IC/66/41

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1966
PIAZZA OBERDAN
TRIESTE

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ON THE DYNAMICAL ORIGIN OF SU(6) †

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26 April 1966

† To be submitted to Physics Letters

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ON THE DYNAMICAL ORIGIN OF SU(6)

It is now generally accepted that if the spectrum of elementary particles exhibits traces of higher spin-containing symmetries, such symmetries must have a dynamical origin. In this note we consider a <u>relativistic</u> dynamical model for massive quark-antiquark binding which gives rise to the rest symmetry $U(6)_W \otimes O(3)$ for the meson bound states. The model may or may not be realistic: it does however show how higher symmetries may emerge from dynamical approximations; it throws some light on the role of non-compact groups and it leads to a definite representation mixing.

We consider the Bethe-Salpeter (B. S) equation for quarks binding [1] through the exchange of a massless neutral singlet; let p and q stand for the total and relative momentum components in the B. S wave function. We treat the two cases:

(A) <u>Spinless quarks</u>

$$[(\tfrac{1}{2}p+q)^2 - m^2]\,[(\tfrac{1}{2}p-q)^2 - m^2]\,\phi(p,q) = \lambda m^2 \int d^4q' \,\frac{\phi(p,q')}{(q-q')^2}$$

(B) <u>Spin $\frac{1}{2}$ quarks</u>

$$[\tfrac{1}{2}p+q-m]_A^C\,\phi_C^D(p,q)\,[\tfrac{1}{2}p-q+m]_D^B = \lambda \int d^4q' \,\frac{\phi_A^B(p,q')}{(q-q')^2}$$

With the WICK [2] boundary condition on the bound state wave function ϕ, a passage may be made to a Euclidean metric for all vectors p and q, by the transformation to $x_4 = i x_0$. Following CUTKOSKY [3] and SCHWINGER [4] we then make the projective transformation to a unit 5-dimensional hypersphere:

$$q_\mu = [m^2 - \tfrac{1}{4}p^2]^{\frac{1}{2}} \; [\eta_5 \pm 1]^{\mp 1} \, \eta_\mu$$

$$\vec{\eta}^2 = \eta_5^2 - \eta^2 = 1 \qquad ,$$

and define a new wave function

$$\Phi(p, \vec{\eta}) = \phi(p, q) / (\eta_5 + 1)^3$$

The symmetries of the equation are best exhibited by proceeding in all cases to a particular frame.

(A) Spinless quarks

Take the rest frame $\underline{p} = 0$.

$$\left[1 - \frac{p^2}{4m^2}(1 - \eta_4^2)\right] \Phi(\vec{\eta}) = \lambda \int d^5\vec{\eta}' \, \delta(\vec{\eta}'^2 - 1) \, \frac{\Phi(\vec{\eta}')}{1 - \vec{\eta}\cdot\vec{\eta}'}$$

There is an obvious symmetry of the wave function under the following η-rotations:

(i) O(5) for $p = 0$ (maximal binding). To each level (N) is associated a degeneracy $\frac{1}{6}N(N+1)(2N+1)$. Each level corresponds to a component of a fully-symmetric traceless tensor $\Phi\mu_1\mu_2 \ldots$ ($\mu_1, \mu_2, \ldots = 1, \ldots 5$). When arranged in a tower it is easily seen that such symmetric tensors correspond to a single irreducible representation of the non-compact group O(5, 1).

(ii) O(4) for $\eta_1, \eta_2, \eta_3, \eta_5$ rotations for the realistic case $p^2 \neq 0$. This case has been completely solved by CUTKOSKY [3] who finds that each O(5) level is split and labelled by two quantum numbers n and k ($N = n + k$). It is easy to see that the levels may now be arranged according to the following irreducible representations [5] of O(4, 1)

$k = 0 :$ $(0,0) \oplus (\frac{1}{2},\frac{1}{2}) \oplus (1,1) \oplus \ldots$

$k = 1 :$ $(\frac{1}{2},\frac{1}{2}) \oplus (1,1) \oplus \ldots$ etc.

$k = 2 :$ $(1,1) \oplus \ldots$ etc.

i.e., the original $p = 0$, $O(5,1)$ representation splits as $O(5,1) = \Sigma \oplus O(4,1)$.

(B) Quarks with spin

Even though we started with no more than U(3) symmetry, the assumption of singlet exchange leads immediately to the extended group of U(6,6) transformations. After a Wick rotation this group structure appears as a compact U(12). To obtain the symmetry subgroup we choose the frame $p = (p_4, p_3, 0, 0)$. $q = (q_4, 0, 0, 0)$.

The equation then reads:

$$\left[(\tfrac{1}{2}\not{p} + m)(\eta_5 + 1) + \gamma_4 q_4 \sqrt{m^2 - \tfrac{1}{4}p^2}\right] \Phi(\vec{\eta}) \left[(\tfrac{1}{2}\not{p} - m)(\eta_5 + 1) - \gamma_4 q_4 \sqrt{m^2 - \tfrac{1}{4}p^2}\right]$$

$$= \lambda \int d\Omega(\vec{\eta}') \frac{\Phi(\eta')}{(1 - \vec{\eta}\cdot\vec{\eta}')}$$

The solution is evidently symmetric under O(3) transformations of $\eta_1\ \eta_2\ \eta_3$. Moreover, there is an invariance of the equation under spinor index transformations that commute with $\not{p}$ and $\not{\eta}$; thus we arrive at the following symmetry subgroups:

(i) $[U(6) \otimes U(6)]_{\gamma_0} \otimes O(3)$ for $p = 0$

(ii) $[U(6)]_{\gamma_0 \gamma_3} \otimes O(3)$ for $p \neq 0$.

In the second case the collinear $U(6)_W$ symmetry implies that there will be a mixing of the $U(6) \otimes U(6)$ components appearing in the first case for each internal orbital level. To be specific, the 144 ($\ell = 0$) component spinor splits into

$$\begin{pmatrix} (1,1)+(35,1) & (6,\bar{6}) \\ (\bar{6},6) & (1,35)+(1,1) \end{pmatrix}$$

at the U(6) ⊠ U(6) stage and finally to a set of four (1 ⊕ 35) physical particles. The equations for the $U(6)_W$ singlets, for example, are

$$\left[\tfrac{1}{4}p_0^2-(iq_4-m)^2\right]\phi(36,1)-\tfrac{1}{4}p_3^2\,\phi(1,36)=\lambda\int d^4q'\,\frac{\phi(36,1)}{(q-q')^2}$$

$$\left[\tfrac{1}{4}p_0^2-(iq_4+m)^2\right]\phi(1,36)-\tfrac{1}{4}p_3^2\,\phi(36,1)=-\lambda\int d^4q'\,\frac{\phi(1,36)}{(q-q')^2}$$

$$\left[(\tfrac{1}{2}p_0-m)^2+q_4^2\right]\phi(6,\bar{6})-\tfrac{1}{4}p_3^2\,\phi(\bar{6},6)=-\lambda\int d^4q'\,\frac{\phi(6,\bar{6})}{(q-q')^2}$$

$$\left[(\tfrac{1}{2}p_0+m)^2+q_4^2\right]\phi(\bar{6},6)-\tfrac{1}{4}p_3^2\,\phi(6,\bar{6})=-\lambda\int d^4q'\,\frac{\phi(\bar{6},6)}{(q-q')^2}$$

the singlets themselves being linear combinations of [$\phi(1,36)$, $\phi(36,1)$] and [$\phi(6,\bar{6})$, $\phi(\bar{6},6)$] . The notation $\phi(36,1)$, e.g., implies that a trace over the appropriate 6-valued indices is taken. For the 35-folds there is a simultaneous mixing among all the U(6) ⊠ U(6) multiplets [6] .

We may now remark on various modifications to the kernels and the resulting dynamical symmetries:

(a) If the exchanged meson is spinless but massive the symmetry reduces to

(A) Spinless quarks; O(3) for all p .

(B) Spin $\frac{1}{2}$ quarks; O(3) ⊠ $[U(6) ⊠ U(6)]_{\gamma_0}$, $p = 0$

O(3) ⊠ $[U(6)_{\gamma_0\gamma_3}$, $p^2 \neq 0$.

(b) If the quarks have unequal masses there is (A) O(4) symmetry both when $p^2 \neq 0$ and $p = 0$ for spinless quarks, while (B) the symmetry groups for quarks with spin are unchanged.

(c) We have not considered the extension of the above results to the case of 3-quark binding. It is clear, however, that the spin part of the wave function ψ_{ABC} (p, q, r) will exhibit the coplanar symmetry $[U(3) \otimes U(3)]_{\gamma_0 \gamma_3 \gamma_2}$. A given coplanar representation becomes a mixture [7] of the appropriate 12^3 components of ψ_{ABC}.

REFERENCES AND FOOTNOTES

[1] The study of relativistic bound-state equations for the quark systems was first initiated by Bogolubov and co-workers at Dubna who considered the case of general scalar kernels of the B.S. equation with particular reference to the role of the group $\tilde{U}(12)$ [For references see A. TAVKHELIDZE, Proceedings of the Seminar on High-Energy Physics and Elementary Particles, Trieste, 1965 (IAEA, Vienna), page 763.] The study was pursued by J. DABOUL and R. DELBOURGO (to be published in Nuovo Cimento) for a spinorial 4-Fermi kernel. In this note we are however concerned with the maximal symmetries one may expect on the basis of the particular model chosen.

[2] G.C. WICK, Phys. Rev. 96, 1124 (1954).

[3] R.E. CUTKOSKY, Phys. Rev. 96, 1135 (1954).

[4] J. SCHWINGER, J. Math. Phys. 5, 1606 (1964).

[5] R. RĄCZKA (private communication).

[6] The problem of the mass spectrum has not been dealt with in this note. However we can hope that the four multiplets are widely separated in mass.

[7] Such mixing has been advocated on the basis of current algebras by various authors. See for instance
H. HARARI (Stanford preprint);
N. CABIBBO and H. RUEGG (CERN preprint);
G. ALTARELLI, R. GATTO, L. MAIANI and G. PREPARATA (Florence preprint).
Most of these authors consider $U(3) \otimes U(3)$ as the chiral group whereas our formulation characterizes it as the coplanar group which, for particle states, is the helicity group rather than the chiral group.

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