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## ADDENDUM

After this paper was written the authors became aware that Prof. P.G.O. Freund (Phys. Rev. Letters 15, 929 (1965) ) had already expressed the view that the $2^+$ mesons may have stress tensors as their source.

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# STRESS TENSOR AND THE $2^+$ MESONS

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SUMMARY:

It is suggested that the recently discovered $2^+$ particles have the stress tensor $\theta_{\mu\nu}$ and its octet generalisations $\theta^i_{\mu\nu}$ as their sources. Consequences of this hypothesis are examined.

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§.1 It is an attractive conjecture that the singlet $f^o(2^+)$ current may be associated with the energy-momentum tensor $\theta_{\mu\nu}$ – the hypothesis of "strong gravitation". If this conjecture is tame, the algebra of $\theta_{\mu\nu}$ may provide a clue to the structure of the strong Hamiltonian $\int \theta_{oo}(x)d^3\vec{x}$. One may make the further hypothesis that the remaining members of the $2^+$ nonet are likewise associated with a tensor $\theta^i_{\mu\nu}$ ($i = o,1,......8$) possessing appropriate SU(3) transformation properties. In this note we examine how far such a hypothesis can be carried, basing ourselves on a quark model[1] for the strong stress-tensor as well as for the structure of the $2^+$ and other particles.

§.2 A $2^+$ particle is described by a symmetrical, conserved, traceless tensor. The conventional stress tensor is symmetric and conserved but not traceless. Also, although it is easy to find the SU(3) generalisation $\theta^i_{\mu\nu}$ of a given stress tensor $\theta_{\mu\nu}$, such generalisations are conserved only in special circumstances for $i \neq o$. The physical identification of the $2^+$ currents with $\theta^i_{\mu\nu}$ must therefore take account of this lack of conservation as well as of the trace conditions. However before we make this identification, consider the type of sum rule one may obtain in an idealised situation where one assumes the existence of a hypothetical $F^0$ with the current $J_{\mu\nu}$ proportional to the traceless part of $\theta_{\mu\nu}$.

The most immediate consequence of the assumption that the $F^0$ current is proportional to the stress tensor is that $F^0$ couples with all particles through their mass[2]. This could of course be directly tested by comparing decay rates such as $(F^0 \to AA)/(F^0 \to BB)$. In practice since the only relevant decays are into pseudoscalar mesons with the effective coupling always written as -

$\frac{G}{2M} F_{\mu\nu}(q) (p+p')_\mu (p+p')_\nu \varphi(p) \varphi(p')$, the test is not a stringent one. A better place for testing the hypothesis is to compare $2^+$ exchange processes. For example, assuming that $F^0$ lies on the Pomeranchuk trajectory, we would obtain the relative magnitude of the coupling constants of $F^0$ to mesons (M) and nucleons (N) by comparing high energy total cross sections for MN and NN scattering. In making any such test it is quite crucial that we keep track of all mass factors and we shall therefore present the argument in some detail.

The matrix elements of $\theta_{\mu\nu}$ between one-particle states have been discussed by Pagels.[5] For scalar states,

$$(1) \quad < p' \,|\, \theta_{\mu\nu}(o) \,|\, p > = \tfrac{1}{2} P_\mu P_\nu F_1(q^2) + (q^2 g_{\mu\nu} - q_\mu q_\nu) F_2(q^2)$$

For spinor states

$$(2) \quad <p' \,|\, \theta_{\mu\nu}(o) \,|\, p> = \bar{u}(p') \left[ \begin{array}{l} \frac{1}{4} (\gamma_\mu P_\nu + \gamma_\nu P_\mu) \; G(q^2) \\ + \frac{1}{4m} \; P_\mu P_\nu \; G_2(q^2) \\ + \frac{1}{m} \; (q^2 g_{\mu\nu} - q_\mu q_\nu) \; G_3(q^2) \end{array} \right] u(p)$$

where $P = p+p'$, $q = p-p'$ and the boundary values $F_1(o) = 1$, $G_1(o) + G_2(o) = 1$ assure that the diagonal matrix element is $<m, \underline{o} \,|\, \theta_{oo} \,|\, m, \underline{o}> = 2m^2$ in accordance with our normalisation convention :

$$<p' \,|\, p> \delta(p^2 - m^2) = (2\pi)^3 \delta^4(p-p') \; ; \quad \bar{u}u = 2m.$$

In order to fix invariant factors that occur in Regge formulas, we first consider the conventional Feynman diagram for $F^o$ exchange with our basic assumption that the $F^o$ vertex equals

$$(3) \qquad \Gamma_{\mu\nu}(p',p) = \langle p' | \theta_{\mu\nu}(o) | p \rangle (G/M)$$

where $G/M$ is a universal factor by hypothesis. Thus the Born term is

$$T(p'k',pk) = \tfrac{1}{4}\,\Gamma_{\mu\nu}(p',p)\,\Delta_{\mu\nu\kappa\lambda}(q)\,\Gamma_{\kappa\lambda}(k',k)$$

with

$$(q^2-m_F^2)\,\Delta_{\mu\nu\kappa\lambda}(q) = \tfrac{1}{2}\left[\,d_{\kappa\mu}(q)d_{\lambda\nu}(q) + d_{\kappa\nu}(q)\,d_{\lambda\mu}(q)\right] - \frac{1}{3}\,d_{\kappa\lambda}(q)\,d_{\mu\nu}(q)$$

$$d_{\mu\nu}(q) \equiv g_{\mu\nu} - \frac{q_\mu q_\nu}{m_F^2} \quad .$$

For M-M and M-N scattering, the $P_2(\cos\theta_t)$ terms are

$$(4) \qquad T_{MM} = \left[\gamma_M(t)\right]^2 \frac{P_2(\cos\theta_t)}{t - m_F^2}$$

$$T_{MN} = \left[\gamma_M(t)\,\gamma_N(t)\right] \frac{P_2(\cos\theta_t)}{t - m_F^2}$$

where

$$\gamma_\mu(t) = \sqrt{\tfrac{2}{3}}\ (\tfrac{1}{4}t - \mu^2)\ F_1(t)\left(\frac{G}{M}\right)$$

$$\gamma_N(t) = \sqrt{\tfrac{2}{3}}\ \frac{\bar{u}(p')u(p)}{2m} \cdot (\tfrac{1}{4}t - m^2)\ [\ G_1(t) + G_2(t)\ ]\left(\frac{G}{M}\right)$$

According to the standard Reggeising prescriptions we replace $\dfrac{P_2(\cos\theta)}{t - m_F^2}$ by $\frac{1}{2}\pi\alpha' \cdot \dfrac{(1 + e^{i\pi\alpha(t)})}{\sin\pi\alpha(t)}\ \ P_{\alpha(t)}(\cos\theta_t)$ to obtain the Regge amplitude for the $F^0$ trajectory. As stated before we here make the further hypothesis that this is the Pomeranchuk trajectory itself and will dominate all high energy elastic scattering. By the optical theorem $\mathrm{Im}\,T \approx s\,\sigma^{total}$, we get

$$(5) \qquad \frac{\sigma^{tot}_{\pi\pi}}{\sigma^{tot}_{\pi N}} = \frac{\mu^2 (\cos\theta_t)_{\pi\pi}}{m^2\ (\cos\theta_t)_{\pi N}} \approx \frac{\mu}{m}$$

By the factorisation theorem, this equals $\sigma^{tot}_{N\pi}\ / \sigma^{tot}_{NN}$ .

In the above, $(\cos\theta_t)_{AB} = \dfrac{s - u}{[\ (t-4m_A^2)(t-4m_B^2)\ ]^{\frac{1}{2}}} \approx \dfrac{s}{2m_A m_B}$

It is crucial to note that in most discussions the mass factor $2m_A m_B$ in the denominator of $\cot\theta_t \approx {}^{s}/2m_A m_B$ is replaced by a universal length $s_o$ thereby obscuring the significant role played by the particle masses. In fact our treatment of the hypothetical $F^o$ would give the general prediction :

$$(6) \qquad \frac{\sigma_{AB}^{tot}}{\sigma_{CD}^{tot}} = \frac{m_A m_B}{m_C m_D} \qquad \text{as } s \to \infty$$

We shall use a modified version of this formula in Section 5 for the actual physical situation.

§.3 In this section we derive low-energy theorems for $F^o$ production using current algebra methods. (A more general approach using gauge techniques is described in the appendix). Our basic notion is that the source of the $F^o$ is the strong stress tensor which is made up of quark fields. This stress tensor does not contain explicitly the $F^o$ field itself because we assume $F^o$ (as well as other particles) to be quark composites. The quark structure of $F^o$, however, is irrelevant for considerations of this section.

For any local operator $\phi(x)$ we expect that ,

$$\int d^3\vec{y}\ [\theta_{0\nu}(y),\ \phi(x)\ ]\ =\ -i\partial_\nu\phi(x)$$

It follows therefore that

$$(7)\quad \delta(x_0-y_0)\ [\ \theta_{0\nu}(x),\ \phi(y)\ ] = -i\delta^4(x-y)\ \partial_\nu\phi(x) + \Gamma_{\nu j}R_j(x,y).$$

where $\Gamma_{\nu j}R_j$ must consist of terms containing spatial derivatives of $\delta$-functions as well as of other operators(3) and such that $\int \Gamma_{\nu j}R_j(x)\ d^3\vec{x} = 0$. Let $\phi(y)$ stand for the $\pi^{0}$-meson field and consider the production process $\pi^- + p \to F^0 + n$. Our identification of the $F^0$ current with $\theta_{\mu\nu}$ gives the sum rule,

$$(8)\quad [(p-p')^2 - \mu^2]\,[k^2-\mu^2]^{-1}\quad q_\mu M_{\mu\nu}(p',q;p,k) = -\frac{G}{2M}(p-p')_\nu\ (p',p)+\Gamma_{\nu j}R_j - \text{terms}$$

~~terms~~ where conventionally,

$$(9)\quad (2\pi)^4\,\delta^4(p+k-p'-q)\ M_{\mu\nu}\ (p',q;p,k)$$

$$= \frac{G}{2M}\int d^4x\, d^4y\ \ e^{iqx-iky}.i\ (\partial_y^2 - \mu^2) < p'|T\ [\theta_{\mu\nu}(x)\ \phi(y)]|\ p >$$

$$\Gamma(p',p)\ = <p'\,|\,\phi(o)\,|\ p>\ [\ (p-p')^2 - \mu^2\ ]$$

For a spinless field like $\phi$, all noncovariant spatial derivative terms $\Gamma_{\nu j} R_j$ on the right of (9) can be dropped. This uses the ansatz of Feynman, Bjorken and Nauenberg[4], which asserts that the conventional definition (9) for S-matrix amplitudes needs to be modified by subtracting from it precisely the noncovariant terms like $\Gamma R$ in order to restore covariance. We are thus left with

$$(10) \quad [(p-p')^2-\mu^2]\, q_\mu M_{\mu\nu}(p',q;p,k) = -\frac{G}{2M}(k^2-\mu^2)\,(p-p')_\nu \Gamma(p',p)$$

A low energy theorem follows when we take derivatives of (10) at q = o, keeping the nucleons on the mass shell[5] :-

$$(11) \quad (k^2-\mu^2)\, M_{\mu\nu}(p',o;p.k) = \frac{G}{M}\, k_\mu k_\nu \Gamma(p',p)$$

Relation (11) states that the soft production amplitude equals the peripheral contribution[6] ($o^-$ exchange). Although we should not strictly identify the physical $f^o$ with the hypothetical $F^o$ for reasons given in the next section, it is amusing that the production of $f^o$ in the collisions considered is almost wholly peripheral[6] at the extrapolated point $q^2 = m_f^2$.

§.4 So far we have discussed a hypothetical $F^0$ with the exact $(\theta_{\mu\nu})$ strong as its source. It is an open question whether a singlet represented by such a particle exists in addition to the known nonet, or whether $F^0$ can be identified with the singlet contained therein. In this section we wish to make a suggestion which treats all the members of the nonet on the same footing.

In a Lagrangian quark model where for example[1],

$$(12) \qquad L = L_{free} + L_{int} = \bar{\psi}(\tfrac{1}{2}i\overleftrightarrow{\not\partial} - m)\psi + h(\bar{\psi}\psi)^2,$$

the stress tensor has the form

$$(13) \qquad \theta_{\mu\nu} = \frac{i}{4}\,\bar{\psi}(\gamma_\mu\overleftrightarrow{\partial}_\nu + \gamma_\nu\overleftrightarrow{\partial}_\mu)\,\psi - g_{\mu\nu}\,L$$

We shall assume that the dominant parts of the $2^+$ nonet current are obtained from

$$(14) \qquad \theta^i_{\mu\nu} = \frac{i}{4}\,\bar{\psi}(\gamma_\mu\overleftrightarrow{\partial}_\nu + \gamma_\nu\overleftrightarrow{\partial}_\mu)\,T^i\psi - g_{\mu\nu}\,\bar{\psi}(\tfrac{1}{2}i\overleftrightarrow{\not\partial} - m)\,T^i\psi \quad .$$

$\theta^i_{\mu\nu}$ are bilinear in quark fields; if the $2^+$ particles contain $\bar{\psi}\psi\bar{\psi}\psi$

and higher quark components, these are at this stage being considered small. Observe that the singlet current, $\sqrt{6}\,\theta_{\mu\nu}^{0} = \theta_{\mu\nu} - g_{\mu\nu} L_{int}$, where $\theta_{\mu\nu}$ is the full stress tensor. It is important to note that none of the $\theta_{\mu\nu}^{i}$ is conserved except in the case that the quarks are free. Thus for $L_{int}$ given above,

$$\partial_{\mu}\theta_{\mu\nu}^{i} = 2h\,(\bar{\psi}T^{i}\psi)\,\partial_{\nu}(\bar{\psi}\psi)$$

This lack of conservation could mean that in general $\theta_{\mu\nu}^{i}$ may have associated $1^{-}$ and $0^{+}$ states. It is possible to show that such states possess no propagation character using methods analogous to those of Deser, Trubatch and Trubatch[7].

From the quark commutation rule,

$$(15)\quad \delta(x_0-y_0)\,[\,\theta_{0\nu}^{i}(x),\psi^{+}(y)\,] = -i\delta^{4}(x-y)\partial_{\nu}\psi^{+}(x)\,T^{i} + \tfrac{1}{2}\partial_{\mu}[\psi^{+}(x)\,\Gamma_{\nu\ \mu}^{\ i}\,\delta^{4}(x-y)] + \tfrac{1}{4}\epsilon_{o\lambda\mu\nu}\partial_{\mu}[\psi^{+}(x)\,i\gamma_0\gamma_\lambda\gamma_5 T^{i}\,\delta^{4}(x-y)]$$

it is straightforward to derive the equal time commutation rule,

(16) $\partial_{x\mu}\ \theta(x-y)\ [\ \theta^{i}_{\mu\nu}(x),\ \psi^{+}(y)\ A\psi(y)\ ]$

$$= \ -\tfrac{1}{2}\, i\ \delta^4(x-y)[\partial_\nu(\psi^+\{A,T^i\}\psi) + \psi^+[A,T^i]\overleftrightarrow{\partial}_\nu\psi\ ]$$

$$+\ \tfrac{1}{4}\ \epsilon_{o\lambda\mu\nu}\,\partial_\mu\,[\psi^+[\ i\gamma_o\gamma_\lambda\gamma_5 T^i, A\ ]\psi\ \ \delta^4(x-y)\,]$$

$$+\ \tfrac{1}{2}\ i\,\partial_\mu[\psi\,\{A,\ \Gamma^{\ i}_{\nu\mu}\}\ \psi\ \delta^4(x-y)\ ]$$

$$-\ \tfrac{1}{4}\ i\delta^4(x-y)[\ \partial_\mu(\psi^+\ \{A,\Gamma^{\ i}_{\mu}\}\psi) - \psi^+[\ A,\Gamma^{\ i}_{\ \mu}]\overleftrightarrow{\partial}_\mu\psi\ ]$$

$$+\ \ 2h\theta\ (x-y)\,[\ (\bar\psi T^i\psi)\ \partial_\nu(\psi\psi),]\ \psi^+(y)A\psi\,(y)$$

+ possible Schwinger terms.

Here $\Gamma^{\ i}_{\nu\mu} = (g_{\nu\mu} - g_{o\nu}\gamma_o\ \gamma_\mu)T^i$ is non-covariant.

Once again let us apply the commutator to the pseudoscalar mesons assuming for simplicity that their fields are represented by $\phi^j = \bar\psi\gamma_5 T^j\psi$. Dropping all non-covariant terms ($\Gamma R$), the commutator simplifies to

(17) $$\partial_{x\mu}\,\theta(x-y)\,[\,\theta^{i}_{\mu\nu}(x)\,,\;\phi^{j}(y)\,]$$

$$= -\tfrac{1}{2}\; i\,\delta^{4}(x-y)\,[\,d^{ijk}\,\partial_{\nu}\phi^{k}(x) + i f^{ijk}\,\bar{\psi}(x)\,\gamma_5\overleftrightarrow{\partial}_{\nu}T^{k}\psi(x)\,]$$

$$+ \quad 2h\;(x-y)[\,(\bar{\psi}T^{i}\psi)\,\partial_{\nu}(\bar{\psi}\psi\,),\;\phi(y)\,]$$

The last term in corresponding to $1^{-+}$ entities distinguishes one theory from another; it would vanish for no quark interaction Lagrangian. The terms proportional to $f^{k}_{\nu} = i\bar{\psi}\gamma_5\overleftrightarrow{\partial}_{\nu}T^{k}\psi$ are however present in all cases and may correspond to a possible $J^{PC} = 1^{+-}$ octet ($\partial_{\nu} f_{\nu} \equiv 0$); this might include the B particle if it exists.

Sandwiching (17) between nucleon states, we obtain the covariant Ward-Takahashi-like identity,

(18) $$q_{\mu}M^{ij}_{\mu\nu}(p',q;p,k) = -\tfrac{1}{2}\,d^{ijk}\,\frac{(k^2-\mu^2)}{(p-p')^2-\mu^2}\,(p-p')_{\nu}\,\Gamma^{k}(p',p)$$

$$+ \quad \tfrac{1}{2}\,f^{ijk}\,\frac{(k^2-\mu^2)}{[\,(p-p')^2-\mu^2\,]}\,(p+p')_{\nu}\;F^{k}(p',p)$$

$$+ \; 1^{-+}\text{ contributions from } L_{int}.$$

Here $F^k$ corresponds to the $1^{+-}$ "charge" form factor in that $< p' | f_\nu^k (o) | p > = (p+p')_\nu F^k(p',p) [(p-p')^2 - m^2]^{-1}$. A low energy-theorem (for the case when quark $L_{int} = o$) follows by taking the limit $q \to o$ in (10) :

$$(19) \quad \frac{2M}{G} M_{\mu\nu}^{ij} (p',p;p,k) = d^{ijk} \frac{k_\mu k_\nu}{k^2 - \mu^2} \Gamma^k(p',p)$$

$$+ \frac{1}{2} f^{ijk} \frac{[k_\mu (p+p')_\nu + k_\nu (p+p')_\mu]}{k^2 - m^2} F^k(p',p)$$

Once again the derivation of (19) is subject to ambiguities inherent in the current algebra technique which have been noted in the appendix. As it stands (19) states that the amplitude is described purely by the peripheral diagrams ($0^{-+}$ and $1^{+-}$ exchange) with the appropriate d and f couplings at the $2^+$ meson vertices of the $0^-$ and $1^+$ particles. Without having full data on $2^+$ production and its full analysis in terms of invariant amplitudes, it is of course impossible to say if (18) and (19) are likely to make any experimental sense and what the form of $L_{int}$ should be - even whether it exists or not. It is however noteworthy that all $2^+$ particles are produced peripherally as remarked previously and further that the decays of the $2^+$ nonet single out the SU(3) invariant vertex[8], $d^{ijk} f^i \varphi^j \varphi^k$, from the three possible ones. We may therefore provisionally conclude that the ("abnormal" C) spin one entities $1^{-+}$ and $1^{+-}$ couple weakly to the nucleons.

Commutators of other currents[9] with the new current $\theta^i_{\mu\nu}$ should provide new sum rules. For instance taken with the vector current $\theta^o_{\mu\nu}$ will relate photoproduction amplitudes to vector form factors. Even for pions one could perform more sophisticated calculations by using the P.C.A.C. hypothesis and evaluating $[\ \theta^i_{\mu\nu}\ ,\ A^i_\lambda\ ]$.

§.5 In section 4 we have treated the entire nonet of $2^+$ particles on the same basis. None of their currents as defined in (14) is conserved, except if the quarks are free. It is commonly supposed in most quark calculations that the dominant effect of quark interaction Lagrangian is a mass renormalization. Assuming this, we may make the hypothesis that in $\theta^i_{\mu\nu}$ the m's which appear stand for the 'effective' quark masses and that $\theta^i_{\mu\nu}$ are conserved to the extent that this "mass renormalisation" takes account of $L_{int}$. We believe this is the picture which is most likely to approximate to the physical situation. The diagonal matrix elements of

$\theta^{o}_{\mu\nu}$ in particular are now proportional to the effective quark mass content of the physical particles concerned. Thus relation (6) in this interpretation reads

$$(20) \qquad \frac{\sigma^{tot}_{AB}}{\sigma^{tot}_{CD}} \rightarrow \left(\frac{m_A m_B}{m_C m_D}\right)_{\text{effective}}$$

$$= \frac{n_A\ n_B}{n_C\ n_D}$$ where $n_A$ is the number of quarks making up A.

The special case $\frac{\sigma_{NN}}{\sigma_{MN}} \approx \frac{3}{2}$ is a result common to all quark models.[10]

An alternative model to the above could be to separate the singlet from the octet; to suppose that the singlet is indeed the hypothetical $F^{o}$ of section 2 with the full stress tensor as its current. (This model would certainly possess more exciting cosmological consequences). The remaining octet of currents are then systematically derived by the following construction. Extend the ordinary field displacement by kaing the variation

$$\delta\varphi(x) = \lambda_{\mu i}\ F^{i} \partial_{\mu}\, \varphi(x) \qquad (21)$$

The change in Lagrange density

$$\delta L = \lambda_{\nu\, i}\partial_\mu \left[\frac{\delta L}{\delta(\partial_\mu\varphi)} \quad F^i \partial_\nu \varphi\right]$$

can in general be written as a sum of a gradient term $\lambda_{\mu i}\partial_\mu L^i$ plus a term $\lambda_{\mu i} V^i_\mu$ where V has non-vanishing curl. Then define

$$\theta^{\;i}_{\mu\nu} = \frac{\delta L}{\delta(\partial_\mu\varphi)} \cdot F^i\partial_\nu\varphi - i\,\partial_\lambda \cdot \frac{\delta L}{\delta(\partial_\lambda\varphi)} \; F^i\,\Sigma_{\mu\nu}\varphi$$

$$- g_{\mu\nu}\, L^i$$

Clearly i = o gives the conventional stress tensor. Also $\partial_\mu\theta^{\;i}_{\mu\nu} = V^i_\nu$ ; thus for $L_{int}$ given in (12), $V^i_\nu = h(\bar\psi T^i\psi)\overleftrightarrow{\partial}_\nu(\bar\psi\psi)$. The modifications to be expected in (16) and (17) are trivially obtained since they only affect the interaction term h. Because the conclusions following upon (18) and (19) are unaltered we shall not discuss this situation any further.

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APPENDIX.

To obtain the low energy theorems we employ the gauge method developed by Just and Rossberg[11] for the gravitational case.[12] This gives Green's function expressions for soft graviton emission in processes involving any number of other (e.g. strongly interacting) particles. Our basic idea is to regard the tensor current $f\theta_{\mu\nu}$ which is the source of gravitons in the weak-field-approximation as also the source of the $F^0$ i.e.

$$(22) \qquad (\Box + m_F^2)\ F_{\mu\nu} = \frac{G}{M}\ \theta_{\mu\nu}$$

where G/M denotes a strong coupling constant. The current $\theta_{\mu\nu}$ does not contain the gravitational field and is therefore conserved up to higher order terms in the gravitational constant f.

Underlying our application of this method is the idea that $\theta_{\mu\nu}$ is made up entirely of quark fields. The phenomenological fields $\phi$, $\psi$, $f_{\mu\nu}$ ,..... which characterize the observed pions, nucleons, f-mesons,.... do not enter. We shall assume, however, that these fields transform in the usual local fashion under the general coordinate transformations of the gravitational gauge group. That is, under the infinitesimal transformation

$$x_\mu \to x_\mu + f\lambda_\mu(x),$$

for which the gravitational tensor $h_{\mu\nu}$ transforms as

$$h_{\mu\nu} \to h_{\mu\nu} + \lambda_{\mu,\nu} + \lambda_{\nu,\mu} - g_{\mu\nu}\lambda_{\alpha,\alpha} + \text{ terms proportional to } f,$$

we have according to Just and Rossberg,[11]

(23)
$$\varphi \to \varphi - f\lambda_\mu \psi_{,\mu}$$

$$\psi \to \psi - f\lambda_\mu \psi_{,\mu} - i\frac{f}{4}\lambda_{\mu,\nu}\sigma_{\mu\nu}\psi$$

$$\bar\psi \to \bar\psi - f\lambda_\mu \bar\psi_{,\mu} + i\frac{f}{4}\lambda_{\mu,\nu}\bar\psi\,\sigma_{\mu\nu}$$

$$f_{\mu\nu} \to f_{\mu\nu} - f\lambda_\rho f_{\mu\nu,\rho} + f\lambda_{\rho,\sigma}(g_{\mu\rho}f_{\nu\sigma} + g_{\mu\sigma}f_{\nu\rho} - g_{\rho\sigma}f_{\mu\nu}).$$

(Note that the transformation (21) in the text for i = o is the special case of (23) when $\lambda_\mu$ is independent of x).

In these expression we are using $g_{\mu\nu}$ to denote the Minkowskian metric tensor; diag (+ - - - ). The gravitational field $h_{\mu\nu}$ is defined in terms of the pseudotensor $\mathfrak{g}^{\mu\nu}$ by

$$\mathfrak{g}^{\mu\nu} = g_{\mu\nu} + fh_{\mu\nu} \quad .$$

The Green's function for the process of interest,

$$(24) \qquad \langle T\,(\psi(x)\,\psi(y)\,\varphi(z)\quad \theta_{\mu\nu}(t))\rangle_o \quad =$$

$$= \int \frac{dp}{(2\pi)^4}\,\frac{dp'}{(2\pi)^4}\,\frac{dk}{(2\pi)^4}\,\frac{dq}{(2\pi)^4}\,(2\pi)^4\,\delta\,(p+k-p'-q)\quad e^{-ip'x+ipy+ikz-iqt}\,.$$

$$\cdot\ S(p')\,M_{\mu\nu}\,(p'q;pk)\,S(p)\Delta(k)$$

is approached by considering the behaviour under a special class of gauge transformations of the graviaton function

$$\langle T\,(\psi(x)\,\overline{\psi}(y)\,\varphi\,(z)\,h_{\mu\nu}(t))\quad\rangle_o$$

Following Just and Rossberg(11) we suppose that $\lambda_\mu(x)$ is an operator valued function satisfying the conditions

$$[\lambda_\mu(x)\,,\lambda_\nu(y)\,]\quad \delta(x_o-y_o) \quad = \quad 0$$

$$[\,\lambda_\mu(x)\,,\dot{\lambda}_\nu(y)\,]\quad \delta(x_o-y_o) \quad = \quad 0$$

$$\langle\lambda\mu\rangle o \quad = \quad 0$$

We require, moreover, the existence of a "Landau" gauge wherein

$$\partial_\mu h^L_{\mu\nu} = 0, \quad [h^L_{\mu\nu}(x), h^L_{\rho\sigma}(y)]\,\delta(x_o - y_o) = 0$$

and,

$$[\lambda_\mu(x), \varphi^L(y)] = [\lambda_\mu(x), \psi^L(y)] = [\lambda_\mu(x), h^L_{\nu\rho}(y)] = 0.$$

Finally we require that

$$\partial^2 < T(\lambda_\mu(x)\lambda_\nu(y)) > = s_{\mu\nu}(x-y) \neq 0.$$

These conditions are discussed in detail in ref. (11).

Following a procedure devised by Evans, Feldman and Matthews[(13)] which applies this gauge transformation to the Landau gauge Green's function, and cancels from both sides the factor $s_{\mu\nu}(q)$ - which is non-singular by hypothesis - we arrive at a Ward-Takahashi like rèlation valid for all gauges connected by the above transformations with the Landau gauge. The result is .

$$(25)\quad 2\left(q_\nu M_{\mu\nu}(p'q;pk) - \tfrac{1}{4}\, g_{\mu\nu} M_{\alpha\alpha}(p'q;pk)\right) =$$

$$= S^{-1}(p')\left[(p'+q)_\mu - \tfrac{1}{4}\sigma_{\mu\nu}q_\nu\right]S(p'+q)\,\Gamma_5(p'+q,p)$$

$$- \Gamma_5(p',p-q)\,S(p-q)\left[(p-q)_\mu - \tfrac{1}{4}\,\sigma_{\mu\nu}q_\nu\right]S^{-1}(p)$$

$$- (k-q)_\mu\,\Gamma_5\,(p',p)\,\Delta(k-q)\,\Delta^{-1}(k)$$

where $S(p)$ and $\Delta(k)$ denote respectively the nucleon and meson propagators and $\Gamma_5\,(p',p)$ the meson-nucleon vertex function.

To obtain a low-energy theorem it is necessary to differentiate this expression with respect to $q$ and take the limit $q \to o$. Thus

$$(26)\quad 2\,M_{\mu\nu}\,(p'o;pk) - g_{\mu\nu}\,M_{\alpha\alpha}(p'o;pk) =$$

$$= S^{-1}(p')\left[g_{\mu\nu} - \tfrac{1}{4}\sigma_{\mu\nu}\right]S(p')\,\Gamma_5(p',p) + p'_\mu\,S^{-1}(p')\frac{\partial}{\partial p'_\nu}\left[S(p')\Gamma_5(p',p)\right]$$

$$+ \Gamma_5(p',p)\,S(p)\left[g_{\mu\nu} + \tfrac{1}{4}\sigma_{\mu\nu}\right]S^{-1}(p) + p_\mu\,\frac{\partial}{\partial p_\nu}\left[\Gamma_5(p',p)S(p)\right]S^{-1}(p)$$

$$+ g_{\mu\nu}\Gamma_5\,(p',p) + k_\mu\,\Gamma_5(p',p)\frac{\partial\Delta(k)}{\partial k_\nu}\;\Delta^{-1}(k)$$

Unfortunately this result becomes singular if any of the particles are put on their mass-shells. This is of course not unexpected since the unphysical infra-red[12] limit $q \to o$ brings one up against the pole terms.

But even disregarding the singularity, the origin of the fact that our result differs from the one previously obtained using naive current algebra methods is not difficult to trace. Each term in our expression (25) has a factor $S^{-1}(p')$, $S^{-1}(p)$ or $\Delta^{-1}(k)$ which vanishes when the relevant particle is put onto its mass-shell provided $q \neq o$. These terms remain finite if $q = o$. This means that the limiting process $q \to o$ and $k^2 \to \mu^2$ (say) is ambiguous, the result depending on the order in which the two limits are taken (This has been strongly emphasised by Raman and Sudarshan[14]).

In the absence of any deeper theoretical understanding of this ambiguity we can only choose the method which leads to the least implausible result. Since, because of energy-momentum conservation, we shall have to keep at least one of the masses unphysical: either $p^2 = s$, $p'^2 = u$ or $k^2 = t$, it appears to us that the reasonable course is to set $k^2 = t$ since t is much nearer to $\mu^2$ than either s or u is to

$m^2$ for the process of interest. Thus if we take the limit $q \to o$ <u>after</u> setting $p^2 = p'^2 = m^2$, we recover the result given in the text.

## REFERENCES & FOOTNOTES.

1. R. Delbourgo, A. Salam and J. Strathdee, Phys. Letters 22, 680 (1960).

2. We are grateful to Dr. R. Omnes and H. Jones for discussions on this point.

3. These operators will contain spin matrices and will also include possible Schwinger terms. The precise form of $\Gamma R$ depends on the interaction Lagrangian and is exhibited in eq.(15) for a quark model. Schwinger terms have been shown to be present in certain $[\, \vartheta, \vartheta \,]$ commutators by G. Boulware and S. Deser. (J. Math. Phys., to be published).

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M. Nauenberg, CERN preprint, (unpublished).
If $\varphi$ represents a field with spin, $\Gamma^{\nu} R$ must include terms with spin matrices. The covariant parts of such terms must be isolated and retained since they are essential in producing symmetric $\vartheta_{\mu\nu}$ . The covariant gauge procedure in the appendix automatically takes care of such tedious details and provides independent support of the dovariance ansatz.

5. The Green's function method in the appendix gives additional nucleon pole contribution on the right side of (11), which come about because all particles are treated off the mass shell. This ambiguity is discussed further in the appendix.

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S.U. Chung et. al. ibid. 15, 325 (1965).

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These $1^-$ objects possess C parity + 1, the same as the vacuum, and do not propagate. It is conceivable that a Reggeisation of the $2^+$ particles using the method of M. Gell-Mann, M.L. Goldberger, F. Low, E. Marx and F. Zachariasen, Phys. Rev. 133, B145 (1964). could demonstrate that $\partial_\mu \theta^i_{\mu\nu}$ provide the field theoretic characterisation of the Pomeranchukon.

8. The most general SU(3) invariant coupling of the three nonets is

$$(\tfrac{1}{2} - \alpha)\, d^{ijk} + \sqrt{\tfrac{2}{3}}\;\; \alpha \delta^{jk}\delta^{io} + \beta\,(\delta^{ik}\delta^{jo} + \delta^{ij}\delta^{ko} - 2\delta^{io}\delta^{jo}\delta^{ko})$$

The calculations of S. Glashow and R. Socolow, Phys.Rev. Letters. 15, (1965) 329 strongly indicate that $\alpha = o$, while $\beta = o$ on the basis of supermultiplet theories ( R. Delbourgo, M.A. Rashid and J. Strathdee, Phys. Rev. Letters. 14, (1965) 719. and R. Gatto, L. Maiani and G. Preparata, Nuovo Cimento 39 (1966) 1192).

9. The methods of this paper could also be applied to the spin densities $S_{\mu\nu\lambda}$ appearing in

$$M_{\mu\nu} = \int M_{\mu\nu\lambda}\, d\sigma_\lambda\,;$$

$$M_{\mu\nu\lambda} = x_\nu \theta_{\mu\lambda} - x_\mu \theta_{\nu\lambda} + S_{\mu\nu\lambda}; \quad S_{\mu\nu\lambda} = \frac{-i\delta L}{\delta(\partial_\lambda \varphi)}\, \Sigma_{\mu\nu\varphi}\,.$$

These can be similarly generalised to $S^i_{\mu\nu\lambda}$.

10. N. Cabibbo, Horowitz and Y. Ne'eman (CERN preprint) have obtained a similar result using the $0^+$ residues on the Pomeranchuk trajectory. Since their Regge formulas are expressed in terms of $(s/s_0)^{\alpha(t)}$ rather than the more natural combination $(\cos\theta)^{\alpha} \approx (s/2m\mu)^{\alpha(t)}$ as in our case, the precise relation of the residues assumed by ourselves and these authors is not clear.

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