PHOTO-PRODUCTION OF VECTOR MESONS FROM NUCLEONS

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The photoproduction of rho and omega mesons from nucleons is treated phenomenologically, emphasising the suitability of choosing a particular set of twelve invariant amplitudes of this process. These are obtained in analogy with Compton scattering from nucleons so that the expressions for the physical observables are closely related in the two cases. Contributions to the amplitudes from nucleon exchange in the direct channel and one-pion exchange in the crossed channel are given and the latter diagram is explicitly compared with the few experimental data available at present.
1. INTRODUCTION

With the coming into operation of very high-energy electron synchrotrons in various parts of the world, it has now become experimentally possible to produce rho and omega mesons from nucleons by means of gamma rays. Recently preliminary results on the photo-production of neutral rho-mesons have been published\(^1\).

On the theoretical side, DRELL and BERNAN\(^2\) first presented a diffraction-type calculation in which the essential idea was to employ a multi-peripheral model for the production process. The first few experimental data available seem generally to agree with this model. JOOS and KRAMER\(^3\) have however made a one-pion exchange calculation taking into account the presence of the pseudo-scalar pi and eta and scalar sigma mesons but, as we will show in the following, such a model fails to fit the observed differential cross-section.

MORAVCSIK and the present author\(^4\) have given a detailed account of the general formalism of the spin-1 boson photo-production and very recently MORAVCSIK and his collaborators\(^5\) have further extended this work to include the production of any spin \(s\) mesons.

A detailed treatment of the mechanism of the production process has not however been attempted in any of these works, although ACHUTHAN and SRINIVASAN\(^6\) have very recently reported some preliminary investigation in this direction. In the present work we describe the kinematics of the process in some detail and emphasize the simplicity of the choice of invariants of our previous work\(^4\). The great advantage of working with these invariants is that one can study the Compton scattering from nucleons simultaneously and express differential cross-section, polarization, etc. in manifestly analogous forms in the two cases. In the present stage of the experimental situation, there is of course very little point in making a dispersion theoretic analysis of the amplitudes although it is relatively simple.
to do so in our formalism in exact analogy with the Compton scattering case. We present instead the pole term contributions from the different channels and take the point of view, as we have emphasized elsewhere\(^7\), that it is quite reasonable to approximate the complete amplitude by the pole terms alone.

As pointed out by JOOS and KRÄMER\(^3\), it is customary in such treatments to regard the rho and omega mesons as "particles", although they should strictly be regarded as unstable resonances. This is of course in conformity with the current practice in higher symmetry schemes like \(\text{SU}(6)\) wherein the pseudoscalar mesons and the vector mesons are put in the same 35-dimensional representation of the group. GELL-MANN and ZACHARIASEN\(^8\), on the other hand, have studied a field theoretic model in which some justification can be advanced for treating the unstable resonances as particles in the conventional sense.

In the next section we describe in considerable detail our choice of the twelve invariant amplitudes. Since we have used a slightly different notation for our longitudinal amplitudes, our formulae here are correspondingly different from those of Ref. (4). In this section we work out in detail the expressions for the helicity amplitudes which are necessary for a study of the absorption corrections to the peripheral model as suggested by GOTTFRIED and JACKSON\(^9\). In the third section we give the one-nucleon exchange and one-pion exchange contributions to the invariant amplitudes and confront the latter with the meagre experimental data available. We conclude from this that such a one-pion exchange model cannot account for the steep fall-off of the differential cross-section experimentally observed. This is in accordance with what has been observed in many other reactions but unfortunately absorption corrections à la GOTTFRIED and JACKSON\(^9\) do not appear to improve the situation much. On the other hand, a detailed treatment on the lines of dispersion theory does not seem to be warranted at the present stage of the experimental data.
2. KINEMATICS

We simultaneously consider the three processes

(I) \( \gamma + N \rightarrow N + V \)

(II) \( \gamma + V \rightarrow N + \bar{N} \)

(III) \( \gamma + \bar{N} \rightarrow \bar{N} + V \)

where \( V \) stands for the vector meson \( \rho, \omega \) and \( \varphi \). We denote by \( p_1 \) and \( p_f \) the initial and final four-momenta of the nucleons and by \( q \) and \( k \) those of the vector meson and the photon respectively. We introduce the Mandelstam variables

\[
s = -(p_1 + k)^2, \quad t = -(q - k)^2, \quad u = -(p_2 - k)^2
\]

(2.1)

which are the squares of the total energy in the c.m. system for the three processes considered above. As usual the conservation of energy and momentum \( p_1 + k = p_2 + q \) together with the mass-shell restrictions \( p_1^2 = -m^2 \), \( q^2 = -m^2 \) and \( k^2 = 0 \) lead to the restraint

\[
s + t + u = 2m^2 + m^2_v
\]

(2.2)

In the c.m. system of Channel 1, we have

\[
s = W^2 = (E_1 + k)^2 = (E_2 + \omega)^2
\]

\[
t = m^2_v - 2k\omega + 2qk\cos \theta
\]

(2.3)

where \( W \) is the total c.m. energy, \( q \) and \( k \) are the magnitudes of the vector meson and photon momenta, \( E_1 = (k^2 + m^2)^{1/2} \) and \( E_2 = (q^2 + m^2)^{1/2} \) are the initial and final nucleon energies,
\( \omega = (q^2 + m_v^2)^{1/2} \) is the vector-meson energy and \( \cos \Theta \) is the cosine of the production angle. In terms of \( W \), we have the incident three-momentum

\[
k = \left( W^2 - m^2 \right) / 2W \quad (2.4)
\]

and the final three-momentum \( Q \) given by

\[
4W^2 q^2 = W^4 - 2W^2(m^2 + m_v^2) + (m^2 - m_v^2)^2 \quad (2.5)
\]

The S-matrix for the process is written in the form

\[
S_{fi} = i R_{fi} \quad (2.6)
\]

where \( R \) is related to the Feynman amplitude \( F \) by

\[
R_{fi} = (2 \pi)^4 \delta(4)(p_1 + k - p_2 - q_2) \left( \frac{m^2}{4 \omega k E_1 E_2} \right)^{1/2} F_{fi}
\]

The isotopic decomposition of the \( F \)-matrix is simple and has been given elsewhere (4). We now explain our particular choice of the invariant amplitudes in some detail since we consider our set somewhat more advantageous than that proposed in Ref. (6).

We know from our general considerations in Ref. (4) that twelve invariant amplitudes are necessary to describe the production process ignoring the charge degrees of freedom. The form of these amplitudes is determined from Lorentz and gauge invariance and a general method of constructing these invariants has been given by HEARN (10). From Lorentz invariance we can write the \( F \)-matrix in the form

\[
F = \bar{u}(p_2) \sum_i A_i \mathcal{M}_i(p \xi_1) u(p_1)
\]
where \( A_i \) are the invariant amplitudes and \( M_i \) are functions of momentum parameters \( p \) and spin parameters \( \xi \).

Since the amplitude must in general be a multilinear form in spinors and polarization vectors, we can write

\[
P = \bar{u}(p_2) \gamma_\mu \sum_{i,j} A_{ij} \left[ C_i(p) D_j(\xi) \right]_{\mu \nu} \xi_\nu u(p_1)
\]

where \( \eta_\mu \) and \( \xi_\nu \) are the polarization four-vectors of the vector-meson and the photon respectively, \( D_j(\xi) \) is a co-variant tensor function of \( \xi \) and \( C_i(\xi) \) is a set of momentum functions which are contravariant tensors.

In the present case, the photo-production process involves two fermions and \( D_j(\xi) \) can be taken to be the usual Dirac matrices \( 1, i\gamma_\mu, i\gamma_\nu, i\gamma_\mu_\gamma_\nu, i\gamma_\mu \gamma_\nu \). On the other hand, the choice of possible \( C_i(\xi) \) is not at all unique. Ideally one would like to construct a basis which consists of normalized and orthogonal vectors. For example, in the closely related case of Compton scattering from nucleons, the basis vectors

\[
P'_\mu, K_\mu, Q_\mu, N_\mu
\]

where

\[
P' = p - \frac{p \cdot k}{k^2} k, \quad P = \frac{i}{2} (p_1 + p_2)
\]

\[
K = \frac{i}{2} (k + q), \quad Q = \frac{i}{2} (k - q)
\]

\[
N_\mu = i \epsilon_{\mu \nu \rho \sigma} P'_\nu K_\rho Q_\sigma
\]

form an orthonormal set but, on account of the finite vector-meson mass, this is no longer the case for photo-production of vector mesons.
Despite this difficulty we decide to use the above set of basis vectors principally because we wish to study Compton scattering from nucleons in parallel with the photo-production of vector mesons. The vector-meson photo-production differs from Compton scattering on account of the vector meson mass and its longitudinal polarization so that when we put the vector-meson mass and its longitudinal polarization equal to zero we recover the well-known formulae of the Compton scattering. This has the great advantage that in this way we are able to check all our theoretical expressions at every stage with the corresponding formulae in Compton scattering. Considering the complexity of the problem, we emphasize that this is a very important advantage which should be utilized to the full. In the analysis of preliminary data, we also feel that such an explicit separation of the transverse and longitudinal polarizations will make our approximations more transparent and therefore enable us to study the production mechanism in greater detail.

With this idea in mind, we write the $F$-matrix in the form

$$F = u(p) \left[ \frac{\epsilon \cdot p' \gamma' \rho}{p'^2} (B_1 + i \gamma \beta_2) + \frac{\epsilon \cdot N \gamma' N}{N^2} (B_3 + i \gamma \beta_4) \right. + \left. \frac{\gamma' \rho \epsilon \cdot N - \gamma' N \epsilon \cdot p'}{\sqrt{p'^2 N^2}} \gamma_5 (B_5 + i \gamma \beta_6) + \frac{\gamma' \rho \epsilon \cdot N + \gamma' N \epsilon \cdot p'}{\sqrt{p'^2 N^2}} \gamma_5 (B_7 + i \gamma \beta_8) \right. + \left. \frac{\gamma'^{\mu} \gamma^{\nu}}{\sqrt{p'^2 \rho^2}} (B_9 + i \gamma \beta_{10}) + \frac{\gamma'^{\mu} \gamma^{\nu}}{\sqrt{N^2}} \gamma_5 (B_{11} + i \gamma \beta_{12}) \right] u(p)$$

(2.8)

where $\gamma'^{\mu}$ is the transverse part of the vector-meson polarization four-vector and $\gamma'^{\mu}$ is the fourth component of its longitudinal
part, whose magnitude is \( q/m_v \) as we will presently show. For the photon polarization vector we have

\[
\sum_{s=1}^{2} \epsilon_1^{(s)}(k) \epsilon_2^{(s)}(k) = \delta_{ij} - k_i k_j/k^2
\]

whereas the vector-meson polarization vectors \( \eta_\mu(q) \) \( (s = 1, 2, 3) \) satisfy

\[
\eta^2 = 1, \quad \eta \cdot q = 0
\]

From these relations it follows that

\[
\sum_{s=1}^{3} \eta_\mu^{(s)}(q) \eta_\nu^{(s)}(q) = \delta_{\mu\nu} - q_\mu q_\nu/m_v^2
\]

Taking \( \eta_\mu^{(1)} \) and \( \eta_\mu^{(2)} \) to be the transverse polarization vectors

\[
\eta_\mu = \eta_\mu^{(1,2)} = (\eta^{(1,2)}, 0)
\]

we find that the longitudinal polarization vector \( \eta_\mu^{(3)} \) has the components

\[
\eta_\mu^{(3)}(q) = \frac{1}{m_v} (\omega \hat{q}, q)
\]

where \( \hat{q} \) is the unit vector in the direction of \( q \).

Introducing now the vector \( \hat{e} = \hat{n} \sin \theta \) where \( \hat{n} \) is the normal to the production plane defined by \( k \times q \) we can rewrite the F-matrix in the convenient form

\[
F = \frac{1}{\sin \theta} \bar{u}(k) \left[ \gamma_\mu \epsilon \bar{\psi}\gamma^5 (A_1 + i \gamma^5 A_2) + \epsilon \cdot \hat{e} \gamma^5 \bar{\psi}\gamma_\mu (A_3 + i \gamma^5 A_4) \\
+ \gamma_\mu \epsilon \bar{\psi}\gamma^5 (A_5 + i \gamma^5 A_6) + \gamma_\mu \bar{\psi}\gamma^5 (A_7 + i \gamma^5 A_8) + \right]
\]

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\[ A_{1,2} = (1 + \frac{P \cdot K}{k^2})^2 \left( \frac{-K^2q^2}{w} \right)^{\frac{1}{2}} \frac{B_{1,2}}{qk} \]

\[ A_{3,4} = B_{3,4} \]

\[ A_{5,7} = (1 + \frac{P \cdot K}{k^2}) \frac{\sqrt{-K^2q^2(1 + \frac{k^2}{q^2})}}{w} \frac{B_7 + B_5}{(k, q)} \]

\[ A_{6,8} = (1 + \frac{P \cdot K}{k^2}) \frac{\sqrt{-K^2q^2(1 + \frac{k^2}{q^2})}}{w} \frac{B_8 + B_6}{(k, q)} \]

\[ A_{9,10} = (1 + \frac{P \cdot K}{k^2}) \frac{\sqrt{-K^2q^2(1 + \frac{k^2}{q^2})}}{w} \frac{B_{9,10}}{q} \]

\[ A_{11,12} = B_{11,12} \]

and

\[ \xi = \frac{-(K \cdot q)^2}{K^2q^2} - \frac{(P \cdot q)^2}{P^2q^2} \]

It is useful to note that in the Compton scattering case, \( \xi = 0 \), \( K^2 = -Q^2 \) and using the fact that from time reversal invariance \( A_t = B_t = 0 \), we easily see that Eq. (2.9) reduces to the standard form given in the literature.

The differential cross-section for the photo-production process is given by

\[ \frac{d\sigma}{d\Omega} = \left( \frac{m}{4\pi w} \right)^2 \sum |\langle NV | F | \gamma N \rangle|^2 \]

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where the summation represents an average over initial and a sum over final spins and polarizations. Carrying out the summations, we get

\[
\frac{d\sigma}{d\Omega} = \frac{1}{4} \left( \frac{m}{4\pi W} \right)^2 \left( -\frac{4m^2}{2m^2} \right) \left\{ |A_1|^2 + |A_3|^2 + \left( \frac{m}{\omega} \right)^2 |A_9|^2 \right\} \\
+ \left( -\frac{t}{2m^2} \right) \left\{ |A_5|^2 + |A_7|^2 + \left( \frac{m}{\omega} \right)^2 |A_{11}|^2 \right\} \\
+ \left( -\frac{kW(u-m^2)}{m^2} \right) \left\{ |A_6|^2 + |A_4|^2 + \left( \frac{m}{\omega} \right)^2 |A_{10}|^2 \right\} \\
+ \frac{(u-5)+t(4m^2-t)}{8m^2} \left\{ |A_6|^2 + |A_4|^2 + \left( \frac{m}{\omega} \right)^2 |A_{10}|^2 \right\} \\
+ \frac{(u-m^2)/2 - kW}{m} \left\{ 2 \text{Re} \left( A_1^* A_2 + A_3^* A_4 + \left( \frac{m}{\omega} \right)^2 A_9^* A_{10} \right) \right\} \\
+ \left( \frac{m}{\omega} \right)^2 \left\{ 2 \text{Re} \left( A_5^* A_6 + A_7^* A_8 + \left( \frac{m}{\omega} \right)^2 A_{11}^* A_{12} \right) \right\}
\]

This expression reduces to the corresponding formula for the Compton scattering as given by LEADER and HEARN\(^{(12)}\). We may mention here that the amplitudes \( Q_i \) defined in Ref. (4) are slightly more advantageous in one respect; in this form the expression for the differential cross-section does not contain any crossed terms and may be more useful for experimental analysis.

Let us now consider the spin and helicity amplitudes. If we
consider the production amplitude as a matrix taken between Pauli spinors rather than Dirac spinors we get

\[
F = \frac{1}{\sin \theta} \chi^\dagger \left[ \gamma' \hat{\sigma} \gamma \hat{\sigma} \varepsilon \cdot \hat{\gamma} \left( F_1 + \sigma \hat{\gamma} \sigma \cdot \hat{\sigma} F_2 \right) + \gamma' \hat{\sigma} \gamma \hat{\sigma} \varepsilon \cdot \hat{\gamma} \left( F_3 + \sigma \hat{\gamma} \sigma \cdot \hat{\sigma} F_4 \right) \\
+ (\gamma' \hat{\sigma} \gamma \hat{\sigma} \varepsilon \cdot \hat{\gamma} (\varepsilon \sigma \cdot \hat{\sigma} F_5 + i \sigma \cdot \hat{\sigma} F_6)) \\
+ (\gamma' \hat{\sigma} \gamma \hat{\sigma} \varepsilon \cdot \hat{\gamma} (\varepsilon \sigma \cdot \hat{\sigma} F_7 + i \sigma \cdot \hat{\sigma} F_8)) \\
+ \varepsilon \cdot \hat{\gamma} \gamma' \hat{\sigma} \gamma \hat{\sigma} \varepsilon \cdot \hat{\gamma} \left( F_9 + \sigma \hat{\gamma} \sigma \cdot \hat{\sigma} F_{10} \right) \\
+ \varepsilon \cdot \hat{\gamma} \gamma' \hat{\sigma} \gamma \hat{\sigma} \varepsilon \cdot \hat{\gamma} \left( i \sigma \cdot \hat{\sigma} F_{11} + i \sigma \cdot \hat{\sigma} F_{12} \right) \right] \chi
\]

where

\[
F_1 = \frac{\sqrt{(E+\text{m})(E-\text{m})}}{2m} \left[ \begin{array}{c} A_1 - (W-\text{m})A_2 \\ \end{array} \right] \\
F_2 = -\frac{q k}{2 m \sqrt{(E+\text{m})(E-\text{m})}} \left[ A_1 + (W+\text{m})A_2 \right] \\
F_3 = \frac{\sqrt{(E+\text{m})(E-\text{m})}}{2m} \left[ \begin{array}{c} A_3 - (W-\text{m})A_4 \\ \end{array} \right] \\
F_4 = -\frac{q k}{2 m \sqrt{(E+\text{m})(E-\text{m})}} \left[ A_3 + (W+\text{m})A_4 \right] \\
F_5 = \frac{\sqrt{(E+\text{m})(E-\text{m})}}{2m} \frac{9}{E_{+\text{m}}} \left[ -A_5 + W A_6 \right] \\
F_6 = \frac{\sqrt{(E+\text{m})(E-\text{m})}}{2m} \frac{k}{E_{+\text{m}}} \left[ A_5 + W A_6 \right] \\
F_7 = \frac{\sqrt{(E+\text{m})(E-\text{m})}}{2m} \frac{9}{E_{+\text{m}}} \left[ -A_7 + W A_8 \right] \\
F_8 = \frac{\sqrt{(E+\text{m})(E-\text{m})}}{2m} \frac{k}{E_{+\text{m}}} \left[ A_7 + W A_8 \right] \\
F_9 = \frac{\sqrt{(E+\text{m})(E-\text{m})}}{2m} \frac{m}{\omega} \left[ A_9 - (W-\text{m})A_{10} \right] \\
F_{10} = -\frac{q k}{2 m \sqrt{(E+\text{m})(E-\text{m})}} \left( A_9 + (W+\text{m})A_{10} \right) \\
F_{11} = \frac{\sqrt{(E+\text{m})(E-\text{m})}}{2m} \frac{9}{E_{+\text{m}}} \left( \frac{m}{\omega} \right) \left[ -A_{11} + W A_{12} \right] \\
F_{12} = \frac{\sqrt{(E+\text{m})(E-\text{m})}}{2m} \frac{k}{E_{+\text{m}}} \left( \frac{m}{\omega} \right) \left[ A_{11} + W A_{12} \right]
\]
The pair-wise relationship of the amplitudes is reminiscent of similar relations in pion-nucleon scattering as has been pointed out by CONTOGOURIS\textsuperscript{(13)}. The usefulness of the amplitude in this form stems from the fact that an angular momentum decomposition of the amplitudes \( F_i \) can be easily performed as in Ref. (10). We will not do this here but will introduce the helicity amplitudes instead.

In the helicity formalism the production process is described by a matrix \( \Phi \) in spin space defined in such a way that the differential cross-section is given by

\[
\frac{d\sigma}{d\Omega} = \left| \langle \lambda'_N, \lambda'_\nu \mid \Phi \mid \lambda_N, \lambda_\nu \rangle \right|^2
\]

where \( \lambda'_N, \lambda'_\nu \) represent the spin states of the outgoing nucleon and vector meson and \( \lambda_N, \lambda_\nu \) those of the incoming nucleon and photon respectively. The expansion of the production matrix \( \Phi \) in terms of amplitudes for transitions between states of given helicities is written in the form

\[
\langle \lambda'_N, \lambda'_\nu \mid \Phi \mid \lambda_N, \lambda_\nu \rangle = \frac{1}{2} \sum_J (2J+1) \langle \lambda'_N, \lambda'_\nu \mid T^{(J)} \mid \lambda_N, \lambda_\nu \rangle \, d^J_{\lambda'_\nu, \lambda_N} (\theta)
\]

where \( J \) is the total angular momentum and \( d^J_{\lambda'_\nu, \lambda_N} (\theta) \) is the reduced rotation matrix with \( \lambda' = \lambda_N - \lambda_\nu \) and \( \lambda = \lambda_N - \lambda_\nu \). The connection between the twelve helicity amplitudes \( \Phi_i \) and the amplitudes \( A_i \) can be established by a straightforward calculation. This gives
\[ 8\pi W\Phi_1 = 8\pi W \langle \frac{1}{2} 1\Phi | \frac{1}{2} 1 \rangle \]
\[ = \frac{\cos \frac{\Phi}{2}}{2\sqrt{(E_{1m})(E_{2m})}} \left[ \left\{ (E_{2m})(E_{1m}) - qk^2 \right\}(A_1 + A_3) + k(W_{1m})(E_{2m+q})(A_2 - A_4 + A_5 - A_7) \\
+ \left\{ k(E_{2m} - q(E_{1m}))^2 \right\}\left\{ (A_5 + A_7) - m(A_6 + A_8) \right\} \right] \]

\[ 8\pi W\Phi_2 = 8\pi W \langle -\frac{1}{2} -1\Phi | \frac{1}{2} 1 \rangle \]
\[ = \frac{\sin \frac{\Phi}{2}}{2\sqrt{(E_{1m})(E_{2m})}} \left[ \left\{ (E_{2m})(E_{1m}) + qk^2 \right\}(A_1 + A_3) - k(W_{1m})(E_{2m+q})(A_2 - A_4 + A_5 - A_7) \\
+ \left\{ k(E_{2m} + q(E_{1m}))^2 \right\}\left\{ (A_5 - A_7) - m(A_6 - A_8) \right\} \right] \]

\[ 8\pi W\Phi_3 = 8\pi W \langle \frac{1}{2} -1\Phi | \frac{1}{2} 1 \rangle \]
\[ = \frac{\cos \frac{\Phi}{2}}{2\sqrt{(E_{1m})(E_{2m})}} \left[ \left\{ (E_{2m})(E_{1m}) - qk^2 \right\}(A_1 + A_3) - k(W_{1m})(E_{2m+q})(A_2 - A_4 + A_5 - A_7) \\
+ \left\{ k(E_{2m} - q(E_{1m}))^2 \right\}\left\{ (A_5 - A_7) - m(A_6 - A_8) \right\} \right] \]

\[ 8\pi W\Phi_4 = 8\pi W \langle -\frac{1}{2} -1\Phi | \frac{1}{2} 1 \rangle \]
\[ = \frac{\sin \frac{\Phi}{2}}{2\sqrt{(E_{1m})(E_{2m})}} \left[ \left\{ (E_{2m})(E_{1m}) + qk^2 \right\}(-A_1 + A_3) + k(W_{1m})(E_{2m+q})(A_2 + A_4 - A_5 - A_7) \\
+ \left\{ q(E_{2m} + k(E_{1m}))^2 \right\}\left\{ (A_5 + A_7) - m(A_6 + A_8) \right\} \right] \]
\[ 8\pi W \Phi_5 \equiv 8\pi W \langle -\frac{1}{2} - 1 \mid \Phi \mid -\frac{1}{2} - 1 \rangle \]
\[ = -\frac{\cos \frac{\theta}{2}}{2\sqrt{(E_{1m})(E_{2m})}} \left[ \left\{ (E_{1m}')(E_{2m}) - 9k \right\} (-A_1 + A_3) - k(W_{1m})(E_{2m} - W_{2m})(A_2 + A_4 + A_6 - A_8) \right. \]
\[ \left. - \left\{ k(E_{2m}) - 9(E_{1m}) \right\} \left\{ (A_5 + A_7) - m(A_6 + A_8) \right\} \right] \]

\[ 8\pi W \Phi_6 \equiv 8\pi W \langle \frac{1}{2} - 1 \mid \Phi \mid -\frac{1}{2} - 1 \rangle \]
\[ = \frac{\sin \frac{\theta}{2}}{2\sqrt{(E_{1m})(E_{2m})}} \left[ \left\{ (E_{2m}')(E_{1m}) + 9k \right\} (A_1 + A_3) - k(W_{1m})(E_{2m} - W_{2m})(A_2 + A_4 + A_6 - A_8) \right. \]
\[ \left. - \left\{ k(E_{2m}) + 9(E_{1m}) \right\} \left\{ (A_5 + A_7) - m(A_6 + A_8) \right\} \right] \]

\[ 8\pi W \Phi_7 \equiv 8\pi W \langle \frac{1}{2} + 1 \mid \Phi \mid \frac{1}{2} - 1 \rangle \]
\[ = \frac{\cos \frac{\theta}{2}}{2\sqrt{(E_{1m})(E_{2m})}} \left[ \left\{ (E_{2m}')(E_{1m}) - 9k \right\} (A_1 + A_3) - k(W_{1m})(E_{2m} - W_{2m})(A_2 + A_4 + A_6 - A_8) \right. \]
\[ \left. + \left\{ k(E_{2m}) - 9(E_{1m}) \right\} \left\{ (-A_5 + A_7) - m(-A_6 + A_8) \right\} \right] \]

\[ 8\pi W \Phi_8 \equiv 8\pi W \langle -\frac{1}{2} - 1 \mid \Phi \mid \frac{1}{2} - 1 \rangle \]
\[ = -\frac{\sin \frac{\theta}{2}}{2\sqrt{(E_{1m})(E_{2m})}} \left[ \left\{ (E_{2m}')(E_{1m}) + 9k \right\} (-A_1 + A_3) - k(W_{1m})(E_{2m} - W_{2m})(A_2 + A_4 + A_6 - A_8) \right. \]
\[ \left. - \left\{ k(E_{2m}) + 9(E_{1m}) \right\} \left\{ (A_5 + A_7) - m(A_6 + A_8) \right\} \right] \]

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\[ 8\pi W \Phi_9 = 8\pi W \left< \frac{1}{2} \mid \Phi \mid \frac{1}{2} \right> \]
\[ = \frac{\cos \theta}{2\sqrt{(E_{1+m})(E_{2+m})}} \left[ -\left\{ (E_{1+m})(E_{2+m}) - qk \right\} A_9 + k(W+m)(E_{1+m}-q)(A_{10} A_{12}) \right. \]
\[ + \left\{ k(E_{1+m}) - q(E_{2+m}) \right\} \left( A_{11} - m A_{12} \right) \]

\[ 8\pi W \Phi_{10} = 8\pi W \left< \frac{1}{2} \mid \Phi \mid -\frac{1}{2} \right> \]
\[ = \frac{-\sin \theta}{2\sqrt{(E_{1+m})(E_{2+m})}} \left[ \left\{ (E_{1+m})(E_{2+m}) + qk \right\} A_9 - k(W+m)(E_{1+m}-q)(A_{10} A_{12}) \right. \]
\[ + \left\{ k(E_{2+m}) + q(E_{1+m}) \right\} \left( A_{11} - m A_{12} \right) \]

\[ 8\pi W \Phi_{11} = 8\pi W \left< \frac{1}{2} \mid \Phi \mid -\frac{1}{2} \right> \]
\[ = \frac{\sin \theta}{2\sqrt{(E_{1+m})(E_{2+m})}} \left[ \left\{ (E_{1+m})(E_{2+m}) + qk \right\} A_9 - k(W+m)(E_{1+m}-q)(A_{10} A_{12}) \right. \]
\[ + \left\{ k(E_{2+m}) + q(E_{1+m}) \right\} \left( A_{11} - m A_{12} \right) \]

\[ 8\pi W \Phi_{12} = 8\pi W \left< \frac{1}{2} \mid \Phi \mid \frac{1}{2} \right> \]
\[ = \frac{\cos \theta}{2\sqrt{(E_{1+m})(E_{2+m})}} \left[ -\left\{ (E_{1+m})(E_{2+m}) - qk \right\} A_9 + k(N+m)(E_{1+m}-q)(A_{10} A_{12}) \right. \]
\[ + \left\{ k(E_{1+m}) - q(E_{2+m}) \right\} \left( A_{11} - m A_{12} \right) \]
The importance of the helicity amplitudes stems from the fact that we can make absorption corrections due to final state interactions using the amplitudes in this form as has been shown by Gottfried and Jackson (9). For this purpose we need the one-pion pole contributions to the invariant amplitudes which we calculate in the next section.

3. THE UNITARITY CONDITION AND THE POLE APPROXIMATION

The usual unitarity condition for the S-matrix $S^* S = S S^* = 1$ when expressed in terms of the $R$-operator leads to the equation

$$i \langle \alpha | R^\dagger - R | \beta \rangle = \sum_{\nu} \langle \alpha | R^\dagger | \nu \rangle \langle \nu | R | \beta \rangle$$

where the sum is over all permissible physical states $\nu$ having the same energy momentum as $\alpha$ or $\beta$. Since the invariants are self-adjoint, we can write

$$\sum_{i=1}^{12} \text{Im} \ B_i (s, t, u) = \rho (w) \sum_{\nu} \langle \alpha | R^\dagger | \nu \rangle \langle \nu | R | \beta \rangle$$

where $\rho (w)$ is the phase-space factor.

At the present stage of the experimental situation, the r.h.s. need only be calculated using some pole approximations. In what follows we consider the most important pole terms coming from the $S$ and $t$ channels.

3.1 Channel I

For this channel we take only the contributions of the one-nucleon intermediate state. The $N\bar{N}V$-vertex function is

$$\bar{u} (p) \left[ g_{VNN} \gamma^\mu + \frac{F_{VNN}}{2m} \sigma_{\mu\nu} q^\nu \right] u (P)$$

where $q = P - \pi_2$ is the momentum transfer and $g_{VNN}$, $F_{VNN}$ are
the Dirac and Pauli coupling constants respectively. The $\gamma_{NN}$ vertex is similarly written in the form

$$\bar{u}(p') \left[ \left\{ F_1(k^2) + F_2(k^2) \right\} i\gamma^\mu - \frac{F_2(k^2)}{m} (p^\mu + k^\mu) \right] u(p)$$

where $k = p_1 - p$ and $F_1(k^2), F_2(k^2)$ are the usual nucleon electromagnetic form factors normalized such that

$$F_1 = F_1(0) = e(1 + \tau_3)/2$$
$$F_2 = F_2(0) = \frac{1}{2}(\mu_p + \mu_n) + \frac{1}{2}(\mu_p - \mu_n) \tau_3$$

Using these vertex functions we get after a tedious calculation the following contributions* to the amplitudes $B_i$:

$$B_1 = C \left[ 2m \left( (1+\lambda')(1+\lambda) \right) \frac{(1+\lambda')(1+\lambda)}{m} \left\{ -m^2 - \frac{m^2}{2} \frac{t-m^2}{t-2m^2} \right. \\
+ \frac{1}{2} \left( t-2m^2 - \frac{t(t-m^2)}{t-2m^2} \right) \left. \right\} + \frac{\lambda'(1+\lambda)}{m} \left( 1-\lambda \right) \left\{ -2m^2 - \frac{m^2}{2} \frac{t-m^2}{t-2m^2} \right. \\
- \frac{m^2}{2} \right\} + \frac{2\lambda(1+\lambda')}{m} \left( m^2 + \frac{m^2}{4} \frac{1}{t-2m^2} \right) \left( 1+\lambda \right) \left( 1-\lambda \right) \right]$$

$$B_2 = C \left[ - (1+\lambda)(1+\lambda') \frac{t}{t-2m^2} + \lambda'(1+\lambda)(1+\lambda) \frac{t-m^2}{t-2m^2} + \right.$$

* There are certain errors in Ref. (4) which are here corrected.
\[ B_3 = 0 \]

\[ B_4 = C (1 + \lambda) (1 + \lambda') \]

\[ B_5 = - \frac{C}{2} \left[ 2m(1+\lambda)(1+\lambda') + \frac{1}{m^2 + \frac{m_{\nu}^2}{4} \frac{1}{t-2m_{\nu}^2}} \right] \left\{ \lambda (1+\lambda)(1+\mu) + \lambda' (1+\lambda)(1+\mu') \right\} \]

\[ B_6 = - \frac{C}{2} \sqrt{\frac{t-2m_{\nu}^2}{t(1+\xi)}} \left[ \frac{1}{2} (1+\lambda)(1+\lambda') \frac{m_{\nu}/4}{m^2 (t-2m_{\nu}^2) + (m_{\nu}/4)} \right. \]

\[ \left. - \frac{1}{m} \left\{ - \lambda (1+\lambda')(1+\mu') + \lambda' (1+\lambda)(1+\mu') \right\} \right] \]

\[ B_7 = - \frac{C}{2} \frac{1}{m} \frac{1}{m^2 + \frac{m_{\nu}^2}{4} \frac{1}{t-2m_{\nu}^2}} \left[ \lambda' (1+\lambda)(1+\mu) - \lambda (1+\lambda')(1+\mu) \right] \]

\[ B_8 = \frac{C}{2} \sqrt{\frac{t-2m_{\nu}^2}{t(1+\xi)}} \left[ - (1+\lambda)(1+\lambda') \frac{2m^2}{m^2 + \frac{m_{\nu}^2}{4} \frac{1}{t-2m_{\nu}^2}} + \frac{(m_{\nu}/4)}{m^2 (t-2m_{\nu}^2) + (m_{\nu}/4)} \left\{ \right. \right. \]

\[ \left. \left. (1+\lambda)(1+\lambda') \left( - \frac{t-2m_{\nu}^2}{4} \right) + (1+\lambda')(1+\lambda) \left( - \frac{t-m_{\nu}^2}{2} \right) \right\} \right] + \lambda (1+\lambda')(1+\mu') + \lambda' (1+\lambda)(1+\mu) \]

\[ - 17 - \]
\[ \frac{m_v}{q} B_{3,10} = \frac{B_{t,2}}{\sqrt{m^2 + \frac{(m_v^2/4)}{t - 2m^2}}} \]

\[ \frac{m_v}{q} B_{11} = -C \sqrt{\frac{t - 2m^2}{t(1 + \xi)}} \sqrt{\frac{(m^2/4)}{t - 2m^2}} \left[ \left( 1 + \lambda \right) \left( 1 + \lambda' \right) \left( m^2 + \frac{t^2}{t - 2m^2} \right) \right] \]

\[ \lambda = \frac{F_{VNN}}{g_{VNN}} \quad \lambda' = \frac{F_{VNN}}{e} \]

\[ \lambda = \frac{G_{VNN}}{s - m^2} \]

\[ \mu = -\frac{m_v^2}{4} \quad \frac{t - m^2}{m^2(t - 2m^2) - (m_v^2/4)} \]

\[ \xi = \frac{m_v^2}{2t} \left[ \frac{1}{t - 2m_v^2} - \frac{(\mu - \xi)^2}{m^2(t - 2m_v^2) + (m_v^2/4)} \right] \]

3.2 **Channel II**

In this channel we are dealing with the process of nucleon-antinucleon annihilation into a photon and a vector meson so that in the unitarity equation we can approximate the r.h.s. by one-
and two-pion states.

Let us first consider the one-pion contribution. This depends on the pion-nucleon vertex \( \langle \pi \mid R \mid NN \rangle \) and the matrix element for the radiative vector meson decay \( \langle \gamma V \mid R \mid \pi \rangle \).

The matrix element \( \langle \pi \mid R \mid NN \rangle \) is related to the pion-nucleon coupling constant \( g_r \) by the expression

\[
\langle \pi(q') \mid R \mid NN \rangle = \frac{g_r}{(2\pi)^3} \sqrt{\frac{m^3}{p_1^0 p_2^0 2q'}} \bar{V}(p^2) \gamma_5 \gamma_\mu \gamma_\tau (R)
\]

where \( g_r \) is the rationalized renormalized pion-nucleon coupling constant \( g_r^2 / 4\pi \approx 15 \). The other vertex function is written in the form

\[
\langle \gamma V \mid R \mid \pi(q') \rangle = \frac{F_{\gamma \pi V}(t)}{(2\pi)^3 \sqrt{8q'} k_0 q_0} \gamma_\mu \varepsilon_{\nu \rho \sigma} (k+q)_\rho (k-q)_\sigma
\]

where \( F_{\gamma \pi V}(t) \) is related to the decay width of the vector meson

\[
F_{\gamma \pi V}(-m^2) = -8\sqrt{\frac{\pi}{m_\pi \tau}}
\]

\( \tau \) being the vector meson decay life-time.

Using these vertex functions we get

\[
A_5 = -\frac{g_r F_{\gamma \pi V}(t)}{t-1} \kappa(q \omega, \omega - \omega)
\]

\[
A_7 = -\frac{g_r F_{\gamma \pi V}(t)}{t-1} \kappa(q - \omega \omega, \omega)
\]
\[ \frac{m}{q} A_{ii} = - \frac{g_i F_{PV}(t)}{t-1} \frac{m}{q} \]

and all other \(A_i\) are zero. The differential cross-section of this one-pion exchange model is given by

\[
\frac{d\sigma}{d\Omega} = \frac{1}{4} \left( \frac{m}{4\pi N} \right)^2 \left[ (|A_5|^2 + |A_7|^2 + \left( \frac{m}{q} \right)^2 |A_{ii}|^2 \right] \left( -\frac{t}{2m^2} \right)
\]

\[
= \frac{1}{4} \left( \frac{m}{4\pi N} \right)^2 \frac{1}{2} \left( \frac{Q_i F_{PV}(t)}{t-1} \right)^2 \left( -\frac{t}{2m^2} \right) \left( \frac{t-m^2}{2} \right)^2
\]

This expression agrees with that given by JOOS and KRAMER\(^{(3)}\) and shows furthermore that we can use the above expressions for the amplitudes in a study of the absorption correction at high energy.

Incidentally the corresponding expression of ACHUTHAN and SRINIVASAN\(^{(6)}\) (their Eq. (5.4)) seems to be in error.

In the figure we compare the calculated differential cross-section with the experimental data available. It is clear from this comparison that such a simple one-pion exchange model fails to explain the data. Modification of this model due to absorptions will be considered elsewhere.
In this work we have shown the explicit advantage of a particular choice of the twelve invariant amplitudes for the process of photo-production of vector mesons from nucleons. In an earlier publication we have listed the 64 physical observables which can be measured in such photo-production experiment. Since we have chosen the twelve invariant amplitudes in complete analogy with Compton scattering (where however there are only six) our expressions for differential cross-section, polarization, etc., reduce to those of Compton scattering when we take the suitable limit. We consider this to be an important advantage since it may enable us to make meaningful approximations to our theoretical expressions. For example, the complicated Born term contributions coming from nucleon exchange can be considerably simplified if we put as a first approximation \( \lambda = \lambda' \) and in fact in this case the formulae are identical with those of Compton scattering (if we put* \( m_v = 0 \)).

In a previous paper we have given the differential cross-section and polarization expressions in terms of the amplitudes \( Q_i \) introduced there. Like the helicity amplitudes given in this paper, the \( Q_i \) amplitudes have the advantage that the differential cross-section takes a simple form in terms of these amplitudes. It is fairly easy to write down expressions for the observables in Compton scattering in terms of these amplitudes \( q_i \). Thus the differential cross-section for Compton scattering is given by

\[
\frac{d\sigma}{d\Omega} = \frac{1}{4} \left( \frac{m}{4\pi W} \right)^2 \frac{1}{s^{\frac{3}{2}}\theta} \left[ \sin^2\theta \left\{ |a_1|^2 + |a_4|^2 + |a_5|^2 + |a_8|^2 \right\} \\
+ |a_6|^2 + |a_7|^2 \right]
\]

and that of the polarization is

\[
I_\pi P_m = \frac{C}{s^{\frac{3}{2}}\theta} \left[ \text{Im} \left( a_1 a_5 + a_4 a_8 \right) + \sin^2\theta \ \text{Im} \left( a_2 a_c + a_3 a_d \right) \right]
\]

* This approximation is not unreasonable in the pole term where \( s = m^2 \).
where $C$ is a certain constant. Using the relationship between these amplitudes $a_i$ and the invariant amplitudes $A_i$, we easily recover from the above expressions the corresponding formula given by Leader and Hearn. However, the simplicity of the above expressions in both Compton scattering and photo-production processes will be of great practical advantage when analysing experimental data.

The most important exchange diagrams have been evaluated by applying the Feynman technique. The pion-exchange amplitude has been specifically confronted with the few experimental data now available but it is well known that such an unmodified one-pion-exchange diagram fails to fit the experimental data in many reactions. It is important to take into account final state interactions in the manner of GOTTFRIED and JACKSON\(^{(9)}\) but even such an absorption model does not follow the observed steep fall-off of the differential cross-section. However, more thorough theoretical analysis can only be undertaken when there are more and accurate experimental data available.

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\[
\frac{ds}{d\Delta^2} = \mu b/GeV^2
\]

**Fig. 1**

\[\text{ONE PION EXCHANGE} = \left[\frac{\Delta^2}{(\Delta^2 + m^2)}\right]^{\frac{1}{2}}\]

\[\Delta^2 \text{ in } GeV^2\]