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ABSTRACT

In this paper we propose a mechanism for explaining the polarization in high-energy charge-exchange πN scattering, based on O(3,1) partial wave analysis. We found that the non-vanishing polarization may be understood in terms of one Lorentz pole \( \sigma_\rho = \sigma_\rho + 1 \) and that it is due to the interference of \( \rho \) with another Regge pole \( \rho' \) belonging to the family \( j_0 = 0 \) and given in terms of \( \sigma_\rho \) by the relation \( \sigma_{\rho'} = \sigma_\rho^{-2} \).
It is well known that in order to explain the polarization phenomenon in high-energy meson-nucleon charge-exchange scattering, one has to modify the single Regge pole model \(^1\) (\(\rho\)-trajectory) either by introducing other singularities in the \(J\)-plane (Regge cuts, \(\rho'\) Regge pole) or by taking into account direct channel effects, for example the effect of tails of low-energy resonances at high energies.

The aim of this work is to study the high-energy behaviour of polarization within a simple model in which only one Lorentz pole, \(\sigma_\rho(t) = \alpha_\rho(t) + 1\), degenerate with respect to various quantum numbers, \((j_0, \text{Lorentz signature } \chi)\) is taken into account.

According to their results, the dominance of a Lorentz pole at \(\sigma_\rho(t) = \sigma_\rho(t) + 1\) may reproduce all the qualitative features of the experimental angular distribution, provided that the \(\rho\)-trajectory belongs to a mixed representation with \(j_0 = 0\) and \(j_0 = 1\).

In what follows we denote by \(f_{++}(s,t)\) and \(f_{+-}(s,t)\) the non-spin flip and the spin flip helicity amplitudes, respectively, corresponding to the direct channel (\(s\)-channel) of the process.
where $X_1, X_3$ are the helicities of the mesons and $X_2, X_4$ the helicities of the nucleons. The $O(3,1)$ expansion of $f_{++}(s,t)$ and $f_{+-}(s,t)$ is given by the following formulae:

$$\frac{1}{2\pi i} \int_{-\infty}^{\infty} d\sigma \ T_{++}^0(\sigma,t) \ d^{0\sigma}_{000}(\xi_t)$$

$$\frac{1}{2\pi i} \int_{-\infty}^{\infty} d\sigma \ [\delta^2 \ T_{+-}^0(\sigma,t) \ d^{0\sigma}_{444}(\xi_t) - (1-\sigma^2) \ T_{+-}^1(\sigma,t) \ d^{1\sigma}_{444}(\xi_t)]$$

where $T_{++}^0(\sigma,t)$, $T_{+-}^0(\sigma,t)$ are the $O(3,1)$ "partial wave" amplitudes ($j_0 = 0, 1$) and

$$\chi_{\xi_t} = \frac{S - u}{(4m^2_{X_N} - t)^{\frac{1}{2}} (4m^2_{X_N} - t)^{\frac{1}{2}} \lambda_0}$$

Before introducing Lorentz poles in the $\sigma$-plane, we modify formulae (2), defining reduced amplitudes $T_{++}^0(\sigma,t)$ and $T_{++}^0(\sigma,t)$ having $O(3,1)$ signature $X$. Following D. AKYEAMPO, J. BOYCE and M.A. RASHID we may write ($\chi = \pm 1$):

$$\frac{1}{2\pi i} \int_{-\infty}^{\infty} d\sigma \ T_{++}^0(\sigma,t) \ d^{0\sigma}_{000}(\xi_t)$$

$$\frac{1}{2\pi i} \int_{-\infty}^{\infty} d\sigma \ [\delta^2 \ T_{+-}^0(\sigma,t) \ d^{0\sigma}_{444}(\xi_t) - (1-\sigma^2) \ T_{+-}^1(\sigma,t) \ d^{1\sigma}_{444}(\xi_t)]$$

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where $T_{++}^0(\sigma,t)$, $T_{+-}^0(\sigma,t)$ are the $O(3,1)$ "partial wave" amplitudes ($j_0 = 0, 1$) and

$$\chi_{\xi_t} = \frac{S - u}{(4m^2_{X_N} - t)^{\frac{1}{2}} (4m^2_{X_N} - t)^{\frac{1}{2}} \lambda_0}$$
We assume now that the helicity amplitudes (3) are dominated by one Lorentz pole \( \sigma \). We also assume that the location of the Lorentz pole is independent of \( j_0 \). Therefore all amplitudes \( T_{++}^{0X} \), \( T_{+-}^{0X} \), \( T_{++}^{1X} \) have a pole at the same point \( \sigma = \sigma(t) \) corresponding to a "degenerate" trajectory in the sense that it accommodates both \( j_0 = 0 \) and \( j_0 = 1 \) states. 3) If we shift the path of integration in formulae (3) to the left, we obtain the following expressions for the contribution of the Lorentz pole \( \sigma \):

\[
\frac{1}{\lambda} \left[ \rho_{++}^{\sigma} \right]_{L=0} = \sum_x \left( \frac{\sigma_9^2}{\sin \pi \sigma_9} \right) \frac{1 - \lambda \cdot e^{i \pi \sigma_9}}{2} \left( \xi_{++}^0 \right) \beta_{++}^{\sigma} (t)
\]

It is easy to show that the signatures of the Regge trajectories, associated with the Lorentz pole \( \sigma \), are given by the formula

\[
\xi_n = \chi (-1)^n
\]

where \( \xi_n \) is the signature of the \( n \)th member of the family: \( \alpha_n = \sigma - n - 1 \). Since the only Regge poles which are allowed in this process are those with the quantum numbers of the \( \rho \)-meson, namely with negative signature \( (\xi = -1) \) and natural parity \( (\eta \xi = +1) \), we distinguish the following coupling schemes (\( \eta \) is the parity of the exchanged particle):

a) The leading Regge particle, associated with the amplitudes
$T^{0X}_{++} \sim \beta^{0X}_{++} d_{000}^0 \rho$ and $T^{1X}_{+-} \sim \beta^{1X}_{+-} d_{111}^1 \rho$, corresponds to the member $n = 0$

of the family $\alpha_n = \sigma - n - 1$ and therefore, according to (5), we must

have $\chi = -1$ since $\xi = -1$ ($\rho$-meson). Hence, the non-spin flip

amplitude $f_{++}$ and the leading part of the spin flip amplitude $(\beta_{++}^{1X})$ are

controlled by the exchange of the $j_0 = 0$ and $j_0 = 1$ components of the

$\rho$-trajectory.

b) The leading Regge trajectory, associated with the amplitude

$T^{0X}_{+-} \sim \beta^{0X}_{+-} d_{111}^1 \rho$ which contributes to the spin flip amplitude (formulae (4)), corresponds to the member $n = 1$ of the family $\alpha_n = \sigma - n - 1$ ,
as can easily be shown by decomposing the $O(3,1)$ d-function $d_{111}^1 \rho$
in terms of $O(2,1)$ matrix elements, following the formalism of

SCIARRINO and TOLLER. The contribution of this Regge pole is not

zero only if $\xi = -1$, i.e., if $\chi = +1$, according to formula (5).

We identify this Regge particle with the $\rho'$ meson which therefore belongs to the following class: $j_0 = 0$, $\eta \xi = +1$, $\xi = -1$.

Moreover, the trajectory $\alpha_{\rho'}(t)$ is given in terms of $\alpha_{\rho}(t)$ by the following simple relation:

$$\alpha_{\rho'}(t) = \alpha_{\rho}(t) - 1$$

(6)

Keeping now the appropriate signatures in formulae (4) and using the asymptotic expressions of the d-functions, we may write the contributions of $\rho$ and $\rho'$ as follows:

$$f_{++}(s,t) = \frac{\beta^{0X}_{++}}{S_{++} \pi \lambda g} \left(1 + \frac{-i \pi \lambda g}{2} \right) \left(\alpha_{\rho'} + 1\right) \left(\frac{s}{2 m_n m_\pi}\right)^{\lambda g}$$

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We observe that, although the angular distribution is dominated by the exchange of the $\rho$-meson alone, the non-vanishing polarization at high energies is due to the interference between the $\rho$ and $\rho'$ terms in eqs. (7). This mechanism coincides with the $\rho + \rho'$ model introduced for the explanation of the polarization within the Regge theory \cite{6,7}, so the identification of the amplitude $T_{++}^{0X} \sim \beta_{++}^{0X} \rho_1^{00}$ with the conventional $\rho'$ meson exchange is therefore justified. The details of the mechanism which explains the polarization data within the present model are as follows.

The differential cross-section $d\sigma/dt$ of the reaction $\pi^- p \to \pi^0 n$ is given by the formula

$$
\frac{d\sigma}{dt} = \frac{1}{2\pi s^2} \left( |\ell_{++}|^2 + |\ell_{++}|^2 \right) \quad (s \to \infty)
$$

and the polarization of the neutron $P$ by the expression

$$
P = -\frac{2 \text{Im} (\ell_{++} \ell_{+-}^*)}{|\ell_{++}|^2 + |\ell_{++}|^2} \quad (s \to \infty)
$$

From eqs. (7) and (9) we easily find that the energy dependence of the polarization $P$ in the asymptotic region $(s \to \infty)$ is $P \sim 1/s$.

The experimental results in $\pi N$ charge-exchange scattering at 5.9
and 11.2 GeV/c give for the mean value of $P$ in a given momentum transfer integral, the value $<P_{11.2}>/<P_{5.9}> = 0.8 \pm 0.2$. This experimental result indicates a slower decrease of $P$ with energy as compared with our prediction but, nevertheless, our result is consistent with the above value, within the experimental errors (Figs. 1 and 2). The explicit form of $P$ is found using eqs. (7) and (9). We obtain

$$
P = \frac{2\sqrt{3} \beta^{0-1}_{++} \beta^{01}_{+-} (\alpha_9 + i)^2 |t|^3/2}{(\alpha_9 + 2) \left[ 1/4 (\beta^{01}_{++})^2 (\alpha_9 + i)^2 + 3 |t| \alpha_9 (\beta^{01}_{+-})^2 \right]^2} \left( \frac{2 m_{\Pi} m_{\Pi}}{s} \right) (10)
$$

The $\rho$-trajectory is taken to be $\rho_\rho(t) = 0.57 + 0.91 t$.

The residues $\beta^{0-1}_{++}$, $\beta^{01}_{+-}$, treated as constants, are determined by fitting the angular distribution. In formula (10) the residue $\beta^{01}_{+-}$ remains a free parameter and it is chosen to fit the polarization data at 5.9 and 11.2 GeV/c in the process $\pi^- p \to \pi^0 n$. The results are shown in Figs. 1 and 2.

To conclude, we have shown that within the O(3,1) partial wave expansion of the $\pi N$ charge-exchange scattering amplitudes, two leading Regge poles emerge and they are identified with the $\rho$ and $\rho'$ mesons. The $\rho$-trajectory corresponds to the first member ($n = 0$) of the family $\rho_\rho = \sigma_\rho - n - 1$ generated by the Lorentz pole $\sigma_\rho$ with quantum numbers $\eta \chi = +1$, $\chi = 1$ ($j_0 = 0, 1$).

The $\rho'$ trajectory corresponds to the leading Regge pole which contributes to the amplitude $T^{0\chi}_{+-} = \beta^{0\chi}_{+-} \sigma_\rho^{00} d_{1-11}$ (formulae (4)). It is
generated by a Lorentz pole with quantum numbers \( j_0 = 0 \), \( \eta \chi = +1 \), \( \chi = +1 \) and lying on the same trajectory \( \sigma_\rho(t) \). Moreover, the \( \rho' \) trajectory is given in terms of \( \alpha_\rho \) by a simple relation \( \alpha_{\rho'} = \alpha_\rho - 1 \). We wish to point out that the \( \rho' \)-trajectory must not be interpreted as a daughter of the \( \rho \)-trajectory, since these Regge poles correspond to different \( O(3,1) \) signatures. From the interference of these two trajectories, a non-zero polarization is obtained and a reasonable fitting of the data is given.

Finally, we compare the present model with the results of Regge theory where the parameters of the \( \rho' \) are determined by fitting the experimental data. In order to satisfy the constraints of both the polarization data and the superconvergence relations, SERTORIO and TOLLER \(^9\) introduced a conspirator \( \rho' \) with Lorentz quantum number \( j_0 = 1 \) in disagreement with the present model according to which \( \rho' \) is a non-conspiring Regge pole with \( j_0 = 0 \). More recently, T. J. GAJDICAR, R. K. LOGAN and J. W. MOFFAT \(^10\), using the \( \rho + \rho' \) Regge model, have analysed in detail the \( \pi N \) charge-exchange scattering by fitting the differential cross-section, the charge-exchange polarization, the \( \pi p \) total cross-section, the real forward amplitude \( D^{(\ast)} \) and the non-spin flip superconvergence relations. They found that it is not necessary to introduce a conspiring \( \rho' \) trajectory, but that it is possible to satisfy the superconvergence relations and the polarization data with a non-conspiring \( \rho' \) (\( j_0 = 0 \)). Moreover, one of the two solutions of their model gives \( \alpha_{\rho'}(0) = -0.5 \), \( \alpha_\rho(0) = 0.57 \), in very good agreement with the prediction \( \alpha_{\rho'} = \alpha_\rho - 1 \).
Finally, H. HÖGAASEN and W. FISCHER $^7$), in their attempt to fit the experimental data on nucleon-nucleon charge-exchange scattering, found for the intercept of the $\rho'$ trajectory the value $\chi_{\rho'}(0) = -0.63$, again in good agreement with the result of our model.

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The polarization of the neutron in the reaction $\pi^- p \rightarrow \pi^0 n$ for pion momentum $P_{\text{lab}} = 5.9 \text{ GeV/c}$ is plotted. The solid line represents our result. Experimental points are also given.
The polarization of the neutron in the reaction $\pi^- p \rightarrow \pi^0 n$ for pion momentum $P_{\text{lab}} = 11.2 \text{ GeV}/c$ is plotted. The solid line represents our result. Experimental points are also given.