EVEN AND ODD PARITY STATES IN $^{9}$Be

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Results of a projected Hartree-Fock calculation are presented both for the even and odd parity states of $^9\text{Be}$. It is found that the even parity states come down very low in the energy spectrum and explain the occurrence of non-normal parity states at low energies.
EVEN AND ODD PARITY STATES IN $^9$Be.

There is now considerable experimental evidence $^1$ for the existence of very low-lying non-normal parity states in $^9$Be, the lowest one being a $^{3/2}_+ \text{state at } 1.7 \text{ MeV}$. It was suggested already several years ago by LANE $^2$ that these levels could be described by weakly coupling an Os or Od-neutron to the lowest states of $^8$Be. This idea was developed more quantitatively by BARKER $^3$ and recently also by ADLER et al. $^4$ in a somewhat modified form, using Elliott's SU$_3$-classification. Although they obtain very satisfactory agreement with the relative experimental energy splittings between the even parity states and with some other spectroscopic data, the position of these states in relation to the odd parity levels is not calculated. The question whether the occurrence of non-normal parity states at very low energies can be understood remains open.

Recent Hartree-Fock (HF) calculations for $^8$Be give some information about the existence of low-lying even parity states in $^9$Be. BASSICHIS, KERMAN and SVENNE $^5$ have used Tabakin's interaction in an unrestricted HF calculation for $^8$Be. For the lowest unoccupied HF orbitals with negative and positive parity they obtain, respectively, the following single-particle energies:

$$\varepsilon(m = \frac{3}{2}, \pi = -) = 3.95 \text{ MeV}$$
$$\varepsilon(m = \frac{1}{2}, \pi = +) = 10.68 \text{ MeV}.$$  

Both these single-particle energies are positive, which would mean that the $^9$Be nucleus is not bound, in contradiction to the experimental situation. The answer to this difficulty is probably that the correlation energy will be much larger for $^9$Be than for $^8$Be. This can be understood from the fact that the energy gap between occupied and unoccupied orbitals is larger for even even $Z = N$ nuclei than for neighbouring odd nuclei. The relative position of positive and negative parity levels in $^9$Be can roughly be estimated as the difference of these single-particle energies. This gives a value of $6.7 \text{ MeV}$ which is already much smaller than the value of $15 \text{ MeV}$ given by the single-particle shell-
model. A more accurate way to obtain this energy difference would require an angular momentum projection from the different intrinsic HF states.

In this letter, we present the results obtained from a projected Hartree-Fock (PHF) calculation for the lowest even and odd parity states of $^9$Be. We start from the Hamiltonian for the internal energy

$$H = \sum_i T_i - T_{cm} + \frac{1}{2} \sum_{i \neq j} V_{ij}$$

and take as nucleon nucleon interaction the potential of Volkov

$$V_{12} = \left\{ -83.34 \exp \left[ -\frac{r_{12}}{1.6}\right] + 144.86 \exp \left[ -\frac{r_{12}}{0.82}\right] \right\} \times
\times (0.4 + 0.6 P_x)$$

where $P_x$ is the Majorana space exchange operator. It was shown in previous PHF calculations that this force gives reasonable energies and wave functions for the low-lying levels of the lighter p-shell nuclei. As we are primarily interested here in the energy difference between even and odd parity states and not in the finer details of the wave functions, we believe that the neglect of non-central forces in the Hamiltonian can be justified.

Following the procedure described previously we take as trial function $\Psi_L = P_L \Phi$ where $P_L$ is a projection operator for the total orbital angular momentum and $\Phi$ is a single Slater determinant belonging to the most symmetric partition [441]. For the negative parity states

$$\Phi = \det \left| \psi_0^4 \psi_0^4 \psi_L^{n+} \right|$$

and for the positive parity states

$$\bar{\Phi} = \det \left| \psi_0^4 \psi_0^4 \chi_0^{n+} \right|$$

Here $\psi_0^4$ means that four orbitals have the spatial function $\psi_0$ with
the four different spin isospin labels, and \( \chi_0^{n+} \) is a neutron orbital with spin +\( \frac{1}{2} \) and spatial function \( \chi_0 \). The spatial functions are chosen as

\[
\psi_0 = |0s0\> + \beta |0do\>
\]

\[
\varphi_0 = |0po\> + \delta |0fo\>
\]

\[
\varphi_1 = |0p1\> + \zeta |0f1\>
\]

\[
\chi_0 = |1so\> - \sqrt{2} |0do\>
\]

The basic states \( |n\ell m\> \) are harmonic oscillator functions with phases and quantum numbers as in Ref.7. Putting \( \beta = \delta = \zeta = 0 \), the Slater determinants \( \Phi \) reduce to the leading states of the lowest representations with negative and positive parity of Elliott's SU\(_3\)-classification.

With the trial function \( \psi_L \), we calculate

\[
E_L (\beta \delta \zeta b b') = \frac{\langle \psi_L | H | \psi_L \rangle}{\langle \psi_L | \psi_L \rangle}
\]

using the method described in Ref.6, and minimize the energy with respect to the variational parameters \( \beta, \delta, \zeta, b \) and \( b' \). Here \( b \) and \( b' \) are the harmonic oscillator length parameters, respectively, of the even parity and odd parity orbitals. Taking them as variational parameters, we allow the different orbitals some freedom in choosing their spatial extension. The other parameters \( \beta, \delta \) and \( \zeta \) determine the deformation of the orbitals. It is seen that the orbital \( \chi_0 \) contains no independent variational parameter, either for its size or for its deformation. The reason for this is that the computer on which this calculation was programmed is not sufficiently large to introduce more variational parameters. For the same reason, the choice of the orbitals (5) is somewhat more restricted than those in our previous work 6).

For this calculation, we also had to truncate the polynomials, called \( F_{AB} \). 
in Ref.6, after terms of degree 6 in $\beta$, $\delta$ and $\zeta$. From our previous work, we estimate the uncertainty on the energy introduced by this truncation to be less than 0.3 MeV.

The results obtained for the negative parity levels with $L = 1,2,3,4$ and for the positive parity levels with $L = 0$ and 2 are presented in Table I. Here, $E^O_L$ is the energy obtained for $\beta = \delta = \zeta = 0$ and for the values of $b$ and $b'$ given in the table. In the last column we also give the deformation energies $E^O_L - E_L$, i.e., the energy gained by introducing the parameters $\beta$, $\delta$ and $\zeta$. This deformation energy is considerably larger for the two lower $L$-values of the negative parity band than for the two upper $L$-values. As a result, the exact rotational band spectrum of $E^O_L = A + B L(L + 1)$ is distorted considerably (Fig.1). As in our previous calculations, we again find that the deformation of the intrinsic state decreases as we go to higher $L$-values. This behaviour is just the opposite of what one might expect from the centrifugal force in a rotational picture.

TABLE I

<table>
<thead>
<tr>
<th>$\pi$</th>
<th>$E_L$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$\zeta$</th>
<th>$b$</th>
<th>$b'$</th>
<th>$E^O_L$</th>
<th>$E^O_L - E_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^9\text{Be}$</td>
<td>1-</td>
<td>-49.7</td>
<td>0.357</td>
<td>0.233</td>
<td>0.205</td>
<td>1.62</td>
<td>1.69</td>
<td>-38.89</td>
</tr>
<tr>
<td></td>
<td>2-</td>
<td>-47.7</td>
<td>0.336</td>
<td>0.220</td>
<td>0.201</td>
<td>1.62</td>
<td>1.69</td>
<td>-37.47</td>
</tr>
<tr>
<td></td>
<td>3-</td>
<td>-42.6</td>
<td>0.258</td>
<td>0.166</td>
<td>0.182</td>
<td>1.61</td>
<td>1.68</td>
<td>-35.35</td>
</tr>
<tr>
<td></td>
<td>4-</td>
<td>-39.6</td>
<td>0.229</td>
<td>0.153</td>
<td>0.181</td>
<td>1.61</td>
<td>1.67</td>
<td>-32.55</td>
</tr>
<tr>
<td></td>
<td>0+</td>
<td>-45.4</td>
<td>0.420</td>
<td>0.315</td>
<td>-</td>
<td>1.74</td>
<td>1.75</td>
<td>-28.89</td>
</tr>
<tr>
<td></td>
<td>2+</td>
<td>-43.0</td>
<td>0.395</td>
<td>0.296</td>
<td>-</td>
<td>1.74</td>
<td>1.75</td>
<td>-28.82</td>
</tr>
<tr>
<td>$^8\text{Be}$</td>
<td>0+</td>
<td>-53.35</td>
<td>0.375</td>
<td>0.239</td>
<td>-</td>
<td>1.61</td>
<td>1.65</td>
<td>-</td>
</tr>
</tbody>
</table>

Energies and wave functions for the low-lying negative and positive parity states of $^9\text{Be}$ and for the ground state of $^8\text{Be}$. 
The most striking result of Table I is the very large
deformation energy for the positive parity levels. Whereas these
levels lie well above the four odd parity states when the deformation
is neglected, they drop down very low between the negative parity
levels when the deformation attains its stable value. The energy
difference between the lowest negative and lowest positive parity level comes
down from 10 MeV to 4.3 MeV. Since the deformation for the \( L = 0 \)
state is very large, it is to be expected that an additional freedom
for the orbital \( \gamma_0 \) to adjust its deformation to that of the other
orbitals will give an additional energy gain, so that the experimental
value 1.7 MeV may well be reached.

In the last row of Table I, we have also given, for comparison,
the energy and wave function obtained for the ground state of \(^8\)Be,
using the trial function \( \Psi_L = 0 = \Phi_L = 0 \) \( \Phi \) , where

\[
\Phi = \det \begin{vmatrix} \psi_0^4 & \psi_0^4 \\ \psi_0^4 & \psi_0^4 \end{vmatrix}
\]

(7)

and \( \psi_0 \), \( \psi_0 \) are given by (5). A larger binding energy is found for
\(^8\)Be than for \(^9\)Be, just as in the HF-calculation of Bassichis et al.,
mentioned above. The same remark about the correlation energy is valid
here. From Table I, it is seen that the size and deformation of the
negative parity states of \(^9\)Be is quite similar to those of the ground
state of \(^8\)Be. The positive parity orbitals have, however, both a
larger size and deformation.

In view of the large deformations obtained in this calculation
it is also interesting to calculate the quadrupole moment of the
ground state of \(^9\)Be. The experimental value is not well known, the
different determinations \(^8\) yielding values ranging between 2.9 and
6.9 fm \(^2\). The most reliable value seems to be obtained from recent high-
energy electron scattering experiments \(^6\) , giving the value \( Q = 3.8 \) fm \(^2\).
The same measurements also determine the r.m.s. radius of the charge
distribution to be \( R = 2.5 \) fm.

On the simple shell-model in LS-coupling, one gets \( Q = \frac{3}{5} b^2 \).
Choosing \( b \) so that the experimental radius is reproduced (\( b = 1.77 \) fm)
gives a quadrupole moment \( Q = 1.88 \) fm \(^2\). The PHF-function gives
\( R = 2.41 \) fm and \( Q = 2.73 \) fm \(^2\). From intermediate coupling calculations \(^9\)\(^10\),
it is well known that even a weak spin orbit force brings about an important increase of the quadrupole moment. This is due to the fact that the lowest partition \[ \{441\} \] of \(^9\text{Be}\) contains \(L = 1,2,3,4\) and a weak spin orbit force will mix some \(L = 2\) state with the \(L = 1\) ground state. In a detailed calculation for \(^9\text{Be}\), it is consequently not allowed to neglect the spin orbit forces. This neglect was possible in our previous calculations \(^6\) for \(4 \leq A \leq 8\) since the lowest partition for these nuclei only contains \(L\)-values which increase by values of 2. However, since the deviation from pure LS-coupling is known \(^9\) to increase \(Q\) by somewhat more than 1 fm\(^2\), our calculated value favours the recent experimental value. A more detailed calculation, including the effect of the spin orbit forces, is in progress.

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Plot of the energies $E^0_L$ and $E_L$ for the odd-parity states as a function of $(L + 1)$.

Fig. 1