ONE-NEUTRON TRANSFER REACTIONS IN EVEN AND ODD TIN ISOTOPES AND A REALISTIC NUCLEON-NUCLEON POTENTIAL

R. ALZETTA
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+\) Istituto di Fisica Teorica dell'Università, Trieste, Italy.
++\) International Centre for Theoretical Physics, Trieste, and Istituto di Fisica Teorica dell'Università, Trieste, Italy.
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ABSTRACT

Low-lying energy levels and wave functions of the even and odd isotopes of tin (A = 116 - 121) are calculated with 0, 2 and 4 and 1 and 3 quasiparticle Tamm-Dancoff theories with the realistic two-nucleon potential of Tabakin renormalized for core polarization by Gmitro et al. All the spectra are in good over-all agreement with experiment. The corresponding spectroscopic factors for the one-neutron stripping and pick-up reactions are calculated and found to be in reasonable general agreement with the data of Schneid et al. and of Yagi et al. No adjustable parameters are involved in the theory (the single-particle input parameters are those of Bando).
I. INTRODUCTION

One- and two-nucleon transfer reactions are an important tool of analysing details of microscopic nuclear wave functions. Spectroscopic structure factors (of fractional parentage) for one-nucleon transfer reactions (stripping and pick-up) give information on states of both the even and the odd nucleus involved and on their most important "parentage" interrelation. The even and odd tin isotopes have recently received very much attention both experimentally and theoretically as typical representatives of the so-called vibrational nuclei. In particular, successful descriptions of the spectra of nearly all the Sn isotopes have been obtained in terms of quasiparticle (qp) Tamm-Dancoff methods 1)-7). The qp configuration mixing must be limited to taking only the five valence neutron subshells explicitly into account. On the other hand, such a description can also be justified for all the low-lying states provided we take into account the effects of the excited configurations of at least the most important subshells of the 50-50 core nucleons through a renormalization of the residual nuclear force. The states of the odd isotopes can be understood as superpositions of one- and three-qp excitations 2) out of the qp vacuum |0>, while those of the even isotopes are described in terms of excitation of zero-, two- and four-quasiparticles 3)-7). We call the respective approximations quasiparticle Tamm-Dancoff 13 (QTD13), and quasiparticle second Tamm-Dancoff (QSTD). All the basic spurious kets due to the nucleon-number non-conservation in the qp formalism must be projected
out before diagonalizing the corresponding secular matrices.

In Refs. 2) and 8) the spectroscopic factors for the one-neutron transfer reactions in Sn were calculated assuming the QTD13 eigenvectors for the states of the odd Sn isotopes while the ground states of the corresponding even Sn isotopes were taken to be the qp vacuum $|0\rangle$, and simple one-qp (independent quasiparticle model [IQM]) states - for the odd isotopes - in describing $(d,p)$ reactions on odd isotopes, while the excited final states of the corresponding even isotopes were described in the simple two-qp Tamm-Dancoff (QTD) approximation.

One of us 9) has generalized the method to calculate the same spectroscopic factors systematically with the QTD13 description for the odd Sn isotopes and with the QSTD description for the even Sn isotopes. The numerical calculations of Ref. 9) were, like those of Refs. 2), 3) and 8), limited to a simple purely phenomenological Gaussian Wigner two-body residual nuclear force.

It is the aim of the present note to extend the calculations of Refs. 2), 8) and 9) to a realistic nucleon-nucleon potential renormalized for core polarization 4). We have chosen to use the nonlocal potential of Tabakin 10) with the second-order core polarization terms as calculated in Ref. 4) (the variant labelled "S2" in Ref. 4)); the unperturbed single-particle (s.p.) energies are the same as those of Ref. 4) taken from Bando 11). The renormalized Tabakin force of Ref. 4) gives rather satisfactory QTD and
QSTD results for the spectra of Sn$^{116}$ and Sn$^{120}$ (except that the $2^+_1$ state lies slightly too high). In the present calculations we have used the QSTD eigenvectors of Sn$^{116}$ of Refs. 4) and 12) but the ground state ($|0^+_4\rangle$) eigenvector is modified (and other $|0^+_n\rangle$ vectors are only slightly modified). In fact, with the spurious states $|\psi_{\text{sp}}\rangle$ of Refs. 3), 4) which, at least with our particular choice of the s.p. basis and of the pairing force, have large vacuum $|0\rangle$ components and thus, when projected out of the $0^+$ QSTD secular matrix, cause an appreciable depletion of the $|0\rangle$ component in $|0^+_4\rangle$. This indicates the possibility of a similar situation in the $J^\pi \neq 0^+$ states when the higher-order spurions $|\psi_{\text{sp}}\rangle$ involving six-qp components are projected out in a six-qp (0, 2, 4 and 6) Tamm-Dancoff calculation. Namely, the projection of such spurions may significantly deplete the two-qp components of the corresponding physical eigenvectors. This question is still open and under investigation. It is also evident that the unprojected BCS ground state is not a very satisfactory starting point (reference ground state) for a microscopic spectroscopy in the circumstances. An exactly particle-number-conserving model with the same dimensions of QSTD becomes highly desirable 13). In our QSTD case, in the situation described above, one would have a model in which the theory of the $J^\pi \neq 0^+$ states would be somehow divorced from that of the QSTD $0^+$ states. On the other hand, it is just the interrelations between the $|0^+_4\rangle$ state and all the other states $J^\pi \neq 0^+$ of both even and odd isotopes which are of main interest for our theory of nuclear reactions. In particular, in our present applications, we have the QTD13 eigenvectors
for the states of the odd Sn isotopes in which the
spurions \( | \psi_{sp5} \rangle \) containing one-, three- and five-qp
components could have appreciable one-qp components,
thus being able to deplete such components in a five-qp
Tamm-Dancoff theory rendering them substantially differ-
ent from our \( \text{QTD13} \) eigenvectors. This question also is still
open and under investigation. Here again an exactly
number-conserving \( \text{QTD13} \) theory (of the \( \text{QTD13} \) dimensions)
would be highly desirable.

Part of the blame for such an uncomfortable
situation in a given particular case (nucleus, s.p.
input parameters, nuclear force, etc.) may be due to
the lack of the Hartree-Fock-Bogolubov (HFB) self-
consistency.

In this situation we have chosen to project out
the spurions
\[
| \psi_{sp4} \rangle = \mathcal{N}' A^{\dagger}_{JM} (cc') (\hat{N} - N_0) | 0 \rangle
\]
where \( A^{\dagger}_{JM} (cc') \) is a normalized two-qp creation operator
of spin \( \hat{J} \); \( \hat{N} \) and \( N_0 \) are the nucleon-number operator
and its correct eigenvalue, respectively; \( \mathcal{N}' \) is the
normalization constant. These spurions differ from \( | \psi_{sp4} \rangle \)
of Refs. 3, 4) by the absence of the vacuum (\( | 0 \rangle \))
components. Such a (somewhat arbitrary) prescription for
spurion projection (definition of four-qp spurions) has
been proposed and applied in Refs. 6, 7). This variant
of \( \text{QSTD} \) involves then the ansatz that a given particular
BCS solution is essentially almost non-spurious and that
the only four-qp correlations in \( | 0^+ \rangle \) are due to the dynamical effect of the \( H_{40} \) term of the qp-transformed inter-
action Hamiltonian $H_{\text{int}}$. Although this ansatz is, at least in principle, unjustified, it has the advantage of a unique, coherent description of all the $0^+$ and other QSTD states and of their relation to the QTD13 states of the odd isotopes. Moreover, the spurious effects of the vacuum $|0\rangle$ itself (such as of the fluctuation of $\hat{N}^2 - N_o^2$, etc.), may have only very little influence on spectroscopic factors for nucleon transfer reactions in this variant where the spurions in QTD13 and in QSTD are both treated on the same footing. Clearly, the whole question of whether or not to project out the $|0\rangle$ component of $|\Psi_{\text{sp4}}\rangle$ in QSTD exclusively concerns the $0^+$ states and, in practice, only the $|0^+\rangle$ eigenvector. In Appendix I we compare our spectroscopic factors computed with $\langle 0|\Psi_{\text{sp4}}\rangle = 0$ with those which result when $|\Psi_{\text{sp4}}\rangle$ are used ($\langle 0|\Psi_{\text{sp4}}\rangle \neq 0$) and we find only small numerical differences between the two variants; agreement with experiment is sometimes even better with $\langle 0|\Psi_{\text{sp4}}\rangle \neq 0$.

In the expressions derived in Ref. 9) for the spectroscopic factors there are, in addition to the $(0\,qp - 1\,qp)$ and $(1\,qp - 2\,qp)$ terms calculated in Refs. 2) and 8), also the terms $(2\,qp - 3\,qp)$ and $(3\,qp - 4\,qp)$.

We compute the QTD13 and QSTD eigenvectors for the renormalized Tabakin force, and then the corresponding spectroscopic factors utilizing the formulae of Ref. 9) for the following stripping and pick-up reactions:

- Sn$^{116}$ (d, p) Sn$^{117}$, Sn$^{118}$ (p, d) Sn$^{117}$,
- Sn$^{118}$ (d, p) Sn$^{119}$, Sn$^{120}$ (d, t) Sn$^{119}$, Sn$^{120}$ (d, p) Sn$^{121}$. 

-6-
for the states of the odd Sn isotopes in which the spurions $| \psi_{sp5} \rangle$ containing one-, three- and five-qp components could have appreciable one-qp components, thus being able to deplete such components in a five-qp Tamm-Dancoff theory rendering them substantially different from our QTD13 eigenvectors. This question also is still open and under investigation. Here again an exactly number-conserving QTD13 theory (of the QTD13 dimensions) would be highly desirable.

Part of the blame for such an uncomfortable situation in a given particular case (nucleus, s.p. input parameters, nuclear force, etc.) may be due to the lack of the Hartree-Fock-Bogolubov (HFB) self-consistency.

In this situation we have chosen to project out the spurions

$$| \psi_{sp4} \rangle = \mathcal{W}' \hat{A}^+_{JM} (cc') (\hat{N} - N_0) | 0 \rangle \quad (1)$$

where $\hat{A}_{JM} (cc')$ is a normalized two-qp creation operator of spin $J$; $\hat{N}$ and $N_0$ are the nucleon-number operator and its correct eigenvalue, respectively; $\mathcal{W}'$ is the normalization constant. These spurions differ from $| \psi_{sp4} \rangle$ of Refs. 3), 4) by the absence of the vacuum ($| 0 \rangle$) components. Such a (somewhat arbitrary) prescription for spurion projection (definition of four-qp spurions) has been proposed and applied in Refs. 6), 7). This variant of QSTD involves then the ansatz that a given particular BCS solution is essentially almost non-spurious and that the only four-qp correlations in $| 0^+_4 \rangle$ are due to the dynamical effect of the $H_{40}$ term of the qp-transformed inter
action Hamiltonian $H_{\text{int}}$. Although this ansatz is, at least in principle, unjustified, it has the advantage of a unique, coherent description of all the $0^+$ and other QSTD states and of their relation to the QTD13 states of the odd isotopes. Moreover, the spurious effects of the vacuum $|0\rangle$ itself (such as of the fluctuation of $\hat{N}^2 - N_0^2$, etc.), may have only very little influence on spectroscopic factors for nucleon transfer reactions in this variant where the spurions in QTD13 and in QSTD are both treated on the same footing. Clearly, the whole question of whether or not to project out the $|0\rangle$ component of $|\Psi_{\text{sp}}\rangle$ in QSTD exclusively concerns the $0^+$ states and, in practice, only the $|0^+_1\rangle$ eigenvector. In Appendix I we compare our spectroscopic factors computed with $\langle 0 | \Psi_{\text{sp}}' \rangle = 0$ with those which result when $|\Psi_{\text{sp}}\rangle$ are used ($\langle 0 | \Psi_{\text{sp}} \rangle \neq 0$) and we find only small numerical differences between the two variants; agreement with experiment is sometimes even better with $\langle 0 | \Psi_{\text{sp}} \rangle \neq 0$.

In the expressions derived in Ref. 9) for the spectroscopic factors there are, in addition to the $(0 \, qp - 1 \, qp)$ and $(1 \, qp - 2 \, qp)$ terms calculated in Refs. 2) and 8), also the terms $(2 \, qp - 3 \, qp)$ and $(3 \, qp - 4 \, qp)$.

We compute the QTD13 and QSTD eigenvectors for the renormalized Tabakin force, and then the corresponding spectroscopic factors utilizing the formulae of Ref. 9) for the following stripping and pick-up reactions:

- Sn$^{116}$ $(d, p)$ Sn$^{117}$, Sn$^{118}$ $(p, d)$ Sn$^{117}$,
- Sn$^{118}$ $(d, p)$ Sn$^{119}$, Sn$^{120}$ $(d, t)$ Sn$^{119}$, Sn$^{120}$ $(d, p)$ Sn$^{121}$. 

-6-
(the first five levels of the final nucleus) and
\( \text{Sn}^{117} (d,p) \text{Sn}^{118} \), \( \text{Sn}^{117} (p,d) \text{Sn}^{116} \),
\( \text{Sn}^{119} (d,p) \text{Sn}^{120} \), \( 0^+_1,2,3 \), \( 2^+_1,2,3 \).

Our results for the spectra of QTD13 and QSTD and for
the spectroscopic factors are compared with the recent
experimental data of Schneid et al.\(^\text{14}\) and of Yagi et
al.\(^\text{15}\).
II. EIGENSTATES OF QTD13 AND QSTD

The computation of QTD13 eigenvalues and eigenvectors for the odd tin isotopes follows exactly the method of Kuo et al. 2) The QSTD results obtained for the same renormalized Tabakin force of Ref. 4) are exactly the same as in Ref. 4) except that the $0^+$ states are now computed with the spurions $\Psi_{sp4}'$ rather than with $\Psi_{sp4}$ (no $0^+$ -components in the new four-qp spurions of $J^\pi = 0^+$ ) 12).

For the odd Sn isotopes 117, 119 and 121 we give our QTD13 excited energy levels in Table I and Figs. 1 - 3. The three-qp total (percentage) weights in the corresponding eigenvectors are given in parenthesis. The general over-all agreement with the data of Refs. 14) - 16) is reasonably good given the fact that there are no ad hoc adjustable parameters in the present calculation. From the comparison with the QTD13 results of Kuo et al. 2) given in Figs. 1 - 3 we see that they are as good as ours in spite of the fact that they use a purely phenomenological nucleon-nucleon interaction with parameters adjusted on odd-even mass differences and other data. The QSTD eigenvalues for the states $0^+_1, 2, 3$ and $2^+_1, 2, 3$ of Sn $116, 118, 120$ are given in Figs. 4 - 6 and compared with the data of Refs. 14) and 15). Again, as in Ref. 4), a general good agreement with experiment is obtained, the best for the 120 isotope. The four-qp percentages are generally small, much smaller than those of Ref. 3) based on the Gaussian Wigner force of Kuo et al. 2) One has to exclude the "two-phonon" interpretation of such states where a "phonon" is defined as a low-lying collective QTD mode.
III. SPECTROSCOPIC FACTORS FOR ONE-NEUTRON TRANSFER REACTIONS

The spectroscopic factor for a \((d, p)\) reaction on an even isotope, \(S_{\gamma}^{(0^+, J_n)}\), is defined as:

\[
S_{\gamma}^{(0^+, J_n)} = \left| \langle J_n^\pi (A+1) | c_\alpha^\dagger | 0^+_1 (A) \rangle \right|^2
\]  

where \(c_\alpha^\dagger\) creates a neutron in the s.p. state \(\alpha\); \(|0^+_1 (A)\rangle\) is the QSTD ground state of the target (\(A\)) nucleus, and \(|J_n^\pi (A+1)\rangle\) is the final QTD state of the odd isotope. The detailed explicit expressions for the matrix element \(\langle c^\dagger | \rangle\) are given in Ref. 9). A quite similar expression defines the spectroscopic factor \(S_{\gamma}^{(-)}(J_n^\pi, 0^+_1)\) for the pick-up reaction on the same (\(A\)) even target to the odd (\(A-1\)) isotope. In fact, \(S_{\gamma}^{(-)}(J_n^\pi, 0^+_1)\) can be derived from the corresponding \(S_{\gamma}^{(0^+, J_n)}\) of Eq. (2) by a simple transformation; explicit expressions for this case are given in Eqs. (2), (5), (5') of Ref. 9).

Our final results for these cases are summarized in Table II and in Figs. 7 and 8. We use the notation \(S_{\gamma}^{(-)}(J_n^\pi, 0^+_1) = \gamma^{-2} S_{\gamma}^{(0^+, J_n)}(J_n^\pi, 0^+_1)\). As seen from Table II and from Figs. 7 and 8, the general agreement with the data of Refs. 14), 15) is generally good, even somewhat better than that of Refs. 2) and 8). The data on the pick-up reaction with \(^{119}\)Sn in the final state refer to the reaction \(^{120}(d, t) \rightarrow^{119}\) as quoted in Ref. 2) rather than to the corresponding \((p, d)\) reaction.

A second series of our calculations concerns odd targets and even isotopes in the final states. In Fig. 9, we compare our calculated spectroscopic factors \(S_{\gamma}^{(-)}(J_n^\pi, 0^+_1)\) for...
and $S_{\ell=2}^{(+)} \left( \frac{4^+}{2^+}, \frac{4^+}{2^-} \right)$ for the $^{117,119}_{\text{Sn}}(d,p)^{118,120}_{\text{Sn}}$ reactions with the corresponding data of Ref. 14). Again a reasonable over-all agreement with experiment is obtained. In Fig. 10 our calculated $S_{\ell=0}^{(+)} \left( \frac{0^+}{2^+}, \frac{1^+}{2^-} \right)$ and $S_{\ell=2}^{(+)} \left( \frac{2^+}{2^+}, \frac{3^+}{2^-} \right)$ for the pick-up reactions $^{117}_{\text{Sn}}(p,d)^{116}_{\text{Sn}} \left( \frac{0^+}{2^+}, \frac{2^+}{2^+} \right)$ are compared with the data of Ref. 15). A reasonable semiquantitative agreement with experiment is obtained.

The $\ell$-spectroscopic factors are here defined as:

$$S_{\ell}^{(+)}(J,J) = \frac{2J+1}{2\ell+1} \sum_{j=\ell-\ell/2}^{\ell+\ell/2} S_{j}^{(+)}(J,J)$$

and

$$S_{\ell}^{(-)}(J,J) = \sum_{j=\ell-\ell/2}^{\ell+\ell/2} S_{j}^{(-)}(J,J)$$

Explicit formulae for $S_{j}^{(+)}(J,J)$ and $S_{j}^{(-)}(J,J)$ are given in Eqs. (2), (6'), and (7') of Ref. 9).

In Table III we give the exact numerical values of the calculated spectroscopic factors of Figs. 9 and 10.
IV. CONCLUSIONS

There are rather considerable uncertainties in the "experimental" spectroscopic factors for the one-nucleon stripping and pick-up reactions, since the extraction of such factors from the data involves some assumptions about the reaction mechanism and DWBA amplitudes which are far from being well founded and understood. One should then be rather careful not to attach too much importance to a detailed quantitative agreement of calculated and observed spectroscopic factors. Still, it is quite enjoyable to see that, in addition to fitting reasonably well the observed spectra of both the even and the odd isotopes of Sn, the calculated spectroscopic factors are generally consistent with those extracted from the stripping and pick-up experiments. An analysis was previously performed for the two-neutron transfer reaction in even tin isotopes \(^{17}\) (although the spurious parts of \(|0\rangle\) are removed in the QSTD 0\(^+\) states in Ref. 17), in contrast to the present calculation; it would be interesting to perform calculations on Sn\((t,p)\) and Sn\((p,t)\) reactions with the 0\(^+\) QSTD eigenvectors as defined in the present paper and to compare the spectroscopic factors both with those of Ref. 17 and with experiment. The comparison of our results for the one-neutron transfer reactions obtained with \(\langle 0|\psi_{sp4}\rangle = 0\) with the corresponding ones calculated with \(\langle 0|\psi_{sp4}\rangle \neq 0\) given in Appendix I shows that numerical differences between the two variants are generally small. This shows that our results are not appreciably affected by the
uncertainty in the treatment of the four-qp QSTD spurions. The realistic residual interaction potential used in our nuclear structure calculation involves no adjustable parameter either in its "bare" or its core polarization parts of the matrix elements. All the parameters involved in our theory are "externally" determined.

A calculation in which the single-qp input parameters are determined directly from the experimental level spacings of odd Sn isotopes by a combination of the inverse gap equations and QTD13 is now in preparation in collaboration with Y.K. Gambhir. Since there exist contaminations of our eigenvectors by higher-order spurious states due to the nucleon-number non-conservation and some ambiguities in the methods of projecting out the basic spurions, calculations based on exactly number-projected Tamm-Dancoff theories would be highly desirable. Unfortunately, such calculations will be much more complicated that the present ones.

ACKNOWLEDGEMENTS

We are happy to acknowledge several useful discussions with J. Hendeković, R. Arvieu, M. Gmitro, A. Rimini and T. Weber. One of us (J.S.) is grateful to Professors Abdus Salam and P. Budini and to I.A.E.A. for their kind hospitality at the International Centre for Theoretical Physics at Trieste. All our numerical computations were performed on the IBM 7044 computer of Centro di Calcolo dell'Università di Trieste.
APPENDIX I

Component of Quasiparticle Vacuum in Four-Quasiparticle $0^+$ QSTD Spurions

We compare the spectroscopic factors for the one-neutron stripping and pick-up reactions on even and odd isotopes of tin calculated with the QSTD $0^+$ eigenvectors obtained after projecting out the four-qp spurions $|\Psi_{sp4}^\prime\rangle$ of Eq. (1) with those which result after projecting out the "usual" spurions $|\Psi_{sp4}\rangle$ of Eq. (22) of Ref. 3), i.e., including the qp-vacuum $|0\rangle$-components ($<0|\Psi_{sp4}\rangle \neq 0$).

From the comparison of the $0^+$ QSTD energy levels of the present paper with those of Ref. 4) one can see that they are almost identical. The only difference is an only very small depletion of the $|0\rangle$ component in $|0_f^+\rangle$ of the present paper.

In Table IV we compare the data of Refs. 14) and 15), respectively, with the spectroscopic factors $S_{\pi}^{(+)}(0^+, J_{\pi})$ for the reaction Sn$^{116}$ (d, p) Sn$^{117}$ and $S_{\pi}^{(-)}(0^+, 1^+)$ for the reaction Sn$^{117}$ (p, d) Sn$^{116}$ calculated with the $|0^+\rangle$ eigenvectors of Sn$^{116}$ corresponding to the two definitions of the four-qp spurions.

We see that with the $|\Psi_{sp4}^\prime\rangle$ definition of the four-qp spurions ($<0|\Psi_{sp4}^\prime\rangle \neq 0$) the calculated spectroscopic factors are in good over-all agreement with the data - not worse than that of the factors calculated with $|\Psi_{sp4}\rangle$ ($<0|\Psi_{sp4}\rangle = 0$), in some cases of $J_{\pi}$ the agreement is even better when one calculates with $|\Psi_{sp4}\rangle$. 

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REFERENCES.


18) Y.K. Gambhir, cf., e.g., ICTP, Trieste, preprint IC/68/21, IC/68/32; and R. Alzetta et al., in preparation.
The lowest-lying QTD13 energy levels (in MeV) calculated for tin isotopes with \( A = 117, 119 \) and 121. The total three-qp weights (in \%) of the corresponding eigenvectors are given in parenthesis. The corresponding experimental levels of Ref. 14) are also given for comparison.

<table>
<thead>
<tr>
<th></th>
<th>117</th>
<th></th>
<th>119</th>
<th></th>
<th>121</th>
<th></th>
</tr>
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<tbody>
<tr>
<td></td>
<td>QTD13</td>
<td>exp,Ref.(^{44})</td>
<td>QTD13</td>
<td>exp,Ref.(^{44})</td>
<td>QTD13</td>
<td>exp,Ref.(^{44})</td>
</tr>
<tr>
<td>((1/2)^+) (_1)</td>
<td>0.0 (5.6)</td>
<td>0.0</td>
<td>0.0 (5.1)</td>
<td>0.0</td>
<td>(3/2)^+ (_1)</td>
<td>0.0 (3.2)</td>
</tr>
<tr>
<td>((11/2)^-) (_1)</td>
<td>0.003 (2.0)</td>
<td>0.32</td>
<td>0.09 (3.6)</td>
<td>0.024</td>
<td>(11/2)^- (_1)</td>
<td>-0.14 (0.9)</td>
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<tr>
<td>((3/2)^+) (_1)</td>
<td>0.17 (4.0)</td>
<td>0.16</td>
<td>-0.08 (1.3)</td>
<td>0.08</td>
<td>(1/2)^+ (_1)</td>
<td>0.01 (4.6)</td>
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<tr>
<td>((7/2)^+) (_1)</td>
<td>0.48 (4.7)</td>
<td>0.72</td>
<td>0.62 (5.8)</td>
<td>0.79</td>
<td>(7/2)^+ (_1)</td>
<td>0.74 (7.2)</td>
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<tr>
<td>((5/2)^+) (_1)</td>
<td>0.86 (4.8)</td>
<td>1.03</td>
<td>1.02 (6.7)</td>
<td>0.93</td>
<td>(5/2)^+ (_1)</td>
<td>1.11 (76.7)</td>
</tr>
</tbody>
</table>

**TABLE I**
### Calculated spectroscopic factors for stripping and pick-up reactions

<table>
<thead>
<tr>
<th>$^{116}\text{Sn} (d,p) ^{117}\text{Sn}$ ($J^\pi_n$)</th>
<th>$^{118}\text{Sn} (p,d) ^{117}\text{Sn}$ ($J^\pi_n$)</th>
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<tr>
<td>$J^\pi_n$</td>
<td>$S_x(0^+,J^\pi_n)$</td>
</tr>
<tr>
<td>$(1/2)^+_4$</td>
<td>0.55</td>
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<tr>
<td>$(3/2)^+_4$</td>
<td>0.68</td>
</tr>
<tr>
<td>$(11/2)^+_4$</td>
<td>0.67</td>
</tr>
<tr>
<td>$(7/2)^+_4$</td>
<td>0.31</td>
</tr>
<tr>
<td>$(5/2)^+_4$</td>
<td>0.12</td>
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<table>
<thead>
<tr>
<th>$^{118}\text{Sn} (d,p) ^{119}\text{Sn}$ ($J^\pi_n$)</th>
<th>$^{120}\text{Sn} (d,t) ^{119}\text{Sn}$ ($J^\pi_n$)</th>
</tr>
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<tr>
<td>$J^\pi_n$</td>
<td>$S_x(0^+,J^\pi_n)$</td>
</tr>
<tr>
<td>$(1/2)^+_4$</td>
<td>0.47</td>
</tr>
<tr>
<td>$(3/2)^+_4$</td>
<td>0.63</td>
</tr>
<tr>
<td>$(11/2)^+_4$</td>
<td>0.59</td>
</tr>
<tr>
<td>$(7/2)^+_4$</td>
<td>0.26</td>
</tr>
<tr>
<td>$(5/2)^+_4$</td>
<td>0.09</td>
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<table>
<thead>
<tr>
<th>$^{120}\text{Sn} (d,p) ^{121}\text{Sn}$ ($J^\pi_n$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J^\pi_n$</td>
</tr>
<tr>
<td>$(3/2)^+_4$</td>
</tr>
<tr>
<td>$(1/2)^+_4$</td>
</tr>
<tr>
<td>$(11/2)^+_4$</td>
</tr>
<tr>
<td>$(7/2)^+_4$</td>
</tr>
<tr>
<td>$(5/2)^+_4$</td>
</tr>
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Collected spectroscopic factors for stripping and pick-up reactions on the even tin isotopes with $\Lambda=116, 118$ and 120 leading to the lowest-lying states (with spin parity $J^\pi_n$) of the corresponding residual odd-mass nuclei. Experimental values of Refs. 14, 15 are also listed for comparison.
Calculated spectroscopic factors for stripping and pick-up reactions on odd tin targets with $A=117$ and 119 leading to the first three $0^+$ and $2^+$ states in the corresponding residual even-mass isotopes.

<table>
<thead>
<tr>
<th>$J_n^+$</th>
<th>$^{117}<em>{\text{Sn}}(d,p),^{118}</em>{\text{Sn}}$</th>
<th>$^{119}<em>{\text{Sn}}(d,p),^{120}</em>{\text{Sn}}$</th>
<th>$^{117}<em>{\text{Sn}}(p,d),^{116}</em>{\text{Sn}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+_{1}$</td>
<td>0.45</td>
<td>0.54</td>
<td>0.55</td>
</tr>
<tr>
<td>$0^+_{2}$</td>
<td>0.49</td>
<td>0.41</td>
<td>0.38</td>
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<tr>
<td>$0^+_{3}$</td>
<td>0.01</td>
<td>0.01</td>
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<td>$2^+_{1}$</td>
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<tr>
<td>$2^+_{2}$</td>
<td>1.02</td>
<td>0.88</td>
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<tr>
<td>$2^+_{3}$</td>
<td>0.34</td>
<td>0.36</td>
<td>0.16</td>
</tr>
</tbody>
</table>
### Table IV

| $\bar{n}$ | $\mathcal{F}_n$ | QSTD $\langle 0 | \Psi_{sp4}^+ \rangle = 0$ | QSTD $\langle 0 | \Psi_{sp4}^+ \rangle \neq 0$ | Exp. | Reaction |
|----------|----------------|----------------------------------|----------------------------------|-----|----------|
| 1/2$^+$  | 0.55           | 0.36                             | 0.65                             |     | Sn$^{116}$ (d, p) Sn$^{117}$ |
| 3/2$^+$  | 0.68           | 0.50                             | 0.55                             |     |          |
| 11/2$^-$ | 0.67           | 0.46                             | 0.81                             |     |          |
| 7/2$^+$  | 0.31           | 0.16                             | 0.13                             |     |          |
| 5/2$^+$  | 0.12           | 0.07                             | 0.061                            |     |          |
| 0$^+$    | 0.55           | 0.36                             | 0.45                             |     | Sn$^{117}$ (p, d) Sn$^{116}$ |
| 0$^+$    | 0.38           | 0.38                             | 0.087                            |     |          |
| 0$^+$    | 0.02           | 0.02                             | 0.092                            |     |          |

Comparison of the experimental spectroscopic factors of Refs. 14, 15 for the reactions Sn$^{116}$ (d, p) Sn$^{117}$ and Sn$^{117}$ (p, d) Sn$^{116}$ with the spectroscopic factors calculated using the two different definitions of the four-qp operators discussed in this paper.
Fig. 1

Comparison of QTD13 energy levels for Sn$^{117}$ with the experimental data of Refs. 14) and 15) and the values obtained by Kuo et al. (Ref. 2).
Fig. 2

Comparison of QTD13 energy levels for Sn$^{119}$ with the experimental data of Refs. 14) and 16) and the values obtained by Kuo et al. (Ref. 2).
Fig. 3

Comparison of QID13 energy levels for Sn$^{121}$ with the experimental data of Ref. 14) and with the levels calculated by Kuo et al. (Ref. 2).
Comparison of the QSTD energy levels $0^+$ and $2^+$ of Sn$^{116}$ with the experimental data of Refs. 14) and 15).

![Energy Level Diagram]

Fig. 4

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Comparison of the QSTD energy levels $0^+$ and $2^+$ of Sn$^{118}$ and the experimental levels of Ref. 14.

Fig. 5
Fig. 6

Comparison of the QSTD energy levels $0^+$ and $2^+$ of Sn$^{120}$ with the data of Ref. 14).
Spectroscopic factors $S_{J}(0^{+}, J^{π})$ for $(d, p)$ reactions on the even tin targets with $A=116, 118, 120$ leading to the lowest state of spin $J^{π}$ of the odd mass isotopes. The present results are connected by a solid line. The experimental values (Ref. 14) are connected by a dashed line, and the results of Kuo et al. (Ref. 2) are indicated as $O$. 

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Fig. 8. Spectroscopic factors $S_{j}^{(-)}(j, 0)$ for pick-up reactions on the even tin targets with $A=118$ and 120 leading to the lowest state of spin $j$ of the odd-mass isotopes. The present results are connected by a solid line; those obtained in Ref. 2) are connected by a dashed line; the data of Ref. 15) for Sn$^{118}$ $(p, d)$ Sn$^{117}$ are marked $\square$; the data for $(d, t)$ reactions quoted in Ref. 2) are marked $\times$. 
Fig. 9  Spectroscopic factors \( S_{e(j)} \left( \frac{1^+}{2^+} \right) \) for the \((d,p)\) reactions on the odd tin targets with \(A=117\) and 119 leading to the states of \(\text{Sn}^{118}\) and \(\text{Sn}^{120}\), respectively, with \(J^\pi = 0^+\) and \(2^+\) up to 3 MeV of excitation energy. Present results are plotted in the centre, results of Ref. 8) to the right. Among the experimental data 14), plotted on the left, the states not labelled explicitly have an ambiguous \((1^+, 2^+, 3^+)\) spin assignment.

Fig. 10  Spectroscopic factors \( S_{e(j)} \left( \frac{1^+}{2^+} \right) \) for the \((p,d)\) reactions on \(\text{Sn}^{117}\), leading to the states of \(\text{Sn}^{116}\) with \(J^n = 0^+\) and \(2^+\) up to 3 MeV excitation. Present results are plotted to the right and the experimental values of Ref. 15) to the left.

Captions for Figs. 1 - 8 appear on the same pages as the figures.
\[ S_{l}(\pm J) \left( \frac{1}{2}, J_n^+ \right) \]

Experiment (Ref. 14)

Present results

Ref. 8

\[ S_{l}(\pm J) \left( \frac{1}{2}, J_n^+ \right) \]

Sn\textsubscript{117} (d, p) Sn\textsubscript{118} 1.0

\[ S_{l}(\pm J) \left( \frac{1}{2}, J_n^+ \right) \]

Sn\textsubscript{119} (d, p) Sn\textsubscript{120}
Experiment (Ref. 15)

Present work

$S_l (\varepsilon J)$ (n,$\frac{1}{2}$)

$\text{Sn}^{117}\,(p,d)\,\text{Sn}^{116}$

Fig. 10
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