PARTIAL SYMMETRY
AND MESON COUPLINGS

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FOOTNOTE AND ERRATA

FOOTNOTE 6) (on p. 8)
Should be replaced by the following:
For \( m_8 \) we use the mass of the \( \rho \)-meson, and for \( m_0 \), that of the \( \phi \)-meson. However, for the phase space we use the actual masses of the particles as given in Rosenfeld et al., Rev. Mod. Phys. 32, 1 (1967).

ERRATA

p. 3: In eqs. (2) and (3), \( f \) should be multiplied by \( (m_8^2/g) \).
In eq. (5), the factors multiplying \( \sin^2 \chi \) and \( \cos^2 \chi \) are respectively 2.20 and 3.30.
As a result, in the last line but one on p. 3, make the replacements 0.55 \( \rightarrow \) 2.20, 8.4 \( \rightarrow \) 3.30.

p. 4: In line 5, 11.5 MeV should read 27.3 MeV. In eq. (8) (second line), the last two terms should also have the factor \( \frac{1}{\sqrt{2}} \).

p. 5: Just before eq. (10), insert the words "(apart from the factor \( g/m_\phi^2 \))".
In third line from bottom, 152 MeV \( \rightarrow \) 38 MeV.

p. 6: In first line, 7.6 MeV \( \rightarrow \) 1.9 MeV.
In line 5, after "-ment", insert "except for \( f^0 \rightarrow \pi^0\pi^0 \)."
In paragraph 2, make the replacements 45 \( \rightarrow \) 11.2, 28 \( \rightarrow \) 7.0 and "small" \( \rightarrow \) "bad".
In paragraph 3, replace "parallel .... mesons" by "a universal coupling of tensor mesons with the same strength as vectors."

p. 7: In Table I, the figures are, for \( f^0 \pi\pi \), 135 \( \rightarrow \) 34; for \( f^0 \pi\pi \), 7.6 \( \rightarrow \) 1.9.
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Schwinger's hypothesis of partial symmetry is used for calculating the couplings of tensor, axial and scalar mesons to vectors and pseudoscalars. The decay widths of principal modes, evaluated on the basis of a universal coupling equal to the isospin coupling, are in good agreement with experiment.
PARTIAL SYMMETRY AND MESON COUPLINGS

Recently SCHWINGER \(^1\) has proposed partial symmetry as a simple but effective method for the treatment of physical processes which depend primarily on the utility of kinematical transformation groups. The method is based on the use of an effective numerical Lagrangian function for a set of processes, which remains invariant under the group \(U(4)\) of transformations (involving internal and spin degrees of freedom) which is now re-interpreted as a partial symmetry group, as a replacement for the techniques of current algebra \(^2\) characterized by the appearance of chiral groups like \(SU(2) \otimes SU(2)\). The great simplicity of the approach is felt through the calculational ease with which it is possible to obtain physical results (where applicable) by methods strongly reminiscent of covariant perturbation theory for lowest order processes. We present here an application of this method to the coupling of tensor, axial and scalar mesons to vectors and pseudoscalars, using \(U(6)\) as the partial symmetry group. Our results for the various decay modes of these mesons agree rather well with experimental values, with no free parameters, except an overall coupling constant whose value is quite close to the "universal" \(\rho \pi \pi\) coupling constant within this theory. The same coupling constant gives very good results for the strong decay modes of all the vector mesons except \(K^*K\pi\).

We first make a straightforward \(SU(3)\) extension of Schwinger's even parity meson matrices and define

\[
M, M' = (\nu^a_i \pm \nu^a_i) \sigma_i \lambda^a + W_i^a \sigma_i + m_0 W_0 = m_8 V_0^a \lambda^a, \tag{1}
\]

where the \(SU(3)\) indices are denoted by \(a, b, c\) and the three-dimensional indices by \(i, j, k\). \(^3\) With the help of these matrices we can define the \(VVP\) and \(VPP\) couplings in a \(G\)-invariant form as
\[ \frac{g}{m_8^2} \frac{1}{4} \text{Tr}(\frac{1}{2} \text{MM} + \frac{1}{2} \text{MM}^\dagger) \]
giving the interaction

\[ \mathcal{L} = \epsilon_{ijkl} \int d^4x \ V_j^{\mu e} P^a_i P^e_k + 2m_0 W_0 V_i^{\mu a} P^a_i + 2m_8 W_k^a V_0^a P_k^a \]

and its Lorentz-invariant extension \(^4\)

\[ \mathcal{L} = -\frac{m_8^2}{8} V^a_{\mu} P^b_{\mu} P^c_{\mu} + m_0 \epsilon_{\mu \nu \lambda \rho} P^a_{\mu} V^a_{\nu \lambda} W_{\rho} + m_8 \epsilon_{\mu \nu \lambda \rho} P^a_{\mu} W_{\nu \lambda} V^a_{\rho} \]

\(^3\)

This expression predicts the decay widths for \( \rho\pi, \varphi K\bar{K}, \varphi\rho\pi \) and \( K^+K^- \). However, this simple derivation misses the correction factor of 3/4 which SCHWINGER obtained by a more careful analysis of the interaction based on chiral dynamics. \(^5\) Keeping this limitation in mind we must normalize our "universal constant" \( g \) to a lower value for \( \rho\pi \) width, say 90 MeV, which gives as our input \(^6\)

\[ 2^{-\frac{1}{2}} \left( \frac{m_\pi}{m_\rho} \right) g = 1.1 \]

\(^4\)

compared with Schwinger's 1.0. The same interaction now produces the results (in MeV)

\[ \Gamma(\varphi \to \pi\pi) = 0.55 \text{ MeV}^2 \alpha, \quad \Gamma(\varphi \to K\bar{K}) = 8.4 \cos^2 \alpha \]

\(^5\)

in terms of an \( \omega-\varphi \) mixing angle \( \alpha \), whose "ideal value" \( \cos^{-1} \sqrt{2/3} \) yields for these widths 0.18 MeV and 5.6 MeV, to be compared with the experimental numbers of 0.36 ± 0.2 and 4.0 ± 1 respectively. It
is interesting to note that the \( \varphi \rightarrow \rho \pi \) width is not exactly zero in this model, but it comes out to be automatically small, presumably because the mixing effect is partly included in the meson matrix. The only bad prediction of this model concerns the \( K^* K \pi \) width which is merely 11.5 MeV as against the experimental value of 49 MeV. However, the results of current algebra in this respect seem to be no more encouraging and one presumably requires more specific physical effects to bridge this discrepancy.

It is now possible to extend the Schwinger method to include the couplings of \( P \) and \( V \) mesons to tensor and scalar (octet plus singlet) and axial (octet) mesons denoted respectively by the fields \((T_{\mu \nu}^{a}, F_{\mu \nu}^{a}), (S^{a}, \Phi)\) and \((B_{\mu \nu}^{a}, A_{\mu \nu}^{a})\), the last two including the possibility of both types of G-parity among axial-mesons of \( I = 1 \). Since there are now two space indices in the field components, one must make use of a higher representation of the spin group. Defining the operators

\[
\sum_{i,j}^{(\lambda)} = \frac{1}{2} \left( \sigma_{i}^{(1)} \sigma_{j}^{(2)} + \sigma_{i}^{(2)} \sigma_{j}^{(1)} - \frac{2}{3} \delta_{i,j} \sigma_{\lambda}^{(1)} \sigma_{\lambda}^{(2)} \right)
\]

\[
\sum_{i,j}^{(\alpha)} = \frac{1}{2} \left( \sigma_{i}^{(1)} \sigma_{j}^{(2)} - \sigma_{i}^{(2)} \sigma_{j}^{(1)} \right)
\]

in terms of two spin vectors \( \sigma^{(1)} \) and \( \sigma^{(2)} \), one assembles the even-parity components of these fields into the meson matrix

\[
\tilde{N} = (m_{+} T_{\alpha}^{a} + \delta_{i} T_{0 i}^{a}) \sum_{\alpha}^{(\lambda)} \lambda_{\alpha} + m_{F} F_{ij} \sum_{\alpha}^{(\lambda)} \lambda_{\alpha} + \frac{i}{\sqrt{3}} (m_{q} \Phi + m_{s} S^{a} \lambda_{a}) \sigma_{\lambda}^{(1)} \sigma_{\lambda}^{(2)}
\]

\[
+ \left( \frac{1}{12} m_{B} B_{ij}^{a} + \frac{1}{12} m_{A} A_{ij}^{a} + \delta_{i} B_{0 j}^{a} + \delta_{j} A_{0 i}^{a} \right) \sum_{\alpha}^{(\lambda)} \lambda_{\alpha}
\]

where the numerical coefficients have been adjusted by the requirement that the mass terms of these field components appear with correct
coefficients in the free Lagrangian which is obtained from \( \frac{1}{16} \mathrm{Tr} N^2 \), in exact analogy with Schwinger's corresponding requirement for the sum of contributions from the \( M \) and \( M' \) matrices. The couplings of these fields to \( V \) and \( P \) mesons are governed by the expression

\[
\left( \frac{G}{m_{\pi}^2} \right) \frac{1}{8} \mathrm{Tr} \left[ M^{(1)} N M^{(2)} \right]
\]

(9)

where the superscripts on the \( M \)-matrices indicate the space on which the spin-matrices operate. This expression is so normalized that the constant \( G \) is directly comparable with the earlier constant \( g \) of eqs. (2)-(4). The evaluation of (9) leads to a sum of \( G \)-conserving vertices whose Lorentz-invariant extension for the cases of physical interest is given by

\[
-f_{abc} \epsilon_{\alpha \beta \gamma \delta} \partial_\beta (T^a_{\lambda \nu}) V^b_{\gamma \delta} P^c_\lambda + m_\tau d_{abc} T^{a}_{\lambda \nu} P^b_\mu P^c_\nu - m_\sigma F^{a}_{\mu \nu} P^b_\mu P^c_\nu \\
+ \frac{1}{12} m_\tau \Phi P^a_\mu P^b_\mu + \frac{1}{12} d_{abc} m_5 S^{a}_{\mu \nu} P^b_\mu P^c_\nu + \frac{1}{12} d_{abc} m_5 B^{a}_{\mu \nu} P^b_\mu V^c_\nu
\]

(10)

This expression contains the predictions for most of the interesting decay rates except \( A_1 \rightarrow \rho \pi \). For comparison of these predictions with experiments one must take into account (i) the actual phase-space available for each mode and (ii) the \( SU(3) \)-mixing effects at least among the \( f^0 \) and \( f^{0'} \) mesons. A rough idea of this mixing is obtained from a straightforward evaluation of the \( f^{0'} \rightarrow \pi \pi \) width, assuming \( f^{0'} (1500) \) to be an \( SU(3) \) singlet, which gives (for \( G \approx g \)) the value 152 MeV, against an experimental figure of \( < 10 \) MeV!

Indeed an "ideal mixing angle" which would make this width zero is \( \tan^{-1} (\sqrt{5}/2\sqrt{2}) \approx 40^0 \), while an angle \( \sim 30^0 \) makes this width...
7.6 MeV. We therefore use an angle of 30° for the singlet-octet mixing between $2^+$ mesons. The decay modes of physical interest which are shown in Table I calculated with $G = g$ (the same value as eq. (4)) seem to indicate a very good pattern of agreement with experiment. Particularly noteworthy are the modes $A_2\sigma\pi$, $A_2\bar{K}K$ and $B\omega\pi$ for which accurate measurements are available, taking into account the fact that there are no free parameters. Good agreement is seen for several other modes requiring the use of the above SU(3) mixing angle. The $A_2\eta\pi$ mode is large, presumably because $\eta\chi^0$ mixing was ignored in the calculation. As for the strange modes, $K_\gamma(1420)$ widths are consistently lower (by about a factor of two) than experiment. This situation is closely analogous to the essentially SU(3) suppression of the $K^*\pi$ mode in relation to $\rho\pi\pi$. Finally, the almost "good" agreement for $K_A(1320) \rightarrow (K^*\pi + K\rho)$ could well be the result of interference between two compensating effects: (i) the usual SU(3) suppression of the "kaon-like" modes and (ii) the neglect of mixing between the "kaon-components" of the two $1^+$ octets assumed in the N-matrix of eq. (8).

We note in passing that $\sigma\pi\pi$ width in this approach is rather small, being 45 MeV and 28 MeV for octet and singlet assumptions, respectively.

Judging from the decay widths, the near equality of $G$ to the universal isospin coupling constant, $g$ is suggestive of parallel Regge trajectories for $1^-$ and $2^+$ mesons. Inasmuch as we have worked within the lowest approximation of Schwinger's chiral dynamics, the above results may well indicate a bigger universality principle.

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Decay widths of $2^+$ and $1^+$ mesons (in MeV) with a universal coupling constant $g = 8.3$ and an $f^0 - f^0'$ mixing angle $= 30^\circ$. The experimental results are taken from ROSENFIELD et al. 6)
REFERENCES AND FOOTNOTES


2) M. GELL-MANN, Physics 1, 63 (1964).

3) For the even-parity components of the vector octet $V$ and singlet $W$ we use the notation $V_{ia}^{H} = \frac{1}{2} \xi_{ijk} V_{jk}^{a}$ where $V_{jk}^{a} = \delta_{j}^{k} \times V_{i}^{a} - \delta_{k}^{i} V_{j}^{a}$, etc., $m_{0}$ is a central mass for the SU(3) octet and $m_{0}$ that of singlet.

4) This type of approach is, however, quite different from the boosted SU(6) theories where a strict symmetry of the Lagrangian under the boosted group was demanded. For reference see, Symmetry Groups by F. J. DYSON, W.A. Benjamin, N.Y. (1966).


6) On the basis of this value the Schwinger correction would have given a width of 160 MeV. But even this large value does not seem to be inconsistent with the figure quoted in ROSENFELD et al., Rev. Mod. Phys. 39, 1 (1967).


9) Note that the axial-field components $B_{ij}$ and $A_{ij}$ are degenerate in the double-index representation. We have therefore divided by the relative factor of $\sqrt{2}$ to avoid double counting of terms in the free Lagrangian.

10) In eq. (10) we have left out couplings of the $f_{0} \rho \rho$, $f_{0} \omega \omega$, $A_{2} \rho \omega$, etc., which are of little physical interest.

11) This angle is in accord with the one assumed by GLASHOW and SOCOLOW, Phys. Rev. Letters 15, 329 (1965), and about 10° higher than the corresponding result of a recent quark-model evaluation: A.N. MITRA and P.P. SRIVASTAVA, ICTP, Trieste, preprint IC/67/31 (1967).