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DUE TO IMPURITY IONS

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It is well known that the two greatest barriers to controlled fusion are the stability problem and the impurity problem - the latter being of great importance since even a small quantity of impurity greatly enhances the radiation losses of the plasma. Such impurities are likely to arise near the wall of a plasma container, especially in the absence of a divertor. In this note we point out that the presence of impurity ions, especially if their density gradient is different from the plasma density gradient, i.e., if they are peaked near the wall, can generate a new instability which is difficult to stabilize and which leads to diffusion of the impurities into the plasma.

Here we consider a one-dimensional model with density varying in the x-direction and the main magnetic field along z. A situation with plasma pressure p much less than magnetic pressure $B^2 / 8 \pi$ is assumed. Then, the electric field perturbations can be taken as potential $\vec{E} = - \nabla \psi$. Considering the wavelength of such perturbations to be greater than the typical Larmor radii of plasma particles, we make use of the Vlasov equation.

$$\frac{\partial f_i}{\partial t} + V_z \frac{\partial f_i}{\partial z} + c \frac{E \times B}{B} \cdot \frac{\partial f_i}{\partial \gamma} + \frac{ZeE_z}{M_i} \frac{\partial f_i}{\partial \nu_z}$$

for the distribution functions $f = f (v_z, \vec{r}, t)$, describing the particle motion in terms of the drift of the guiding centres. The second term in Eq. (1) represents the motion of particles along magnetic field lines, which is influenced only by electric field component along $\vec{B}$ entering the fourth term. The third term represents the electric drift $c \frac{E \times \vec{B}}{B^2}$ of the guiding centres. Thus, in equilibrium, the distribution functions for all species have a form $f_i^0 (x, v_x, v_y)$. Here index $i = I, H, e$ labels the impurity, hydrogen ions and electrons, respectively. The linear stability analysis is then reduced to solving the eigenvalue problems for the linearized equations.
\[ \frac{i(\omega + k_z v_z)}{B_0} \frac{\delta f_i}{\partial x} + \frac{e E_i}{B_0} \frac{\delta f_0^i}{\partial x} + Z_i e \frac{E_x}{M_i} \frac{\delta f_0^i}{\partial v_z} = 0 \]  

(2)

and

\[ Z_i \int dv_z \delta f_i + \int dv_x \delta f_i = \int dv_x \delta f_e \]  

(3)

where

\[ \delta f_i = \sum \delta f_i(x) \exp (i k_y y + i k_z z) \]

\[ E_x = \sum E_x(x) \exp (i k_y y + i k_z z) \]

and \( Z_i \) represents the charge number of each species. Eq. (3) is given by the quasi-neutrality condition.

The substitution of \( \delta f_i \) from Eq. (2) into Eq. (3), since \( E = -\nabla \Phi \), gives the final dispersion relation

\[ k_y \frac{e}{B_0} \int \frac{dv_x}{\omega + k_z v_z} \left\{ \frac{\partial \delta f_i}{\partial x} - \frac{\partial \delta f_e}{\partial x} + Z_i \frac{\partial \delta f_i}{\partial x} \right\} + e k_z \int \frac{dv_x}{\omega + k_z v_z} \left\{ \frac{1}{M_i} \frac{\partial \delta f_e}{\partial v_z} + Z_i^2 \frac{\partial \delta f_i}{\partial v_z} + \frac{1}{m_i} \frac{\partial \delta f_i}{\partial v_x} \right\} = 0 \]  

(4)

For the sake of definiteness we choose \( f_o^i(x) \exp (-M_i v^2 / 2 T_i) \). In this case the integrals may be performed to give

\[ \sum_i Z_i^2 \frac{n_i}{T_i} \left[ 1 - (1 + W_i) \left( 1 + \frac{\omega_i^*}{\omega} \right) \right] = 0 \]  

(5)
Here \( n_i \) represents the density, \( T_i \) the temperature, \( \omega_i = k_y v_{di} \), \( v_{di} \) being the diamagnetic velocity \( v_{di} = v_{ri} / (2 n_i \Omega_i) \), \( \Omega_i = Z_i e B / |H_i| c \) being the gyro-frequency and

\[
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\]

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\[
W_i = -\frac{1}{\sqrt{\pi}} \int_0^{\infty} \frac{3 e^{-\frac{x^2}{2}}}{(\xi + \omega)/k_y v_{thi}}
\]

Notice that in general, when terms of order \( k_y v_{thi} / \Omega_i \) are taken into account and isotropic temperatures are assumed, the second term in brackets is to be multiplied by \( I_0(b_i) e^{-b_i} \) with \( b_i = k_y^2 T_i / (M_i \Omega_i^2) \), and \( I_0 \) being the modified Bessel function of zero order.

We observe that for \( \omega / k_y > v_{th} \) the presence of a small concentration of impurity ions \( n_i' \ll n_i \) is irrelevant and we obtain the usual stable drift oscillations for the case of zero Larmor radius and no temperature gradient. Then the more interesting case is that given by

\[
V_{thI} < \frac{\omega}{k_y} < V_{thH} \ll V_{th}\epsilon
\]

Note that this may come about even for \( T_h = T_i \) if \( M_i > M_h \). In this case we may use the proper asymptotic forms for the \( W \) functions occurring in Eq. (5).

\[
W_\epsilon = -1 + i \sqrt{\pi} \frac{\omega}{k_y v_{th\epsilon}}
\]

\[
W_H = -1 + i \sqrt{\pi} \frac{\omega}{k_y v_{thH}}
\]

\[
W_I = \frac{1}{2} \frac{k_y^2 v_{thI}^2}{\omega^2}
\]

Noting that charge neutrality requires \( \Sigma Z_i n_i \omega_i^\ast = 0 \) we may solve Eq. (5) to find

\[
\omega = \frac{Z_i \omega_i n_i \frac{h_i}{T_i}}{n_e/T_e + n_H/T_H - Z_i^2 n_i \frac{W_I}{W_\epsilon} - i \sqrt{\pi} \frac{(\omega + \omega_H^\ast)}{k_y v_{thH}^2} \frac{n_H}{T_H}}
\]

\[
(6)
\]
Here \( W_1 \) is real since \( \omega > k_z v_{in} \) and marginal stability obtains for \( \omega = -\omega^*_H \). The condition for instability is then:

\[
\frac{n_H Z_1}{n_e \frac{T_H}{T_e} + n_H - n_H^2 \frac{T_H}{T_I}} + \frac{n_H^1}{n_H^1} < 0
\]  

(7)

Two types of instability can now occur. If the number of impurity ions is not too small the impurity ion sound wave can be made to go unstable if the denominator of the first term of Eq. (7) is negative. As the maximum value of \( W \) is about 0.2 the condition for this instability is

\[
0.2 \frac{n_H^2 \frac{T_H}{T_I}}{n_e + n_H} > n_e + n_H
\]

(8)

Due to the possible large value of \( Z_1^2 \) this may occur at relatively modest impurity densities.

For the other mode we may consider the case in which \( n_1^1 \ll n_H \). Then the instability condition is given by

\[
\frac{n_H^1}{n_1^1} + \frac{n_H Z_1}{n_e T_H (n_e + n_H)} < 0
\]

(9)

For the case of a hydrogen plasma with \( T_e/T_H \approx 1 \) this reduces to

\[
\frac{n_H^1}{n_1^1 Z_1} + \frac{1}{2} < 0
\]

(10)

For this instability to occur, the density gradient of impurity ions must be in the opposite sense to that of the plasma ions and not too large. This condition would be likely to be met near the walls where the plasma density decreases and the impurity density increases towards the wall. We see from Eq. (6) that for the worst case

\[
\text{Im} (\omega) \approx \text{Re} (\omega) \approx \frac{Z n_H^1}{n_H^1} \omega^*_H
\]
We notice that if the density gradient of the impurity species is of the order of that for the hydrogen species $n_e^+ + n_i^0 = 0$ the same instability as resulting from Eq. (5), is of fluid type in the sense that it does not involve wave-particles resonance and occurs in the limit $v_{th} > \omega/\kappa > v_{th} u_i > \sqrt{\kappa_i}$.

This instability is very reminiscent of the temperature gradient instability as it occurs for long wavelengths and involves ion rather than electron Landau damping. As in that case, it can be shown that shear is rather ineffective for stabilization. We note that what is relevant for this instability is not the temperature gradient but the gradient of mean parallel velocity, which may be due either to a temperature gradient or to a gradient of mean mass number as in our case. The quasi-linear effect of this instability is of course to cause an outward diffusion of plasma ions and an inward diffusion of impurity ions.

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