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CONDITIONS FOR COMPOSITENESS IN FIELD THEORY † *

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ABSTRACT

The Fried-Jin derivation of the condition $Z = 0$ is re-examined making explicit use of the HNZ construction of a composite particle field. This leads to another condition for compositeness originally due to Nishijima. The latter condition is shown to have important implications concerning the vertex renormalization constants and the mass shift.

CONDITIONS FOR COMPOSITENESS IN FIELD THEORY

I. INTRODUCTION

Nowadays there seems to be an increasing interest in the $Z_3 = 0$ condition for compositeness, after its rather obscure (in the sense of having been largely unnoticed) start more that ten years ago.[1] Until very recently the condition $Z_3 = 0$ was considered only within the context of Lagrangian field theory but after a brief letter by FRIED and JIN[2] it has also been studied within the framework of the "axiomatic" approach. In this context there has been a great deal of discussion concerning i) whether any meaning can be attached to the FRIED-JIN[2] derivation of the JOUVET[1] condition and ii) whether any other additional condition for compositeness can be formulated.

This note is an attempt at answering these questions. To this end, some necessary basic assumptions and conventions of notation are first given in Sec. II. In Sec. III, which is essentially based on the LSZ formalism, two conditions for compositeness are given, the JOUVET condition[1] $Z_3 = 0$ and the NISHIJIMA condition[3] which, in Lagrangian field theory, is equivalent to saying that the self-energy of the particle concerned is divergent. Here the key tool used is the HNZ construction[4), 5), 6)] of a composite particle field. The two conditions mentioned above are, of course, only necessary conditions; the important point, however, is that both can be justified outside Lagrangian field theory. This point must be emphasised here because the relevant arguments in Ref. 2 are based on Ref. 3 which explicitly deals with Lagrangian field theory.

In Sec. IV an explicit Lagrangian is used to illustrate the neat analogy, if not equivalence, of the HNZ construction[4), 5), 6)] with the work of BROIDO and TAYLOR.[7] Further, an additional justification of the Nishijima condition is given and it is shown that it implies that the composite particle, b say, must satisfy the condition

$$\lim_{Z_3 \to 0} \frac{Z_1}{Z_3\, \delta b^2} = 0 \,, \qquad (I.1)$$

if the particle $\underline{b}$ is the limit of an elementary particle with renormalization constant Z_3. The symbol Z_1 stands for the vertex renormalization constant. Finally, the conditions under which relation (I. 1) holds when Z_1 vanishes or $Z_3 \, \delta b^2$ diverges are also studied. Assuming that particle $\underline{b}$ has two constituents 1 and 2 it is shown that $Z_3 \, \delta b^2$ is necessarily divergent in the case when particles 1 and 2 have at least one quantum number different, whereas if this is not the case, any one of the two conditions can be satisfied. In particular, if $Z_3 \, \delta b^2$ is finite or vanishes, then Z_1 itself must vanish, as was first conjectured by SALAM.[8)]

Only stable, massive, neutral and spinless particles are considered throughout.

II. BASIC ASSUMPTIONS AND NOTATION

The axiomatic formalism of Lehmann, Symanzik and Zimmermann is assumed. Further, it is postulated that to every particle $\underline{a}$ there can be associated one and only one field operator $\phi_a(x)$, called interpolating field, which satisfies the canonical commutation relations

$$[\phi_a(x) \, , \, \phi_b(y)]_{x_0 = y_0} = 0 \qquad \text{for all } a, b,$$

and

$$[\phi_a(x) \, , \, \dot{\phi}_b(y)]_{x_0 = y_0} = \delta_{ab} \, i \, Z^{-1}_{(a)_3} \, \delta^{(3)}(\underline{x} - \underline{y}) \, . \tag{II. 1}$$

It is well known that if the τ-functions

$$\tau_{a \ldots b}(x_1 \ldots x_n) = \langle 0 | \, T \, \phi_a(x_1) \ldots \phi_b(x_n) \, | 0 \rangle \, , \tag{II. 2}$$

are given, the dynamics of the theory is completely determined. Conversely, if the off-shell S-matrix is known the τ-functions and the field operators themselves can, in principle, be determined.

The Fourier transform of the τ-functions is

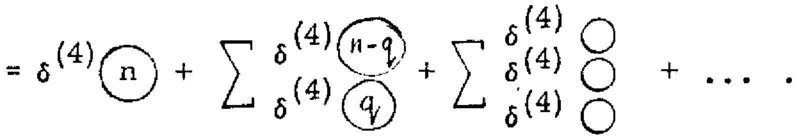
$$\tilde{\tau}_{a \ldots b}(k_1 \ldots k_n) = \langle 0 | \, T \, \tilde{\phi}_a(k_1) \ldots \tilde{\phi}_b(k_n) \, | 0 \rangle \tag{II. 3}$$

and these new functions have a unique decomposition[9] into connected parts $\tilde{\eta}$,

$$\tilde{\tau}(k_1 \ldots k_n) = \delta^{(4)}(\Sigma k_i)\, \tilde{\eta}(k_1 \ldots k_n) +$$

$$+ \sum \delta^{(4)}(\Sigma k_i)\, \delta^{(4)}(\Sigma k_i)\, \tilde{\eta}(k_1 \ldots k_q)\, \tilde{\eta}(k_{q+1} \ldots k_n) + \ldots$$

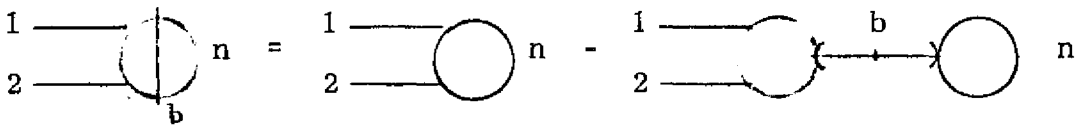
which, diagrammatically, is written as*)



(II. 4)

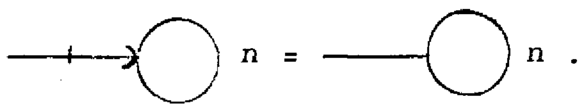
These $\tilde{\eta}$ functions have no δ-singularities and in perturbation theory they correspond to infinite sums of connected Feynman diagrams. Co-ordinate space η-functions can be defined by an inverse Fourier transform.

If the system of two particles 1 and 2 has the quantum numbers of particle <u>b</u> then the <u>irreducible</u> $\tilde{\eta}$ function



(II. 5)

has no pole at $(k_1 + k_2)^2 = b^2$. In eq. (II. 5) the notched line ——+—— denotes the b-particle propagator and)◯ is defined by the equation



III. COMPOSITE PARTICLES

It is natural to expect a composite particle to be logically dependent on its constituents. Mathematically this has been made explicit by assuming that the interpolating composite particle field can be expressed

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*) This diagrammatic notation has been introduced by SYMANZIK[10] and developed by TAYLOR.[11]

in terms of the interpolating fields of the constituents [4), 5), 6), 12)].

In what follows only the simple case

$$\phi_b(x) = \lambda \phi_1(x)\, \phi_2(x) \qquad \text{(III. 1)}$$

is considered. In this equation the local product is defined by a very specific (though not specified) limiting procedure, namely

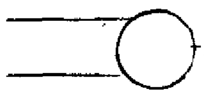
$$\phi_b(x) = \lim_{\xi \to 0} \frac{\phi_b(x,\xi)}{F_K(\xi)} \qquad \text{(III. 2a)}$$

where

$$\phi_b(x,\xi) = \phi_1(x + \alpha_2 \xi)\, \phi_2(x - \alpha_1 \xi) \quad , \qquad \alpha_1 + \alpha_2 = 1 \qquad \text{(III. 2b)}$$

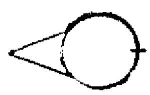
and

$$F_K(\xi) = <0|\, \phi_b(x,\xi)\, |K>$$

$= \begin{matrix} \alpha_2 \xi \\ -\alpha_1 \xi \end{matrix}$  $K \Big|_{K^2 = b^2}$. (III. 2c)

The ket $|K>$ is a one-b-particle state (as defined in Ref. 7) of momentum K , and the limit is such that $\phi_b(x)$ does not depend on it. The limit

$$\lambda^{-1} = \lim_{\xi \to 0} F_K(\xi)$$

$=$  $K \Big|_{K^2 = b^2}$ (III. 3)

does not necessarily exist.

The way in which ξ approaches zero may be crucial for the definition of $\phi_b(x)$. For simplicity it has been assumed that there are selection rules which ensure that

$$<0|\, \phi_1(x)\, \phi_2(y)\, |0> \; = \; 0 \; . \qquad \text{(III. 4)}$$

The postulate given at the beginning of Sec. II ensures that the field $\phi_b(x)$ is the only field associated with the particle b and must satisfy the commutation relations (II. 1). In general, however, a commutator of the form

$$[\phi_C(x)\,\phi_D(x)\,,\,\frac{d}{dy_0}\left\{\phi_C(y)\,\phi_D(y)\right\}]_{x_0=y_0}\quad,$$

with two arbitrary particle fields, ϕ_C and ϕ_D, is not a C-number and hence the commutation relations become a condition for compositeness *) in the case of the field ϕ_b defined by eq. (III. 2). This condition has to be imposed using the explicit limiting procedure of definition (III. 2),

$$\lim_{\substack{\xi\to 0\\ \eta\to 0}} \frac{[\phi_b(x,\xi)\,,\,\dot\phi(y,\eta)]_{\text{equal times}}}{F_K(\xi)\,F_{K'}(\eta)} = i\,Z_3^{-1}\,\delta^{(3)}\,(\underline{x}-\underline{y})\ .$$

Now, as this is a c-number it is equal to its vacuum expectation value. Taking first $\xi_0=\eta_0=0$ the "equal times" condition is exactly $x_0=y_0$ and the previous relation reduces to **)

$$\lim\left[\frac{Z^{-1}_{(1)_3}\,\Delta^{+'}_{(2)}\,(\eta-\xi)+Z^{-1}_{(2)_3}\,\Delta^{+'}\,(\xi-\eta)}{F_K(\xi)\,F_{K'}(\eta)}\right] i\,\delta^{(3)}(x-y)=i\,Z_3^{-1}\,\delta^{(3)}(\underline{x}-\underline{y})$$

after the limit is exchanged with the vacuum expectation value operation. Hence

$$Z_3=\lim_{\substack{\xi\to 0\\ \eta\to 0}}\left[\frac{F_K(\xi)\,F_{K'}(\eta)}{Z^{-1}_{(1)_3}\,\Delta^{+'}_{(2)}\,(\eta-\xi)+Z^{-1}_{(2)_3}\,\Delta^{+'}_{(1)}\,(\xi-\eta)}\right]\ . \qquad \text{(III. 5)}$$

As the function $\Delta^{+'}(x)$ is known to diverge in the origin at least as $1/x^2$, if the function $F_K(\xi)$ stays finite for $\xi^2\to 0$ the condition $Z_3=0$ is guaranteed without further analysis. If, on the other hand, the function $F_K(\xi)$ diverges at the origin, as claimed below , the two limits ($\xi\to 0$ and $\eta\to 0$) must be taken simultaneously; otherwise the first limit

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*) Thus one of the arguments given by BRANDT et al.[13] against the FRIED and JIN[2] derivation of $Z=0$ is explicitly excluded.

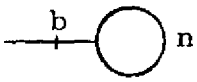
**) In fact one has to set $\xi_0=\eta_0$ before taking the limit operations outside the commutator.

makes expression (III. 5) diverge before the denominator in the right-hand side does.

NISHIJIMA[3] has given very good reasons to believe that the function $F_K(\xi)$ diverges at the origin and this makes λ in definition (III. 2) equal to zero, contrary to the view maintained by BROIDO and TAYLOR.[7] A vanishing λ is not paradoxical, since the interesting limit is (III. 2a) and it is well known that products of field operators at the same point are highly divergent.

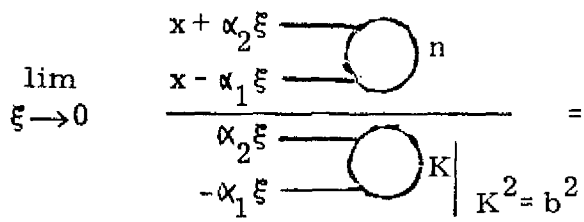
To support the claim that $F_K(\xi)$ diverges at the origin, a slightly modified version of Nishijima's arguments is now given.

The uniqueness of the field $\phi_b(x)$ implies that



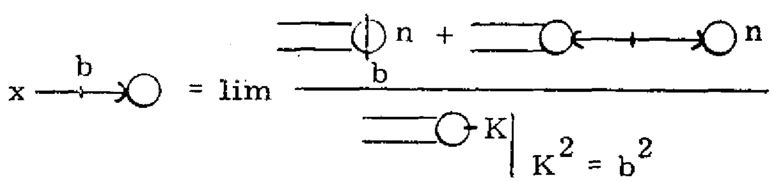
$$= \langle 0 | T\, \phi_b(x)\, \phi_C\, \phi_D \ldots | 0 \rangle_{conn}$$

is equal to



$$= \lim_{\xi \to 0} \langle 0 | T \frac{\phi_b(x,\xi)}{F_K(\xi)} \phi_C \phi_D \ldots | 0 \rangle \Big|_{conn}$$

where the subindex "conn" means "connected part of". Using relation (II. 5) the numerator in the left-hand side of the last equation can be decomposed into two parts, yielding,



(III. 6)

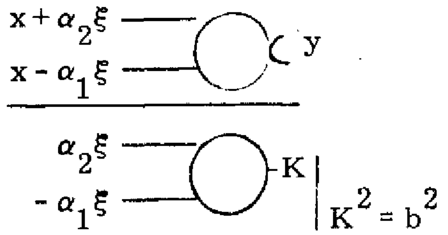
Roughly speaking, this is an equation of the form

$$A = \frac{B + FA}{F} \qquad \text{(III. 7)}$$

which, for A and B different from zero, has as the only solution $F = \infty$ and thus

$$\lim_{\xi \to 0} F_K(\xi) = \infty . \qquad \text{(III. 8)}$$

This relation will be called the Nishijima condition for compositeness.*) It must be remarked that in eq. (III. 7) two different quantities were denoted by F ; this is justified because[3)]

$\lim_{\xi \to 0}$  $= \delta^{(4)}(x - y)$,

which in the loose notation of eq. (III. 7) means that this quotient is "one". By a model-dependent analysis it is also possible to prove that $F_K(\xi)$ diverges logarithmically at the origin[3)] and therefore the condition $Z_3 = 0$ in eq. (III. 5) is ensured.

The Nishijima condition eq. (III. 8) is even more direct in Lagrangian field theory because $F_K(0)$,

$F_K(0) =$  , (III. 9)

represents the self-energy of the particle b and self-energies are known to be divergent in all known models with the exception of the photon in Q. E. D. This is precisely the reason given by Nishijima for claiming that the photon is not a composite particle.

To summarize, the two conditions for compositeness given in this section are

$$Z_3 = 0 \qquad \text{(Jouvet condition)} \qquad \text{(III. 10)}$$

and the condition (III. 8) which, when inverted and with the help of eq. (III. 3), yields

$$\lambda = 0 \qquad \text{(Nishijima condition)} . \qquad \text{(III. 11)}$$

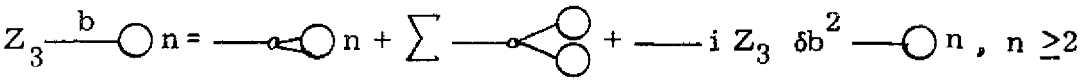
*) Notice that in simple field theoretical models, like the Lee model, the irreducible part B in eq.(III.7) is identically zero and therefore the Nishijima condition cannot be derived.

IV. EXTENSION TO LAGRANGIAN FIELD THEORY [*)]

In this section a composite particle b is assumed to be described in a consistent way as the limit of an elementary particle b with renormalization constant Z_3 set equal to zero. To start with, the initial Lagrangian ($Z_3 \neq 0$) is

$$\mathcal{L} = \sum_{1,2,b} [Z_{(i)_3} \mathcal{L}_i^{free} (\phi_i) + \frac{Z_{(i)_3}}{2} \delta\mu_i^2 (\phi_i)^2] + g Z_1 \phi_1 \phi_2 \phi_b \quad . \tag{IV.1}$$

There are at least two ways, not obviously equivalent, of defining the $Z_3 = 0$ limiting theory: one can either take the limit in the equations of motion corresponding to the Lagrangian (IV.1) or take the limit $\mathcal{L}(Z_3 = 0)$ and afterwards derive the equations of motion from this new Lagrangian. The equations of motion for the particle b derived from the Lagrangian IV.1) are [7)]

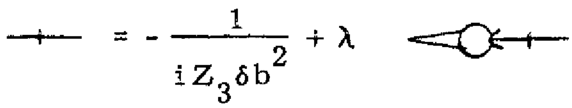


and

(IV.2)

which in the limit $Z_3 \to 0$ give [7)]

and



(IV.3)

with

$$\lambda = - \lim_{Z_3 \to 0} \frac{g Z_1}{Z_3 \delta b^2} \tag{IV.4}$$

and

$$\circ = i\, g\, Z_1 \quad . \tag{IV.5}$$

*) Concerning the description of a composite particle in Lagrangian field theory see, for instance, Ref. 7 and papers quoted therein.

From the second of the eqs. (IV. 3) it is easy to show that

λ  $K\Big|_{K^2 = b^2} = 1$.   (IV. 6)

This equation immediately proves that λ , as defined by eq. (IV. 4), is identical to the λ defined by eq. (III. 3) in the previous section. This is an indication that the HNZ construction, eq. (III. 2), is equivalent to the formalism given in Ref. 7 and therefore that Broido and Taylor are wrong when they assume that λ must be non-zero to allow for the existence of the composite particle. Only in simple models where the self-energy is finite and there are no irreducible diagrams is it possible to violate the Nishijima condition, eq. (III. 11).

Since g , in relation (IV. 4), is the physical coupling constant, the vanishing of λ implies condition (I. 1), which in turn means that either

$$Z_1 = 0 \qquad \text{(IV. 7)}$$

or

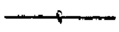
$$Z_3 \, \delta b^2 = \infty \qquad \text{(IV. 8)}$$

Before discussing these relations it is convenient to consider the second way of obtaining the limit $Z_3 = 0$, as explained below eq. (IV. 1).

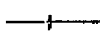
It is very easy to show that the Euler-Lagrange equation for the field ϕ_b coming from the Lagrangian $\mathcal{L} = \mathcal{L}(Z_3 = 0)$ is

$$\phi_b(x) = \lambda \, \phi_1(x) \, \phi_2(x) \ , \qquad \text{(IV. 9)}$$

that is, eq. (III. 1) is re-obtained. Now, using this equation and the definition of the propagator of the particle b , i. e.,

 $= \langle 0 | T \, \phi_b(x) \, \phi_b(y) | 0 \rangle$

it follows that

 $= \lambda$    (IV. 10)

from which two important conclusions can be drawn. First, in momentum space it is possible to cancel the explicit propagators of eq. (IV. 10), thus obtaining the relation

λ  $K = 1$ (IV.11)

off-shell. This is a remarkable result in that it contains eq. (IV.6) as a particular case. However, as the left-hand side of eq. (IV.11) is a function of K^2 and not a constant as the equation suggests, the only alternative is to accept that

 $= \lambda^{-1} + f(K)$

with $\lambda \to 0$. This is again in favour of the Nishijima condition, eq. (III.11), and the fact that in Ref.7 the possibility $\lambda = 0$ was not supported can be understood as due to the fact that the authors did not realize that eq. (IV.11) can be obtained (although they almost obtained it). It is emphasised again that in simple models eq. (IV.11) can perfectly well be a finite equation.

If the two limiting procedures are not equivalent the conclusions are the following. First, it can be shown that, except for the propagator, all Green functions coincide in both limits (compare eq. (IV.3) and eq. (IV.10)). This, however, is self-contradictory since the unitarity relation for the propagator is

$$\mathrm{Im}\, \Delta'_F(s) = \rho(s)\, |\Gamma \Delta'_F|^2 + \text{ higher Green functions}$$

and cannot be valid for two different propagators and the same higher Green functions. Hence, only one of the two limiting theories could hold.

Postulating now that the two limiting procedures which make Z_3 vanish define the same composite-particle theory [7], then a second conclusion is obtained by comparison of eq. (IV.10) with eq. (IV.3),

$$Z_3\, \delta b^2 = \infty \,, \qquad \text{(IV.12)}$$

which is eq. (IV.8). This postulate seems to be a requirement that the composite particle theory be a well-defined one, or at least that the limit be well defined.[*] Some simple models do not give the same result in the two limits, but this could simply be a peculiarity of the models, since they always violate at least one of the general principles of field theory.

Finally, when condition (III.4) is lifted, the derivation of eq. (IV.10) gives an extra constant term which can be identified with $i/Z_3\, \delta b^2$. In such

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*) See Ref. 14 for peculiar results that can be obtained if this is not postulated.

a case eq. (IV. 12) does not necessarily hold but then condition (I. 1) requires that

$$Z_1 = 0 \ ,$$

that is, eq. (IV. 7) is now obtained. This condition, therefore, appears related to the non-existence of selection rules between the constituent fields, whereas eq. (IV. 12) necessarily holds if condition (III. 4) is satisfied, that is, if such selection rules do exist. It appears that the fact that the relative nature of the constituents could fix the vertex function renormalization constant as well as the mass shift has not been realized before. Physically this is a perfectly acceptable situation and it is desirable that this line of investigation be further pursued.

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REFERENCES

1) B. JOUVET, Nuovo Cimento 5, 1 (1957).

2) H.M. FRIED and Y.S. JIN, Phys. Rev. Letter s 17, 1152 (1966).

3) K. NISHIJIMA, Phys. Rev. 133, B 204 (1964);
K. NISHIJIMA, "High-Energy Physics and Elementary Particles",
IAEA, Vienna, 1965, p.137.

4) R. HAAG, Phys. Rev. 112, 669 (1958).

5) K. NISHIJIMA, Phys. Rev. 111, 995 (1958).

6) W. ZIMMERMANN, Nuovo Cimento 10, 597 (1958).

7) M.M. BROIDO and J.G. TAYLOR, Phys. Rev. 147, 993 (1966).

8) ABDUS SALAM, Nuovo Cimento 25, 224 (1962).

9) W. ZIMMERMANN, Nuovo Cimento 13, 503 (1959).

10) K. SYMANZIK, Hercegnovi Summer School, Yugoslavia, 1961.

11) J.G. TAYLOR, Nuovo Cimento 1, 857 (1963).

12) P.P. DIVAKARAN, Phys. Rev. 160, 1468 (1967).

13) R.A. BRANDT, J. SUCKER and C.H. WOO, Phys. Rev. 19, 801 (1967).

14) H. OSBORN, Phys. Rev. Letters 19, 192 , 411(E) (1967).

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