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FOR REGGE TRAJECTORIES

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* Imperial College, London, UK.
** On leave of absence from Pakistan Atomic Energy Commission, Lahore, Pakistan.
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GREEN'S FUNCTIONS FOR REGGE TRAJECTORIES

That Regge trajectories arise from sums of infinite sequences of Feynman diagrams in conventional field theory is well known. Equivalently, they arise also as solutions of Bethe-Salpeter equations for two-particle Green's functions. The problem we wish to consider is the converse: given the trajectory function describing the path of a Regge pole in the J-plane, \( J = \alpha(W) \), where \( \alpha(W) \) satisfies a dispersion relation, we wish to write a (causal) Green's function, \( G(p) \), in momentum space, which should describe the propagation of the trajectory.

Since \( G(p) \) should have poles at the values \( p^2 = W_j^2 \) where \( W_j \) is given by \( \alpha(W_j) = J \) for \( J = \) integer or half-integer, it is evident that in the rest frame we must have

\[
G(p) \sim G(J, W) \propto \frac{i}{J - \alpha(W)}
\]

The problem is to fix on suitable proportionality factors.

The essential clue is contained in the recent work on infinite-dimensional equations\(^1\). In the simplest example the idea exploited there is to incorporate the succession of \( J \) values in a single unitary irreducible representation of \( \text{SL}(2, \mathbb{C}) \) and, in particular, in the Majorana representations\(^2\) \((\frac{1}{2}, 0)\) and \((0, \frac{1}{2})\). We make the same assumption about the trajectory in question. This serves to fix completely the propagator, \( G(p) \), in terms of the rest frame quantities, \( G(J, W) \):
where $L_{\nu}$ denotes a (generally complex) Lorentz transformation which boosts the 4-vector, $(W,0,0,0)$, into $p_{\mu} = W_{\mu}$ where $V_{\mu}$ is the 4-velocity, $V^2 = 1$. The functions $D_{j_{\lambda},j_{\lambda'}}(\Lambda)$ are matrices of the Majorana representation referred to a basis which diagonalizes $\lambda_{j}$ and $J_{3}$. If the function $G(J,W)$ has suitable analyticity properties in the $J$-plane then we can perform a Watson-Sommerfeld transformation on (2), replacing the sum over $J$ by a contour integration in the usual way,

$$G_{j_{\lambda},j_{\lambda'}}(p) = \sum_{j_{\mu}} D_{j_{\lambda},j_{\mu}}(L_{\nu}) \ G(J,W) \ D_{j_{\mu},j_{\lambda'}}(L_{\nu}^{-1})$$  (2)$$

where the discrete terms have their origin in complex $J$-singularities of $G(J,W)$. Now it is of course possible to invent many functions $G(J,W)$ all of which contain the factor $(J - \chi(W))^{-1}$. However, there is a particularly simple one available in the Majorana representation. If we require $G^{-1}(p)$ to be of the form

$$G_{\overline{p}_{\lambda},\overline{p}_{\lambda'}}(p) = \frac{1}{2i} \int_{\text{circ} \ J} \frac{dJ}{2\pi i} \ \sum_{j_{\mu}} D_{\overline{j}_{\lambda},j_{\mu}}(L_{\nu}) \ G(J,W) \ D_{j_{\mu},\overline{j}_{\lambda'}}(L_{\nu}^{-1}) +$$

$$+ \text{discrete terms}$$  (3)
\[ G^{-1}(p) = \Gamma(W^2) + p_{\mu} \Gamma_{\mu}, \]  

where \( \Gamma_{\mu} \) denotes a constant matrix satisfying

\[ \mathcal{D}(\Lambda) \Gamma_{\mu} \mathcal{D}(\Lambda^\dagger) = \Lambda_{\mu\nu} \Gamma_{\nu}, \]  

then we can calculate \( G(J,W) \). The result is

\[ G(J,W) = \frac{1/W}{J - \alpha(W)}, \]  

where

\[ \alpha(W) = -\frac{1}{2} - \frac{\Gamma(W^2)}{W}. \]  

If one is interested in infinite-dimensional local field equations then it is necessary to choose for \( \Gamma(W^2) \) a polynomial. We shall not pursue this line however. Our programme is to develop a propagator for a Regge trajectory and this much has been achieved (at least for the half-integer spins) by employing a Majorana-like equation.

In order to make use of this propagator in a Feynman calculus of Regge poles we must define the vertex functions. The requirement of Lorentz invariance evidently forces these to be...
proportional to the Clebsch-Gordan coefficients of $\text{SL}(2,\mathbb{C})$. With the expressions for these coefficients given by various authors 4), we can couple three Regge trajectories or two Regge trajectories and a finite representation (fixed pole) if necessary.

The physical relevance of the assumptions sketched here can only be decided by reference to experiment 5). We wish merely to point out that one can proceed as in the usual Feynman calculus to compute mass corrections, form factors, etc., and, in particular, the positions and weights of Regge cuts. It may be expected that this quasi-Feynman theory which subsumes strings of normal graphs is more convergent and has fewer infinities than the normal perturbation theory 6).

To summarize we list the Feynman rules for computing the contribution of any diagram. Denote by $|V A j \lambda\rangle$ the 1-particle state with 4-velocity $V_\mu$, intrinsic spin $j$ and helicity $\lambda$. The remaining quantum numbers, $A$, include the mass $m_A$ and Lorentz representation labels $^2 j_{\alpha A}, \sigma_A$ together with any other distinguishing labels which may be required. The rules are:

1. Internal lines:

$$G_{\lambda_1 \lambda_2 \lambda_3 A_1 A_2 A_3} (p) = \sum_{JM} D_{j_1, j_2, j_3}^{A_1} (L) \ G_{j_1, j_2} (j\lambda, \sigma) D_{j_1, j_2, j_3}^{A_2, A_3} (L')$$

2. Vertices:

$$\begin{align*}
\left(2\pi\right)^{n} \delta \left(w_1 N_1 + w_2 N_2 + w_3 N_3\right) \ G_{A_1 A_2 A_3} \left(\lambda_1, \lambda_2, \lambda_3\right)
\end{align*}$$
Sum over all internal quantum numbers and integrate over all internal momenta. The hypothetical information which must be fed into these prescriptions includes the coupling constants, \( \mathcal{g}_{A_1 A_2 A_3} \), and Regge propagators \( G_{AB}(\mathcal{J}, \mathcal{W}) \). Evidently we must choose \( G_{AB} \) such that

\[
det G_{AB}^{-1} \propto (\mathcal{J} - \alpha_1(\mathcal{W}))(\mathcal{J} - \alpha_2(\mathcal{W})) \cdots (\mathcal{J} - \alpha_n(\mathcal{W})),
\]

where \( \alpha_1(\mathcal{W}), \ldots, \alpha_n(\mathcal{W}) \) are the trajectories we wish to propagate.

These rules include only non-derivative couplings involving the \( \text{SL}(2, \mathbb{C}) \) C-Q coefficients, \( \left( \begin{array}{ccc} A_1 & A_2 & A_3 \\ j_1 \lambda_1 & j_2 \lambda_2 & j_3 \lambda_3 \end{array} \right) \). It would be quite easy to allow also for derivatives by coupling in additional 4-vectors.

Questions concerning the discrete symmetries, \( T, C, P \) and also signature will be considered in a more detailed exposition of these ideas.
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2) The representations \( D^\sigma \) of \( SL(2, C) \) have been discussed by a number of authors including:
   M. TOLLER and A. SCIARRINO, Internal Report Univ. of Rome No. 108 (1966);
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4) A. Z. DOLGINOV and I. N. TOPTYGIN, Soviet Phys. -JETP _8_, 550 (1959); _10_, 1022 (1960);
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5) If the Majorana model fails then a more general (reducible) representation is bound to succeed. This is just a question of having sufficient parameters.

6) Notice that the numerator of the propagator (2) contains not polynomials in \( p_\mu \) but the more convergent unitary representations \( D^\sigma (L_V) \).