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GREEN'S FUNCTIONS FOR REGGE TRAJECTORIES

That Regge trajectories arise from sums of infinite sequences of Feynman diagrams in conventional field theory is well known. Equivalently, they arise also as solutions of Bethe-Salpeter equations for two-particle Green's functions. The problem we wish to consider is the converse: given the trajectory function describing the path of a Regge pole in the J-plane, $J = \alpha(W)$, where $\alpha(W)$ satisfies a dispersion relation, we wish to write a (causal) Green's function, $G(p)$, in momentum space, which should describe the propagation of the trajectory.

Since $G(p)$ should have poles at the values $p^2 = W_J^2$ where W_J is given by $\alpha(W_J) = J$ for J = integer or half-integer, it is evident that in the rest frame we must have

$$G(p) \sim G(J,W) \quad \propto \quad \frac{1}{J-\alpha(W)} \tag{1}$$

The problem is to fix on suitable proportionality factors.

The essential clue is contained in the recent work on infinite-dimensional equations[1]. In the simplest example the idea exploited there is to incorporate the succession of J values in a single unitary irreducible representation of SL(2,C) and, in particular, in the Majorana representations[2] $(\frac{1}{2},0)$ and $(0,\frac{1}{2})$. We make the same assumption about the trajectory in question. This serves to fix completely the propagator, $G(p)$, in terms of the rest frame quantities, $G(J,W)$:

$$G_{j\lambda, j'\lambda'}(p) = \sum_{JM} D_{j\lambda, JM}(L_V) \; G(J,W) \; D_{JM, j'\lambda'}(L_V^{-1}) \qquad (2)$$

where L_V denotes a (generally complex) Lorentz transformation which boosts the 4-vector, $(W,0,0,0)$, into $p_\mu = WV_\mu$ where V_μ is the 4-velocity, $V^2 = 1$. The functions[2] $D_{j\lambda, j'\lambda'}(\Lambda)$ are matrices of the Majorana representation referred to a basis which diagonalizes $\underline{J}^2$ and J_3.

If the function $G(J,W)$ has suitable analyticity properties in the J-plane then we can perform a Watson-Sommerfeld transformation on (2), replacing the sum over J by a contour integration in the usual way,

$$G_{j\lambda, j'\lambda'}(p) = \frac{1}{2i} \int \frac{dJ}{\sin \pi J} \sum_M D_{j\lambda JM}(L_V) \, G(J,W) \, D_{JM, j'\lambda'}(L_V^{-1}) + $$

$$+ \text{ discrete terms} \qquad (3)$$

where the discrete terms have their origin in complex J-singularities of $G(J,W)$.

Now it is of course possible to invent many functions $G(J,W)$ all of which contain the factor $(J-\alpha(W))^{-1}$. However, there is a particularly simple one available in the Majorana representation. If we require $G^{-1}(p)$ to be of the form

$$G^{-1}(p) = \Gamma(W^2) + p_\mu \Gamma_\mu \ , \tag{4}$$

where Γ_μ denotes a constant matrix satisfying

$$D(\Lambda)\,\Gamma_\mu\, D(\Lambda^{-1}) = \Lambda_{\mu\nu}\Gamma_\nu \ , \tag{5}$$

then we can calculate $G(J,W)$. The result is[3)]

$$G(J,W) = \frac{1/W}{J - \alpha(W)} \ , \tag{6}$$

where

$$\alpha(W) = -\frac{1}{2} - \frac{\Gamma(W^2)}{W} \ . \tag{7}$$

If one is interested in infinite-dimensional <u>local</u> field equations then it is necessary to choose for $\Gamma(W^2)$ a polynomial. We shall not pursue this line however. Our programme is to develop a propagator for a Regge trajectory and this much has been achieved (at least for the half-integer spins) by employing a Majorana-like equation.

In order to make use of this propagator in a Feynman calculus of Regge poles we must define the vertex functions. The requirement of Lorentz invariance evidently forces these to be proportional to the Clebsch-Gordan coefficients of $SL(2,C)$. With the expressions for these coefficients given by various authors [4] we can couple three Regge trajectories or two Regge trajectories and a finite representation (fixed pole) if necessary.

The physical relevance of the assumptions sketched here can only be decided by reference to experiment[5]. We wish merely to point out that one can proceed as in the usual Feynman calculus to compute mass corrections, form factors, etc., and, in particular, the positions and weights of Regge cuts. It may be expected that this quasi-Feynman theory which subsumes strings of normal graphs is more convergent and has fewer infinities than the normal perturbation theory[6].

To summarize we list the Feynman rules for computing the contribution of any diagram. Denote by $|V\ A\ j\,\lambda\rangle$ the 1-particle state with 4-velocity V_μ, intrinsic spin j and helicity λ. The remaining quantum numbers, A, include the mass m_A and Lorentz representation labels[2] j_{0A}, σ_A together with any other distinguishing labels which may be required. The rules are:

(1) Internal lines:

$$\underset{1}{\cdot}\xrightarrow{\;p_\mu = WV_\mu\;}\underset{2}{\cdot} \quad : \quad G_{A_1 j_1 \lambda_1, A_2 j_2 \lambda_2}(p) = \sum_{JM} D^{A_1}_{j_1\lambda_1, JM}(L_V)\, G_{A_1 A_2}(J,W)\, D^{A_2}_{JM, j_2\lambda_2}(L_V^{-1})$$

(2) Vertices:

$$: \quad i\,(2\pi)^4\,\delta(W_1V_1 + W_2V_2 + W_3V_3)\; g_{A_1A_2A_3}\begin{pmatrix} A_1 & A_2 & A_3 \\ j_1\lambda_1 & j_2\lambda_2 & j_3\lambda_3 \end{pmatrix}$$

(3) External lines:

$$\underset{1}{\bullet\!\!\longrightarrow}\; VAj\lambda \quad : \quad \mathcal{D}^{A}_{j\lambda,\, j_1\lambda_1}(L_V^{-1})$$

$$VAj\lambda \;\underset{1}{\longrightarrow\!\!\bullet} \quad : \quad \mathcal{D}^{A}_{j_1\lambda_1,\, j\lambda}(L_V)$$

Sum over all internal quantum numbers and integrate over all internal momenta. The hypothetical information which must be fed into these prescriptions includes the coupling constants, $g_{A_1A_2A_3}$, and Regge propagators $G_{AB}(J,W)$. Evidently we must choose G_{AB} such that

$$\det G^{-1}_{AB} \quad \propto \quad (J-\alpha_1(W))(J-\alpha_2(W)) \cdots (J-\alpha_n(W)) ,$$

where $\alpha_1(W)$, ... , $\alpha_n(W)$ are the trajectories we wish to propagate.

These rules include only non-derivative couplings involving the SL(2,C) C-G coefficients, $\begin{pmatrix} A_1 & A_2 & A_3 \\ j_1\lambda_1 & j_2\lambda_2 & j_3\lambda_3 \end{pmatrix}$. It would be quite easy to allow also for derivatives by coupling in additional 4-vectors.

Questions concerning the discrete symmetries, T,C,P and also signature will be considered in a more detailed exposition of these ideas.

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2) The representations $D^{j_0\sigma}$ of SL(2, C) have been discussed by a number of authors including:
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5) If the Majorana model fails then a more general (reducible) representation is bound to succeed. This is just a question of having sufficient parameters.

6) Notice that the numerator of the propagator (2) contains not polynomials in $p_\mu = WV_\mu$, but the more convergent unitary representations $D^{j_0\sigma}(L_V)$.

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