ICTP INTERNAL REPORT
5/1967

A CONVERGENT FIELD THEORY ‡

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23 May 1967

‡ Submitted to "Physics Letters"

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A CONVERGENT FIELD THEORY

Consider electrodynamics in three dimensions (two space and one time) with the Lagrangian

$$\overline{\psi}(\sigma_i p_i + m)\psi + ie\overline{\psi}\sigma_i A_i \psi + \tfrac{1}{4}(\partial_i A_j - \partial_j A_i) F_{ij} + \kappa^2 A_i A_i$$

Here $i = 0,1,2,$ $\sigma_0 = \begin{pmatrix} 1 & \\ & -1 \end{pmatrix}$, $\sigma_1 = \begin{pmatrix} & i \\ i & \end{pmatrix}$, $\sigma_2 = \begin{pmatrix} & 1 \\ -1 & \end{pmatrix}$,

$\bar{\psi} = \psi^{\dagger}\sigma_0$.

It is amusing to remark that a perturbation expansion of this theory contains no infinities except second-order logarithmic infinities for self-mass. All higher orders for electron self-mass and the renormalization constants Z_1, Z_2, Z_3 are finite. Unlike the Thirring model in two dimensions, the S-matrix is non-trivial. Unlike the ϕ^3 theory in four dimensions the theory above describes particles and anti-particles with a "spin" quantum number $S = \epsilon_{ijk} p_i J_{jk}$ ($\pm\frac{1}{2}$ for electrons and ± 1 for photons). This quantum number is additive like charge. Since the matrices σ_i and 1 span the 2 x 2 space, any product of propagators can be expressed in terms of the matrices σ_i and 1 again. This means that the complexity of calculations with the theory is not much greater than if the particles carried no spin. The model can be used to check conjectures on Reggeization of the electron and its connection with vanishing Z's.

A non-Lagrangian general theory of particles of arbitrary "spin" in three dimensions can be formulated, the Wigner classification of SL(2,C) representations being replaced by Bargmann's

U(1,1) representations. One may include I-spin (SU(2)) and possible higher symmetries. For example, the analogue of SL(4,C) or U(4,4) non-compact structures in ordinary theory turn out for the above model to be the much smaller groups like O(4,1) or U(2,2) which retain, however, all essential features of higher symmetry schemes.