INELASTIC ELECTRON SCATTERING FROM EVEN TIN ISOTOPES AND MICROSCOPIC THEORIES OF VIBRATIONAL STATES

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ABSTRACT

Two- and four-quasiparticle microscopic theories are applied to fit the most recent data on Sn$^{116}$ $(e,e')$Sn$^{116}$ $(2^+_1,3^-_1)$ reactions. The effective nuclear force is the realistic nucleon-nucleon potential of Tabakin renormalized by the core polarization corrections. The observed angular distributions and the absolute values of the form factors can be reproduced for quite reasonable values of the single-particle parameters.

In a recent Letter BARREAU and BELLICARD$^1$ published the first experimental data on the inelastic electron scattering from the even tin isotopes 116, 120 and 124 with the excitation of the $2^+_1$ and the $3^-_1$ collective states. The bombarding electron energy was 150 MeV and the scattering angle varied between 45° and 80°. The corresponding electric quadrupole and octupole form factors, $|F_{in}(Q)|^2$, have been extracted from the differential cross-sections as:

-1-
Both the absolute values of $|F_{in}|^2$ and their angular distributions should, particularly when combined with the static electromagnetic moments and the corresponding values of $B(E\lambda)$, serve as a sensitive test of any microscopic or other nuclear wave functions of the excited states in question.

The low-lying excited states of even tin isotopes, particularly those of collective character, have been described satisfactorily in terms of the microscopic quasiparticle (qp) second Tamm-Dancoff (QSTD) theory. The eigenvectors of QSTD are superpositions of zero-, two- and four-qp modes of the neutrons of the five valence subshells: $2d_{5/2}, 1g_{7/2}, 3s_{1/2}, 2d_{3/2}$ and $1h_{11/2}$. The 50 neutrons and the 50 protons of the doubly magic core are responsible for a renormalization (core polarization) of the effective residual nucleon-nucleon potential of the valence neutrons, but they do not participate explicitly in the configuration mixing. As for the interaction of the nucleus with an electromagnetic field, the valence neutrons are characterized by effective electric- and magnetic-multipole operators determined by second- and higher-order interactions with the core protons. The concept of the effective operator is in our calculations approximated by that of the effective charge (a constant $e_\lambda$ for the $2\lambda$-pole).

We compare our QSTD form factors $|F_{in}|^2$ with the corresponding ones calculated: 1) with the approximation in which the $0^+$ ground state is the qp-vacuum itself ("uncorrelated", QSTD$_{uncor}$) and 2) with the corresponding pure two-qp Tamm-Dancoff (QTD) theory.

The theoretical results for $|F_{in}|^2$ vary generally only little in passing from one isotope to another. The same is true of the data of ref. 1). For this reason we present in this letter our $|F_{in}|^2$ corresponding to one isotope only, to Sn$^{116}$. -2-
In refs. 4) and 5) the QSTD and QTD theories have been applied to Sn$^{116}$ with the assumption of the realistic non-local nucleon-nucleon potential of TABAKIN 6). The reduced matrix elements for the five valence neutron subshells are renormalized for the core polarization by including the lowest-order RPA particle-hole bubble- and related exchange-diagrams of the core protons and neutrons (we have adopted the variant labelled "S2" in ref. 4) in our calculation reported below). The unperturbed single nucleon energies involved are those of BANDO 7): $2d_{5/2} : 0$, $1g_{7/2} : 0.40$, $3s_{1/2} : 1.90$, $2d_{3/2} : 2.20$ and $1h_{11/2} : 2.40$ for the extra core subshells and $1g_{9/2} : -4.0$, $2p_{1/2}$, $1f_{5/2}$ and $2p_{3/2} : -12.0$ for the core subshells (all the energies in MeV). The single-particle radial wave functions correspond to the harmonic parameter $\sqrt{\nu} = 0.45 F^{-1}$.

The QSTD ground state ($0^+_1$) is represented by a 56-component vector including appreciable 4-qp correlations of the qp-vacuum. The QSTD eigenvector of $2^+_1$ has 71 components in our approximation (the corresponding QTD vector has 9 components only). The $3^-_1$ QSTD eigenvector is approximated by 52 components (two components only in QTD). The QSTD eigenvalues are here $E(2^+_1) = 1.393$, $E(3^-_1) = 2.828$ MeV and the QSTD lowering of the ground state energy is $\Delta E(0^+_1) = 0.07$ MeV.

We have calculated both the Coulomb and the transverse electric parts (spin and current terms) of $|F_{in}|^2$ as defined in eq. (3.64) of ref. 8). Although we apply essentially only Born approximation, we include a correction for the distortion effects as proposed by CZYZ and GOTTFRIED 9) (cf. also eq. (8.13) of ref. 8)).

In Table I we give a comparison of the various different partial contributions to the reduced transition matrix elements of $|F_{in}|^2$.
corresponding to different parts of the eigenvectors involved. We give values of \( \langle J^\pi || M^\text{coul}_J || 0^+_1 \rangle \) and of \( \langle J^\pi || T^\text{el}_J || 0^+_1 \rangle \) where the operators \( M^\text{coul}_\lambda \) and \( T^\text{el}_\lambda \) are defined in eqs. (3.21) and (3.32) of ref. 8), respectively (here \( J = \lambda = 2, 3 \)). We consider only one value of the scattering angle \( \theta = 45^\circ \) since the main characteristics of the decomposition in question remain stable throughout the angular distribution.

The transverse electric terms are found negligible in all the cases not because of a cancellation between the spin and the current terms but rather because both are negligibly small in our case. The scattering is then practically purely Coulomb.

The predominant term in all the cases is the cross-term (AC) of the qp-vacuum component (C) of the ground state with the two-qp components (A) of the final state. The 2-qp- 4-qp (AB) cross-terms are small but non-negligible while the 4-qp- 2-qp (BA) terms are negligible. This is due to the extreme smallness of the 2-qp correlations in the ground state and to the smallness of the 4-qp terms in the particular final states in our problem. The latter fact is also responsible for the smallness of the 4-qp- 4-qp contributions (BB). The 2-qp- 2-qp terms (AA) are always negligible. The AC term is increased considerably and all the other terms disappear when the ground state is replaced by the qp-vacuum \( (\text{QSTD})_{\text{uncor}} \). A quite similar situation is obtained in the QTD case.

For the \( ^2_1 \) state we also compare our predictions with those obtained with a conventional \( P_2 \)-force with the value of the parameter \( X \) of ref. 2). The AC terms are smaller here and the AB- and BB-terms somewhat increased.
Appreciably smaller values of $\langle \| \hat{M} \| \rangle$ are obtained if only pure unperturbed 2-qp excitations of the independent quasiparticle model (IQM) are considered. If the state $2^+_1$ were to be interpreted as the least energetic 2-qp pair one would obtain $\langle 2^+_1 \| \hat{M}_2^{\text{coul}} \| 0^+_1 \rangle (\theta = 45^\circ) = 0.189$ (for $3^-_1$ we have $\langle 3^-_1 \| \hat{M}_3^{\text{coul}} \| 0^+_1 \rangle (\theta = 45^\circ) = 0.058$). For the $P_2$-force we find $\langle 2^+_1 \| M_2^{\text{coul}} \| 0^+_1 \rangle (\theta = 45^\circ) = 0.101$. These numbers are lower than the corresponding ones of Table I but not in a dramatic way. This difference is more marked for the $P_2$-force case ($\| F_{\text{in}} \|^2$ for IQM is 6 times smaller than that for QSTD).

In Fig. 1 are given the angular distributions of $\| F_{\text{in}} \|^2$ for the state $2^+_1$ calculated in QSTD approximation with the forces of Tabakin and $P_2$. Satisfactory agreement with the data of ref. 1) for $E_0 = 150$ MeV is obtained if one chooses the neutron effective charge $e_2 = 1.23$ for Tabakin and $e_2 = 1.57$ for $P_2$. These values are quite reasonable and compatible with those necessary to reproduce the $B(E2)$ data.

In Fig. 2 $\| F_{\text{in}} \|^2$ is given for the state $3^-_1$ as computed with the force of Tabakin. The data of ref. 1) are reproduced for $e_3 = 2.19$.

In concluding, we may state that the data of ref. 1) can be understood in terms of the microscopic quasiparticle theories, both the angular distribution and the absolute values of $\| F_{\text{in}} \|^2$.

ACKNOWLEDGMENTS

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2) P. L. OTTAVIANI, M. SAVOIA, J. SAWICKI and A. TOMASINI, Phys. Rev. 153, 1138 (1967);

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7) H. BANDO, Kyoto University preprint (1967).


TABLE CAPTION

Table 1

Reduced transition matrix elements \( \langle J_1^\pi \hat{M}_J^{\text{coul}} \parallel 0^+ \rangle \) and
\( \langle J_1^\pi \hat{T}_J^{el} \parallel 0^+ \rangle \) for the Sn\(^{116} \) (e,e') Sn\(^{116} \) (2\(^+_1\), 3\(^+_1\)) reactions with \( E_0 = 150 \text{ MeV} \)
and \( \theta = 45^\circ \) calculated with the three theories: QSTD, (QSTD)\(_{\text{uncor}}\) and QTD. The effective nucleon-nucleon force labelled "Tab" refers to the Tabakin potential and that labelled "P\(_2\)" to the P\(_2\)-force. The effective charge is here \( e_2^{\text{eff}} = e_3^{\text{eff}} = 1 \). The partial contributions are: AC:
qp-vacuum \(\rightarrow\) 2qp components of the final state; BA: 2qp \(\rightarrow\) 4qp;
AB: 4qp \(\rightarrow\) 2qp; AA: 2qp \(\rightarrow\) 2qp; BB: 4qp \(\rightarrow\) 4qp terms.

FIGURE CAPTIONS

Fig. 1

Form factor \( | F_{1n} |^2 \) for the Sn\(^{116} \) (e,e') Sn\(^{116} \) (\(^+_1\), \(^+_1\)) reaction at
\( E_0 = 150 \text{ MeV} \). The data are of ref. 1). The theoretical curve labelled "Tab" refers to QSTD theory with the Tabakin potential and \( e_2^{\text{eff}} = 1, 23 \). The curve P\(_2\) gives QSTD results for the P\(_2\)-force and \( e_2^{\text{eff}} = 1, 57 \). The momentum transfer \( Q \) is given in \( F^{-1} \).

Fig. 2

Form factor \( | F_{1n} |^2 \) for the Sn\(^{116} \) (e,e') Sn\(^{116} \) (3\(^-_1\)) reaction at
\( E_0 = 150 \text{ MeV} \). The data are of ref. 1). The theoretical curve refers to QSTD theory with the Tabakin potential and \( e_3^{\text{eff}} = 2, 19 \).
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Fig. 1

\[ |F_{in}|^2 \]

\[ \times 10^{-1} \]

\[ Q(F^{-1}) \]

Fig. 2

\[ |F_{in}|^2 \]

\[ Q(F^{-1}) \]