the present problem. This by no means implies that the $\sigma$-terms are unimportant. In fact, they are quite important in the calculations of the $\pi-\pi$ scattering lengths, in particular the s-wave part; since the available energy of the pions in $K^+_{14}$ decays is small, the effect of $\pi-\pi$ interactions in the final states in $K^+_{14}$ decay may not be large enough. Since the technique of hard pions has been successful in predicting the $\pi-\pi$ phase shifts 23, one is also hopeful of fixing the relative contributions.
from the $U$-type terms in $K_{l4}$ decays purely from the theoretical calculations. We hope to discuss these points elsewhere.

The technique of hard pion and kaon in the context of current algebra has been already quite successful in correlating the strong decays $\rho \to \pi\pi$, $\Lambda_1 \to \rho\pi$ and $K_A \to K^*\pi$, $K^* \to K\pi$ and also in predicting the electromagnetic decays $\eta^0 \to 2\gamma$, $X_0 \to 2\gamma$, etc. The present calculations of the $K_{l4}$ decays along with that of the $K_{l3}$ form factors $8,26$ indicate its further successes in the domain of weak interactions.

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REFERENCES AND FOOTNOTES


8) RIAZUDDIN and A.Q. SARKER, Phys. Rev. to be published.


22) RIAZUDDIN, A.Q. SARKER and FAYYAZUDDIN, Nucl. Phys., to be published.


26) Y. UEDA, University of Toronto preprint, 1968.
Available from the Office of the Scientific Information and Documentation Officer, International Centre for Theoretical Physics, 34100 TRIESTE, Italy