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MULTIPLE FIELD RELATIONS

Three elements have gone into recent calculations in particle theory; 1) higher symmetry multiplets, 2) a postulated algebra for weak currents, 3) PCAC and similar hypotheses.[1] These elements appear somewhat unrelated as used at present. We suggest in the present note that there may be a common basis for these and that this basis may lie in postulating (with Heisenberg) that there exist fundamental fields and all matter is made up from them. For such a fundamental strong field, the simplest assumption is to take a triplet of quarks q_i $(i = 1, 2, 3)$. Such field theories face two problems: 1) what is the fundamental Lagrangian - or at least what symmetry properties it possesses - and, 2) much more difficult, how to express physical fields as multiples of quark fields. This note is concerned with these problems; the particular Lagrangian we choose leads to an approximate higher symmetry classification U(6) x U(6) of strongly interacting particles; using it we are able to obtain a (heuristic) derivation of equal time commutation relations (C.R.) not only for currents but for arbitrary composite fields for use in dispersion theory. These relations are different from those one would get in a canonical formulation of field theory. Further, from the point of view here adopted it appears that relations like PCAC, PCTC, PCBC, etc., essentially express the content of the quark equations of motion.

For the quark Lagrangian take:

$$\begin{aligned}
\mathcal{L} &= \mathcal{L}_{K.E.} + \mathcal{L}_{mass} + \mathcal{L}_{strong} + \mathcal{L}_{weak} + \mathcal{L}_{EM} \\
\mathcal{L}_{K.E} &= \bar{q}\, \not\partial\, q \\
\mathcal{L}_{mass} &= \Sigma\, m_i\, \bar{q}_i\, q_i \\
\mathcal{L}^{(2)}_{strong} &= \lambda\, (\bar{q}\, q)^2 \\
\mathcal{L}_{weak} &= G\, J_\mu\, J_\mu^{+} \\
J_\mu &= \bar{q}\,(1 - i\gamma_5)\gamma_\mu q + \text{lepton currents}
\end{aligned}$$

where only appropriate charged currents are retained in J_μ.

$\mathcal{L}_{K.E} + L_{weak} + L_{E.M.}$ possesses SU(3) x SU(3) symmetry; $\mathcal{L}_{mass} + \mathcal{L}_{strong}$ is SU(6, 6)-invariant if $m_1 = m_2 = m_3$.

For constructing physical particle wave functions, the proper formulation is to consider Bethe-Salpeter amplitudes. For mesons, for example, (leaving out SU(3) indices) one may consider $M_\alpha^\beta(p, k)$ which is Fourier transform of $(0 \mid T(q_\alpha(x_1)\ \bar{q}^\beta(x_2)) \mid p)$. In a set of earlier papers[3] it has been shown that the Lagrangian above can lead to the appearance of a U(6) x U(6) multiplet structure at least for 0^- and 1^- particles as solutions of an approximate Bethe-Salpeter equation. Since it is our intention to write multiple field expressions for physical fields and ultimately to consider equal time commutators, we work with the idealization of the Bethe-Salpeter approach given by NISHIJIMA and ZIMMERMAN.[4] In this approach a local composite field, satisfying the axioms of field theory, is defined (for mesons for example apart from a normalization constant) as:

$$M_\alpha^\beta(x) = \lim_{a \to 0} T(\bar{q}^\beta(x+a)\ q_\alpha(x-a))$$

for zero orbital angular momentum and as

$$\lim_{a \to 0} \sum_\gamma^{\ell} \beta_\gamma \frac{\partial^\gamma}{\partial a^\gamma} \left(T \bar{q}(x+a)\, q\ (x+a) \right)$$

for orbital momentum ℓ . In Zimmerman's formulation a is a space-like or a time-like (but not a light-like) vector. We propose to write equal time C.R. for fields $M_\alpha^\beta(x)$ using the basic quark C.R. with a space-like (in particular $a_0 = 0$). Barring Schwinger terms,[6] this gives:

$$\left[M_\alpha^\beta(X),\ M_\gamma^\delta(Y)\right] \delta(X_0 - Y_0) = -\delta^4(X-Y)\left[(\gamma_0)_\alpha^\delta\ M_\gamma^\beta - (\gamma_0)_\gamma^\beta\ M_\gamma^\delta\right]$$

It is important to stress that unlike a conventional theory of phenomenological fields with its canonical C.R., the composite fields corresponding to two distinct particles do not necessarily commute on a space-like surface. This was first pointed out by THIRRING[5] and constitutes a distinguishing feature of composite model theories. Indeed this may provide a test of the idea whether or not fundamental fields exist at all.

The first problem with this approach is; if we use the completeness of the Dirac algebra to write

$$M_\alpha^\beta(x) = \phi(x) + \gamma_5 \phi_5(x) + i\gamma_\mu \gamma_5 \phi_{\mu 5}(x) + \gamma_\mu \phi_\mu(x) + \tfrac{1}{2}\sigma_{\mu\nu} \phi_{\mu\nu}(x)$$

what physical particles, if any, could correspond to the respective field components ϕ, ϕ_5, ..., etc.? To decide on this we use repeatedly on $M_\alpha^\beta(x)$ the quark equation of motion

$$\not{p}\, q(x) = (m+\sigma)\, q(x) \qquad (1)$$

where $\sigma = (2\lambda\, \bar{q}\, q)$. In the self-consistent <u>approximation</u> (see below) where $\bar{q}q$ on the right of (1) is replaced by its expectation value,[7] we get

$$\int \left(\left\{ \tfrac{\not{p}}{2}, M(p,k) \right\} + \left[\not{k}, M(p,k) \right] \right) = 0 \qquad (2)$$

$$\int \left(\left[\tfrac{\not{p}}{2}, M(p,k) \right] + \left\{ \not{k}, M(p,k) \right\} \right) = \mu \left(\int M dk \right) \qquad (3)$$

Here $\mu = 2(m+\sigma)$ and $\int M(p,k)dk$ is the Fourier transform of $M(x) = \lim_{a\to 0} M(x,a)$. The γ-decomposition of these equations gives

$$p_\mu \Phi_\mu = 0 \qquad (4)$$

$$i\, p_\mu \Phi_{\mu 5} = \mu \Phi_5 \qquad (5)$$

$$i\, \mu \Phi_{\mu 5} = p_\mu \Phi_5 - i \int \varepsilon_{\mu\nu\rho\kappa}\, k_\nu\, \phi_{\rho\kappa}^{(p,k)}\, dk \qquad (6)$$

$$i\, \mu \Phi_{\mu\nu} = p_\mu \Phi_\nu - p_\nu \Phi_\mu + 2i \int \varepsilon_{\mu\nu\rho\kappa}\, k_\rho\, \phi_{\kappa 5}^{(p,k)}\, dk \qquad (7)$$

$$i\, p_\nu \Phi_{\nu\mu} = \mu \Phi_\nu - 2 \int k_\mu\, \phi(p,k)\, dk \qquad (8)$$

$$i\, p_\mu \Phi = -2 \int k_\nu\, \phi_{\mu\nu}(p,k)\, dk \qquad (9)$$

where $\Phi = \int \phi(p,k)dk$ is the Fourier transform of $\phi(x)$ etc. In all these equations it is important to remember that $\sigma = \lambda \bar{q} q$ has been replaced by a constant. This ignores essential dynamics. But even at the kinematical level, note that if in (6) terms on the right under the integral sign are ignored, we recover just the set of Bargmann-Wigner equations for a U(2, 2) multiplet with $\phi_5(x)$, $\phi_\mu(x)$ as independent quantities and $\phi_{\mu 5}(x)$, $\phi_{\mu\nu}(x)$ expressible as their gradients or curls. Further, to this approximation, $p_\mu \phi = 0$, so that ϕ is a constant field consistent with the replacement above of $\phi = \langle\phi\rangle = \langle\bar{q}q\rangle = \sigma$.

We now make the identifications,[8]

$$m_\pi^2 \pi(x) = \alpha \bar{q} \gamma_5 q \quad , \quad m_\rho^2 \rho(x) = \beta \bar{q} \gamma_\mu q$$

Eqs. (4) and (5) read

$$p_\mu \rho_\mu(x) = 0 \tag{10}$$

$$p_\mu J_{\mu A}(x) = p_\mu (i\bar{q} \gamma_5 \gamma_\mu q) - \frac{2\mu}{\alpha} m_\pi^2 \pi(x) \tag{11}$$

Eq. (11) is the analogue of PCAC in our formalism. Eq. (6) further states that $\phi_{\mu 5} = i \bar{q} \gamma_5 \gamma_\mu q$ equals $\frac{2i\alpha\mu}{m_{\pi^2}} p_\mu \pi(x) + R_{\mu 5}(x)$ where[9] $R_{\mu 5}$ may possibly represent a field corresponding to a 1^+ particle. However, there may exist a certain region of frequencies (particularly if the 1^+ particle is much heavier than the π-meson) where the axial-vector field $q \gamma_\mu \gamma_5 q$ may be approximated by the gradient of the π-field, and likewise for the tensor field which may be approximated by the curl of a vector.

Our final conclusion may be stated thus: with the postulated quark Lagrangian, it is possible to write a string of quark-antiquark local field operators. Some of the operators can be identified with independent particles contained in U(2, 2) multiplets (or with inclusion of SU(3), with U(6, 6) multiplets), while the remainder fields possess an approximation in which they can be expressed as gradients or curls of the independent fields mentioned above, following the pattern set by the Bargmann-Wigner formalism plus a residual part which may possess vanishingly small matrix elements for the relevant U(6, 6) free particle states. The completeness of the γ-algebra ensures that all U(2, 2) tensors into which a product like $(qq \ldots \bar{q}\bar{q} \ldots)$ can be decomposed find an interpretation. Further, among these Bargmann-Wigner relations are contained the divergence relations which are analogues of PCAC, PCTC, etc.

To illustrate the utility of the rules formulated above regarding the use of the Bargmann-Wigner ansatz for redundant U(6, 6) tensors, we compute some expressions for the equal time C.R. of the nucleon-fields with meson-fields. To avoid problems of fractional charges and para-statistics (needed to construct a local s-wave symmetric baryon field) we use the Nambu-Han model instead of the pure quark model with three fundamental triplets obeying Fermi statistics. Thus

$$M_A^B(x) = \bar{\psi}^{B\ell}(x)\, \psi_{A\ell}(x)$$

$$B_{ABC}(x) = \epsilon^{ijk}\, \psi_{Ai}(x)\, \psi_{Bj}(x)\, \psi_{Ck}(x) \quad .$$

$\ell = 1, 2, 3$ for the three triplets

It is easy to check that

$$\left[B_{ABC}(x),\ M_E^D(y) \right] \delta(x_0 - y_0) = \delta^4(x-y) \left[(\gamma_0)_C^D\, B_{ABE} + (\gamma_0)_B^D\, B_{ACE} + (\gamma_0)_A^D\, B_{BCE} \right]$$

$$\left\{ B_{ABC}(x),\ \bar{B}^{DEF} \right\} \delta(x_0 - y_0) = \delta^4(x-y) \sum_{A,B,C\ \text{perm.}} (\gamma_0)_A^D \left[M_{BC}^{EF} + (\gamma_0)_B^E\, M_C^F - (\gamma_0)_B^E (\gamma_0)_C^F \right] \tag{12}$$

Here $M_{BC}^{EF} = \bar{\psi}^{E\ell}\, \bar{\psi}^{Fm}\, \psi_{B\ell}\, \psi_{Cm}$ and represents a mixture of 4212 + 5940 multiplets of U(6, 6).

Both B and M may be decomposed relative to SU(3) x U(2, 2) in the usual manner. Denoting SU(3) indices by p, q, r and U(2, 2) indices as α, β, $\gamma \ldots$, and defining the usual projections:-

$$\tfrac{1}{4} \epsilon^{pqr} (C^{-1}\gamma_5)^{\alpha\beta}\, B_{ABC} = 2\delta_r^s\, V_\gamma + 3 N_{\gamma,r}{}^s$$

$$\tfrac{1}{4} \epsilon^{pqr} (C^{-1} i\gamma_\lambda \gamma_5)\, B_{ABC} = 2\delta_r^s\, V_{\gamma,\lambda} + 3 N_{\gamma,\lambda,r}{}^s$$

we obtain from (12) for the baryon commutators with time-component of the axial current the expression

$$\left[2 V_\gamma(x)\, \delta_\gamma^u + 3 N_{\gamma r}{}^u(x) ,\quad M_{05,s}{}^t(y) \right] \delta(x_0 - y_0)$$

$$= \tfrac{1}{4} \delta^4(x-y) \Big[\delta_r^t\, i\gamma_5 \left(2V(x)\, \delta_s^u + 3 N_s^u(x) \right)$$

$$+ \left\{ i \delta_s^t \left(-2\gamma_5 V(x)\, \delta_r^u + 3 \gamma_5 N_r^u + 3i \gamma_\lambda \gamma_5 N_{\lambda,r}{}^u \right) \right.$$

$$\left. - \; t \leftrightarrow u \right\} \Big]$$

The important point is that this gruesome relation reduces to an extremely simple form when one takes account of the Bargmann-Wigner (approximate) ansatz which states that $V \equiv \gamma_5 N + i \gamma_\lambda \gamma_5 N_\lambda \equiv 0$. The commutator then reads:

$$\left[N_p{}^q(x) ,\; M_{05,r}{}^s(y) \right] \delta(x_0 - y_0) \quad = \quad \tfrac{1}{4} \delta^4(x-y)\, \delta_p^s \;\; i\gamma_5 \; N_r^q(x)$$

i.e., (apart from SU(3) complications) the commutator of the time-component of the axial current with the nucleon produces γ_5 times the nucleon. It is important to stress once again that this reasonable result has been obtained using for both the nucleon and the axial current their expressions in terms of quark fields and by getting rid of redundant components using Bargmann-Wigner formalism. One can similarly write C.R. of the field N_λ (which satisfies the PCBC relation $p_\lambda N_\lambda = \mu N$) with M_{05}; the r.h.s. now also involves the decuplet field. With the whole host of new equal time relations available now along with the U(6,6) table giving (at least an approximate) physical interpretation of all composite operators introduced, the range of calculable consequences one can draw is likely to be greatly extended. (Since C.R. in a field theory in general incorporate the the content of kinetic energy terms in a Lagrangian, the use of C.R. will improve pure U(6,6) results to the extent of including parts of kinetic corrections). But before this is done, it is important that one should acquire some feeling for how good the Bargmann-Wigner U(6,6) ansatz in practice turns out to be for the redundant operators and also how consistent such identification of redundant components and the C.R. is. It is indeed with considerable trepidation that we are putting forward this note in so far as our conclusions about C.R. differ very considerably from those accepted in in conventional phenomenological field theories.

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1. In addition to PCAC, other hypotheses like PCTC (W. KRÒLIKOWSKI, Nuovo Cimento 42, A 435 (1966); R. GATTO, M. MAIANI and G. PREPARATA, Nuovo Cimento 20, 622 (1966)) and PCBC (R. HWA and J. NUYTS, Princeton preprint) have recently been proposed. Our formulation gives a unified treatment of such relations. The paper of Hwa and Nuyts contains a treatment of equal time C.R. similar to ours. We are indebted to these authors for sending us a copy of their results prior to publication.

2. $\mathcal{L}_{strong}$ is the simplest 4-Fermi scalar one could write. It is immaterial for future considerations if it has the form $\Sigma \lambda_n (\bar{q}q)^{2n}$ with n arbitrary.

3. R. DELBOURGO, M.A. RASHID, ABDUS SALAM and J. STRATHDEE, Elementary Particles and High Energy Physics (IAEA, Vienna, 1965) p. 455 and ICTP, Trieste, preprint IC/66/60;
J. DABOUL and R. DELBOURGO, Nuovo Cimento (to be published).

4. K. NISHIJIMA, Progr. Theoret. Phys. (Kyoto) 17, 765 (1957);
W. ZIMMERMAN, Nuovo Cimento 4, 129, X (1958).

5. W. THIRRING, Theoretical Physics, (IAEA, Vienna, 1962) p. 467.

6. See however K. JOHNSON and F. E. LOW, M.I.T. preprint.

7. This inclusion of interaction term in mass renormalization is possible only for U(6, 6)-invariant $\mathcal{L}_{strong}$. For a theory with $(\bar{q}\gamma_\mu q)(\bar{q}\gamma_\mu q)$ interaction for example one could not use the simple ansatz above. An infinite constant coming from differentiating the time-ordered product has been dropped on the right of (3). Note that the effective quark mass inside "quark matter" $(m + \sigma)$ could be very different from m (smaller if m and σ are of opposite sign).

8. α and β are normalization constants. In Zimmerman's formalism

$$(m_\pi^2/\alpha) = \lim_{a \to 0} \int e^{ikx}\, dx \, \langle 0 | T\, \bar{q}(a)\, \gamma_5\, q(-a)\, \bar{q}(x+a)\, \gamma_5\, q(x-a) | 0 \rangle \Big|_{k^2 = m_\pi^2} .$$

The integral on the right is essentially the self-mass integral and therefore one may expect that $\alpha \approx 1$.

9. Combining (5) and (6), $R_{\mu 5}$ appears as pion-quark current, presumably with zero matrix element for the single pion state provided $2M \approx$ mean mass of the 0^- multiplet.

10. Note that the relation $p_\mu (i\bar{q}\, \gamma_\mu \gamma_5 q) = \frac{2\mu}{\alpha}\, m_\pi^2\, \pi(x)$ survives even after taking account of weak interactions in the quark equation of motion. The next task of the present formalism is to express $J_{\mu A} = i\bar{q}\gamma_5 \gamma_\mu\, q(x)$ in terms of physical fields (nucleons, pions, etc.) and to derive the Goldberger-Treiman value of the constant $\frac{2\mu}{\alpha} = \frac{g_A}{g_V} = \left(\frac{2M_N}{G_{\pi N}}\right)$. This is a difficult but, we believe, not a completely hopeless task.

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