ON THE RELATION BETWEEN TOTAL AND PARTIAL WIDTHS IN NUCLEAR REACTIONS

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ABSTRACT

The ratio between the total width and the sum of the partial widths as defined in the S-matrix theory has been computed for two resonance levels of $^7\text{Be}$ from a simultaneous fitting of the reactions $^6\text{Li}(p,p)$, $^3\text{He}(\alpha,\alpha)$ and $^6\text{Li}(p,\alpha)$. 
ON THE RELATION BETWEEN TOTAL AND PARTIAL WIDTHS IN NUCLEAR REACTIONS

When real boundary conditions are used to define a set of eigenstates of a nucleus, such as in the R-matrix theory [1], the total width of one such state is by definition the sum of its partial widths for open channels; with familiar notations
\[
\Gamma_n = \sum_{c^+} \Gamma_{cn}
\]  
(1)
The same relation does not hold when complex boundary conditions are used and the total width defined as twice the negative of the imaginary part of the eigenvalue \( \delta_n = E_n - \frac{1}{2} i \Gamma_n \). This is the case in the KAPUR-PEIERLS theory [2]*, the MOLDAUER [3] development of the R-matrix theory and the S-matrix theory [4]. MOLDAUER [3] and HUMBLET and ROSENFELD [4] chose to modify the normalization of the partial widths in order to keep the relation (1) valid. Hence, an extra constant factor \( q_n \) is introduced in the numerator of the resonance terms of the collision matrix expansion.** This plays a controversial part in the statistical theory of nuclear cross section [3].

Confining ourselves to the S-matrix theory, the only theoretical result so far available on the values of \( q_n \), refers to a limiting case, namely [5]
\[
q_n \rightarrow 1 \quad \text{for} \quad \Gamma_n \rightarrow 0.
\]  
(2)
This is a generalization of a result first obtained by WEIDENMÜLLER [6] in his model of many-channel scattering; in a numerical example, he has also obtained \( 0.94 < q_n < 1 \) for \( 1.5 \text{ MeV} \gg \Gamma_n > 0 \). On the other hand, it is possible to compute \( q_n \) from experimental data when they are sufficiently complete. This is done here for two levels of \(^7\text{Be}\).

* See also ref. [1], p. 236.

** In Moldauer notation, this is the quantity \( N_n \) defined according to his eq.(A16); in the S-matrix theory, this is the quantity \( \bar{q}_n \) defined by eq.(6.10) in ref.[5].

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The available differential and integrated cross section for the three reactions [7]

\[ ^6\text{Li}(p, p), \; ^3\text{He}(\alpha, \alpha), \; ^6\text{Li}(p, \alpha), \; (3) \]

have been simultaneously fitted using a two-level approximation [8] for the collision matrix elements. The investigated region of excitation energy of \(^7\text{Be}\) extends from 6.0 to 8.4 MeV; two \(5/2^-\) resonances are observed around 6.40 and 7.20 MeV.

Let \(E_n, E_m, \Gamma_n, \Gamma_m\) be the positions and total widths of these resonances. The resonance terms of the collision matrix elements have numerators proportional to

\[ q_i^{1/2} \Gamma_{pi}^{1/2} \Gamma_{ai}^{1/2} \quad (i = n, m) \quad (4) \]

for the three reactions (3) respectively. Least square adjustments have been performed taking into account the obvious relation existing between the products (4). Good fits were achieved using the values given in the first four lines of Table I.

The detailed analysis will be published elsewhere [9]. Here, Figs. 1 to 3 give only one fit for each reaction. The collision matrix elements introduced in the analysis are the same as those which were used by McCRAY [7]: some channel spins introduce resonance terms, but others only contribute to a non-resonant background term. The fact that the two levels have the same spin and parity introduces a large number of interference terms in the cross sections. Nevertheless, just to exemplify the possible importance of such interference terms, the contribution of the one interference term (represented by the dotted line) corresponding to the interference between the two resonances (pole terms) is also given on Figs. 1 to 3. This interference term only gives a small contribution to the differential cross sections for the elastic reactions, while it appears to bring a very important contribution to the \((p, \alpha)\) reaction.

The relations \(\Gamma_i = \Gamma_{pi} + \Gamma_{ai} \quad (i = n, m)\) together with the third and fourth lines of Table I, easily lead to the results given in the last three
lines of Table I. Both $q_n$ and $q_m$ turn out to be close to unity.

The two levels considered here being rather broad, our results suggest that $q_n$ is likely to be close to unity in the lower part of an excitation spectrum. In view of the great difficulty to estimate $q_n$ from theoretical arguments, it is, however, necessary to perform other analysis of the type described here before making such a general statement.

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<table>
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<th>$i = n$</th>
<th>$i = m$</th>
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<tr>
<td>$E_i$</td>
<td>6.497</td>
<td>7.078</td>
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<tr>
<td>$\Gamma_i$</td>
<td>1.086</td>
<td>0.469</td>
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<td>$q_i \Gamma_{pi}$</td>
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<td>$q_i \Gamma_{\alpha i}$</td>
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<tr>
<td>$\Gamma_{pi}$</td>
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<td>$\Gamma_{\alpha i}$</td>
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<td>0.027</td>
</tr>
<tr>
<td>$q_i$</td>
<td>0.988</td>
<td>0.881</td>
</tr>
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</table>

Energy, total and partial widths in MeV and $q_i$ factors for two resonances in $^7\text{Be}$.
$\sigma(\theta)(\text{mb})$

$\theta_{\text{CM}} = 125^\circ$

$^6\text{Li}(p,p)^6\text{Li}$
\[ \text{Fig. 3} \]

\[ 6\text{Li}(p,\alpha)^3\text{He} \]

\[ \sigma_{\text{tot, C.M.}} \]

\[ E_{p, \text{lab.}} \] (MeV)
FIGURE CAPTIONS

Fig. 1  Differential cross section for the reaction $^6\text{Li}(p, p)$ at $125^\circ$; the dotted line gives the contribution of the cross term corresponding to the two resonances.

Fig. 2  Differential cross section for the reaction $^3\text{He}(\alpha, \alpha)$ at $106,6^\circ$; the dotted line gives the contribution of the cross term corresponding to the two resonances.

Fig. 3  Integrated cross section for the reaction $^6\text{Li}(p, \alpha)$; the dotted line gives the contribution of the cross term corresponding to the two resonances.
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