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ACCORDING TO SL(6,C)

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1. From recent experiments it has been concluded that one cannot understand the electromagnetic form factors of the nucleons with a simple resonance model using the observed masses of only the $\omega$, $\varphi$ and $\rho$ vector mesons. 1) It has been pointed out 2) that an excellent fit to the data may be achieved if one writes

$$G_i(t) = K(t) D_i(t)$$

where $i = E_p, E_N, M_p, M_N$, $D_i(t)$ are the vector meson pole approximations (with the actual observed masses), and $K(t)$ is a phenomenological factor that is the same in all four cases. If $K(t)$ is represented in the form $\frac{\Lambda^2}{(\Lambda^2 - t)}$, then one finds that $\Lambda$ is of the order of the nucleon mass, which suggests that the origin of this factor is kinematical rather than dynamical. That is, $K(t)$ may be associated with the normalization of the external lines in the Feynman diagram, rather than the intermediary states in the dispersion picture.

2. Recently we have reported 3) a calculation of a typical baryon form factor, as predicted by a relativistic extension of SU(6) symmetry. The charge form factor of the proton, for $t < 0$, was written in the form

$$G_{E_p}(t) = K(t) D_{E_p}(t)$$

where

$$K(t) = \left(1 - \frac{t}{4M_p^2}\right)^{-\frac{3}{2}}$$

is a kinematical factor predicted by the symmetry. Using the actually observed masses of the vector mesons in the expression for $D_{E_p}(t)$ we obtained excellent agreement with experiment. Here we report a similar treatment of the charge form factor of the pion.

Again we write, for $t < 0$

$$G_{\pi}(t) = K(\pi)(t) D_{\pi}(t)$$

-1-
where \( D_\pi(t) \) is the vector meson pole approximation, and \( K_\pi(t) \) is the symmetry prediction in the absence of dynamical effects. Again we emphasize that \( K_\pi(t) \) is to be regarded as purely kinematical. A calculation of \( K_\pi(t) \) to the first order in \( t \) is given below. The result is

\[
K_\pi(t) \approx 1 + \alpha \frac{t}{M_\pi^2}
\]

where \( M_\pi \) is the mass of the pion and the slope \( \alpha \) is

\[
\alpha = \frac{1}{12} \left[ 1 - \frac{13C}{12} \right]
\]

The real parameter \( C \) is the Casimir operator of the unitary representation of SL(6, C) to which the mesons are assigned. Unitarity requires that

\[
C < -12
\]

A complete calculation of \( K_\pi(t) \) has been reported elsewhere; the result for two values of \( C \) is shown in Fig. 1.

3. The most striking feature of this result is the prediction of a very large charge radius for the pion. In fact (5) implies that \( <r^2_\pi > 3.8 \text{ F.} \). This large value is to be expected since \( K_\pi(t) \), being kinematical, is determined by the low pion mass. In fact, the large amount of symmetry breaking, evidenced by the meson mass spectrum, introduces a considerable amount of uncertainty as to what is actually the correct mass to use in Eq. (4). We should like to stress, nevertheless, that the presence of a factor \( K_\pi(t) \), introduced by relativistic SU(6) symmetry, implies that the pion charge radius is considerably larger than the simple picture of vector meson dynamics would suggest. Preliminary experimental evidence suggests that this is actually the case.
4. For time-like momentum transfer, Eq. (1) is replaced by

\[ G_i(t) = \frac{4M_N^2 - t}{t} K(4M_N^2 - t) D_i(t) \]

with the same functions \( K \) and \( D_i \) as in Eq. (1). Putting \( t = 5 \) and \( 6.8 \text{ (GeV/c)}^2 \) we find \( G_i(t) = .04 \) and .03, which is in agreement with preliminary experimental results. For the pion, Eq. (3) is replaced by

\[ G_\pi(t) = K_\pi(4M_\pi^2 - t) D_\pi(t) \]

Here too, the factor \( K_\pi \) has a very marked effect on the form factor. We suggest that the measurement of \( G_\pi(t) \), both for \( t < 0 \) and for \( t > 4M_\pi^2 \), is one of the most exciting projects presently becoming experimentally feasible.

5. We give an outline of the calculation that led to the result (4). The complete calculation of \( K_\pi(t) \), the result of which is shown in Fig. 1, was carried out by a quite different method \(^4\) and cannot be shown here.

The representation of \( SL(6, \mathbb{C}) \) associated with the mesons has the tensorial basis \(^7\)

\[ \phi_{\lambda_1, \ldots, \lambda_n; c_1, \ldots, c_n} \]

where \( M \) is a complex number related to the second order Casimir operator characterizing the representation, by \( C = 2M(M + 5) \). The charge part of the electromagnetic vertex is assumed to be given by

\[ \frac{1}{2M} (\rho + \rho') \phi_{\lambda_1, \ldots, \lambda_n; c_1, \ldots, c_n} \phi_{\lambda_1, \ldots, \lambda_n; \rho, \ldots, \rho} \]

where \( Q \) is the SU(3) charge operator.

Expanded in powers of the three-momentum, the 35 multiplet \( \phi_A^B(p) \) in the meson representation is

-3-
Thus, in the rest frame of one of the mesons, so far as it involves the 35\textsuperscript{−} multiplet and to second order in momentum only, the interaction (1) reduces to

\[ \Phi^A (\rho) = \Phi_A (\rho) + (I-N) \frac{C}{D} \left( \frac{\hat{p}_C \cdot \hat{p}_D}{2M_K} \right)^2 \Phi_A (\rho) + \ldots \]

\[ + \frac{1}{2} \left[ (M-N) C \left( \frac{\hat{p}_C \cdot \hat{p}_D}{2M_K} \right)^2 \right] \Phi_A (\rho) \]

The calculation of this expression is a straightforward application of the formulae given, e.g. in ref. 7, and the result is

\[ \frac{1}{2M_K} (\rho + \rho') \mu \left[ 1 + \frac{C}{D} \left( \frac{13M(M+5)-6}{72} \right) \Phi_A (\rho) \Phi_A (\rho) \right] \]

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4) C. FRONSDAL and HARUN AR-RASHID, ICTP, Trieste, preprint IC/66/89.


6) See MASSAM and ZICHICHI, ref. 2.


FIGURE CAPTION

Fig.1

The pion form factor for $M = -\frac{5}{2}$ and $M = -\frac{5}{2} + 2i$, as calculated in ref. 4. The slope at the origin agrees with Eqs. (4) and (5).
Fig. 1

\[ K_\pi(t) \]

\[ M = -\frac{5}{2} \]
\[ M = -\frac{5}{2} + 21 \]

\[ -\frac{t}{M^2 \pi} \]