UNIVERSAL COUPLING
OF VECTOR MESONS REVISITED

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ABSTRACT

It is shown that "the partial conservation of a tensor current" (PCTC) leads to a universality of vector meson interactions, without assuming the Sakurai hypothesis on universal coupling of vector mesons to currents of symmetry generators. Meson electromagnetic decays are discussed on the grounds of PCTC and the results compared with the Sakurai universality.
1. INTRODUCTION

Some years ago SAKURAI formulated an important idea [1] that vector mesons are coupled universally to baryons and mesons by means of currents of symmetry generators like the total isospin $T$, hypercharge $Y$, and baryonic number $B$. In this way the $\rho$ meson would be coupled to the current of the total isospin, whereas two orthogonal mixtures of the $\phi$ and $\omega$ mesons,

\[ \phi_\mu(x) = c_\phi \phi_\mu(x) + c_\omega \omega_\mu(x), \quad \omega_\mu(x) = c_\phi \phi_\mu(x) - c_\omega \omega_\mu(x) \]

to the currents of the total hypercharge and baryonic number respectively. If the Sakurai hypothesis were true, vector meson couplings would provide us with new examples of universality of particle interactions, observed so far in electromagnetic, weak and gravitational couplings. Obviously, this universality would be independent of the SU(3) or SU(6) symmetry, since it would give us relations between couplings of vector mesons both to baryons and mesons.

In a recent paper [2], however, some arguments have been given that only in the case of a strict symmetry, currents of symmetry generators may be identical with source currents of vector mesons. In the case of an approximate symmetry (even if the breaking is caused solely by electromagnetic and weak interactions) both sorts of currents are different, so the vector mesons must not be coupled to currents of symmetry generators. The purpose of the present note is to show that in this case there is still a possibility of a universal coupling of vector mesons. This universality, however, is no longer based on couplings to currents of symmetry generators.
but rather on source currents involved in some partial conservation
relations for tensor currents [2] (see also [3]).

We shall discuss in this note two antisymmetric tensor
currents $T_{\mu\nu}^a(x)$ and $T_{\mu\nu}^{\bar{a}}(x)$ related to the electromagnetic and
baryonic currents $J_{\mu}^a(x)$ and $J_{\mu}^{\bar{a}}(x)$ by the identities [4]

\[(2) \quad J_{\mu}^e(x) = \partial_\nu T_{\nu\mu}^e(x)\]
and
\[(3) \quad J_{\mu}^{\bar{e}}(x) = \partial_\nu T_{\nu\mu}^{\bar{e}}(x).\]

Notice that due to (2) we have the following identity for electromagetic coupling:

\[(4) \quad \langle B(\frac{1}{e} J_{\mu}^e(0)) | A \rangle = -\frac{i}{2} \langle B(\frac{1}{e} T_{\mu\nu}^e(0)) | A \rangle i(k_\mu e_{\nu} - k_\nu e_{\mu}),\]

where $A$ and $B$ are eigenstates of the total four-momentum $P_\mu$, $k = p_B - p_A$, and $e$ denotes a polarisation of the photon, $k_\mu e_{\mu} = 0$.

Assuming now "the partial conservation of the tensor currents"
$T_{\mu\nu}^a(x)$ and $T_{\mu\nu}^{\bar{a}}(x)$ (PCTC) expressed by the relations [4]

\[(5) \quad \partial_\nu T_{\nu\mu}^e(x) = D^3(-\nabla) \phi_\mu^{(\rho)}(x) + D^5(-\nabla) \left[ c_\omega \phi_\mu^{(\omega)}(x) + c_\phi \phi_\mu^{(\phi)}(x) \right]\]
and
\[(6) \quad \partial_\nu T_{\nu\mu}^{\bar{e}}(x) = D^0(-\nabla) \left[ -c_\rho \phi_\mu^{(\rho)}(x) + c_\omega \phi_\mu^{(\omega)}(x) \right],\]

we can write down the electromagnetic and baryonic currents in terms
of source currents of $\rho^0$, $\omega$ and $\phi$ mesons. Here $D^3(k^2)$, $D^5(k^2)$ and
$D^0(k^2)$ are some "gentle" functions of $k^2$ regular at $k^2 = 0$ and
$\phi_\mu^{(\rho)}(x)$ ($x = \rho^0$, $\omega$, $\phi$) denote renormalized fields of $\rho^0$, $\omega$ and $\phi$.
mesons related to the source currents \( j_\mu^{(\pi)}(x) = (-\Box + m_\pi^2) \psi_\mu^{(\pi)}(x) \).

We can see, therefore, that by means of PCTC the well-known universal form of the electromagnetic and baryonic currents implies a universality of couplings of \( \rho^\circ, \omega \) and \( \phi \) mesons. This happens in spite of the fact that these mesons are not coupled to currents of symmetry generators.

2. UNIVERSALITY IMPLIED BY ELECTROMAGNETIC CURRENT

Making use of the formulae (2) and (5) we can write

\[
\langle B^{\mu j}(0)|A\rangle = \frac{D_3^{(0)}}{k^2 + m_\rho^2} \langle B^{\mu j}(0)|A\rangle + D_4^{(0)}\int \frac{d^4k}{k^2 + m_\rho^2} \left[ C_\omega \langle B^{\mu j}(0)|A\rangle + C_\phi \langle B^{\mu j}(0)|A\rangle \right],
\]

where \( A \) and \( B \) are eigenstates of \( P^\mu \) and \( k = p_B - p_A \). Then we have [4]

\[
D_3^{(0)} = \frac{1}{2} \frac{m_\rho^2}{g_{\rho NN}^2 k^{(v)}(0)} \quad \text{and} \quad D_4^{(0)} = \frac{1}{2} \frac{m_\rho^2}{g_{\phi NN}^2 k^{(v)}(0)}
\]

and

\[
\frac{g_{\rho BA}^{(v)} K_{\omega AA}^{(v)}(0)}{m_\rho^2} = \frac{C_\omega}{m_\omega^2} \frac{g_{\phi AA}^{(v)} K_{\phi AA}^{(v)}(0)}{m_\phi^2} + \frac{C_\phi}{m_\phi^2} \frac{g_{\phi AA}^{(v)} K_{\phi AA}^{(v)}(0)}{m_\phi^2},
\]

where \( g_{\rho BA} \) are strong coupling constants, whereas \( K_{\rho BA}^{(v)}(k^2) \) denote vector form factors of the matrix elements \( \langle B^{\mu j}(0)|A\rangle \), normalized to 1 on the mass shell, i.e., \( K_{\rho BA}^{(v)}(-m_\rho^2) = 1 \) (here \( A \) and \( B \) are one-particle states assumed to be eigenstates of \( P^\mu \)). The relation (9) follows solely from (1) and the definitions of \( g_{\rho BA} \) and \( K_{\rho BA}^{(v)}(k^2) \), whereas the relations (8) are consequences of (7) and the formula \( J_\mu^{\rho}(x) = J_\mu^{3}(x) + J_\mu^{8}(x) \), where \( J_\mu^{3}(x) \) and \( J_\mu^{8}(x) \) are currents of the SU(3) generators \( Q^3 = T_3 \) and \( \frac{1}{\sqrt{3}} Q^8 = \frac{1}{2} \gamma \) respectively. Here we assume that \( \psi^{3\mu}(x) = \psi^{(\rho)}(x) \) and \( \psi^{8\mu}(x) = c_\omega \psi^{\mu}(x) + c_\phi \psi^{(\phi)}(x) \) transform under SU(3) as \( Q^3 \) and \( Q^8 \) respectively.
The relationship (7) enables us to relate electromagnetic properties of hadrons to their strong interactions with \( \rho^0, \omega \) and \( \phi \) mesons [4, 5] (see also Section 4). In this note we are interested mainly in a reverse implication of (7); in the consequences of electromagnetic interactions of hadrons for strong couplings of \( \rho^0, \omega \) and \( \phi \) mesons.

To this end let us write down relations for isovector and isoscalar (vector) form factors following from (7), (8) and (9). For \( A = B \) at zero momentum transfer they have the form

\[
\frac{T_{3A}}{3A} = F_{AA}^{(V)}(0) = \frac{1}{2} \frac{\delta_{\rho AA} K_{\rho AA}^{(V)}}{\delta_{\rho NN} K_{\rho NN}^{(V)}}
\]

and

\[
\frac{1}{2} \gamma_A = F_{AA}^{(S)}(0) = \frac{1}{2} \frac{\delta_{\gamma AA} K_{\gamma AA}^{(V)}}{\delta_{\gamma NN} K_{\gamma NN}^{(V)}}.
\]

Hence we conclude that for all hadrons \( \frac{\delta_{\rho AA} K_{\rho AA}^{(V)}}{\delta_{\gamma AA} K_{\gamma AA}^{(V)}} \) and \( \frac{\delta_{\delta AA} K_{\delta AA}^{(V)}}{\delta_{\gamma AA} K_{\gamma AA}^{(V)}} \) are proportional to the eigenvalues \( T_{3A} \) and \( \gamma_A \) respectively. In particular we have

\[
\frac{\delta_{\rho \pi \pi} K_{\rho \pi \pi}^{(V)}}{\delta_{\gamma \pi \pi} K_{\gamma \pi \pi}^{(V)}} = 2 \frac{\delta_{\rho NN} K_{\rho NN}^{(V)}}{\delta_{\gamma NN} K_{\gamma NN}^{(V)}}, \quad \delta_{\gamma \pi \pi} = 0.
\]

These results are exactly the same as in the case of the Sakurai hypothesis, although now the vector mesons \( \rho^0 \) and "8" are not coupled to currents of \( Q^3 = T_3 \) and \( Q^8 = \frac{\sqrt{3}}{2} \gamma \) respectively. This is due, of course, to the fact that in our case these currents are linearly expressed by source currents of \( \rho^0 \) and "8" through PCTC.
3. UNIVERSALITY IMPLIED BY BARYONIC CURRENT

Using now the formulae (3) and (6) we obtain the relation

\[ \langle B | f^\text{bar}_\mu(0) | A \rangle = D^0(0^+) \left[ -\frac{C^0_\phi}{k^2 + m^2_\phi} \langle B | j^{(0)}(0) | A \rangle + \frac{C_\omega}{k^2 + m^2_\omega} \langle B | j^{(\omega)}(0) | A \rangle \right] \]

(13)

Here we get

\[ D^0(0) = \frac{m^2_\phi}{g^0_{\text{NN}} K^{(v)}_{\text{NN}}(0)} \]

and

\[ \frac{g_{\phi A A} K_{\phi A A}^{(v)}(0)}{m^2_\phi} = - \frac{C_\phi g_{\phi A A} K^{(v)}_{\omega A A}(0)}{m^2_\omega} + \frac{C_\omega g_{\phi A A} K^{(v)}_{\phi A A}(0)}{m^2_\phi} \]

(15)

where the label "0" refers to the vector meson "0" defined by (1) and \( J^\mu(x) \) denotes the current of the baryonic number B. Here we assume that \( \phi^0_{\mu}(x) = -C_\phi \phi^{(\omega)}_{\mu}(x) + C_\omega \phi^{(\phi)}_{\mu}(x) \)

transforms under SU(3) as a singlet.

From (13), (14) and (15) we obtain the following relation

\[ B^\phi_A = F^{(v)0}_{\phi A A}(0) = \frac{g_{\phi A A} K^{(v)}_{\phi A A}(0)}{g_{\phi A A} K^{(v)}_{\text{NN}}(0)} \]

(16)

Hence we conclude that for all hadrons \( g_{\phi A A} K^{(v)}_{\phi A A}(0) \) is proportional to the eigenvalue \( B^\phi_A \). In particular we have

\[ g_{\phi \pi \pi} = 0 \]

(17)

\[ g_{\phi \pi \pi} = g_{\phi \pi^+ \pi^-} \]

Applying (9) and (15) to the case of \( A = \pi^+ \) and making use of (12) and (17) we can see that

\[ g_{\omega \pi \pi} = 0 \quad \text{and} \quad g_{\phi \pi \pi} = 0 \]

(18)

The relations (18) forbid the strong decays \( \omega \to \pi^+ \pi^- \) and \( \phi \to \pi^+ \pi^- \) without referring to the G-parity conservation.
Of course, they do not exclude the electromagnetic decays $\omega \rightarrow \pi^+ + \pi^-$ and $\phi \rightarrow \pi^+ + \pi^-$, since the formula (11) leading to (12) comes out by splitting the electromagnetic current into isovector and isoscalar parts. In fact, these decays can proceed via the virtual processes $\omega \rightarrow \gamma \rightarrow \pi^+ + \pi^-$ and $\phi \rightarrow \gamma \rightarrow \pi^+ + \pi^-$. [6].

4. MESON ELECTROMAGNETIC DECAYS

In this section we will describe some applications of PCTC to meson electromagnetic decays and compare results with the Sakurai universality. These results are essentially the same as those obtained on the grounds of the Sakurai hypothesis and the polology. A difference is caused by form factors (taken at zero momentum transfer).

First, consider the vertices $\gamma \rightarrow \gamma$, where $\gamma = \gamma^0, \omega, \phi$.

Using the notation $\bar{j}_\mu(x) = e \int_\mu (x)$ we can write

$$\langle 0 | \bar{j}_\mu (0) | \gamma \rangle = \frac{i}{(2\pi)^{3/2}} \frac{e^\mu}{\sqrt{2k_0}} \gamma_{\gamma} (k^2),$$

where $K_{\gamma} (-m^2_\gamma) = 1$. Then from (7) we obtain the formulae

$$\gamma_{\gamma}^{\phi} = e \frac{3}{2} (-m^2_\phi) \approx \sqrt{2} \left( \frac{\delta_{\gamma\gamma} K_{\gamma\gamma}^{(v)}(0)}{\sqrt{m_\gamma}} \right)^{-1} m^2_\gamma,$$

and

$$\gamma_{\omega} = e \frac{8}{3} (-m^2_\omega) c_\omega \approx \sqrt{2} \left( \frac{\delta_{\gamma\omega} K_{\gamma\omega}^{(v)}(0)}{\sqrt{m_\omega}} \right)^{-1} m^2_\omega,$$

Here we assumed that $\frac{3}{2} (-m^2_\phi) \approx D^{3,6}(0)$. The formula (20) was derived some years ago from the Sakurai hypothesis by GELL-MANN.
and ZACHARIESEN [7, 8]. The formulae (21) are essentially those found on the "universality" basis by DASHEN and SHARP [9], differing from the latter by factors $m^2_\pi / m^2_{\omega}$ and $m^2_\rho / m^2_{\phi}$ respectively. In both cases K's at zero momentum transfer are replaced by 1.

If the decay of $\nu$ into a charged lepton pair $\ell^- + \ell^+$ proceeds via the virtual process $\tau \rightarrow \gamma \rightarrow \ell^- + \ell^+$, we have the decay rate [6, 8]

$$\Gamma (\nu \rightarrow \ell^- \ell^+) = \frac{1}{3} \frac{\gamma_\nu^2}{m_\nu^4} m_\nu \left(1 - \frac{4 m^2_\nu}{m^2_{\pi^+}} \right)^2 \left(1 + \frac{2 m^2_\pi}{m^2_{\pi^+}} \right).$$

This result is consistent with experimental estimates for $\Gamma (\rho^0 \rightarrow \ell^- \ell^+)$ [10, 11], if $\gamma_{\rho^0}$ is given by (20) with $K^{(\nu)}_{\rho NN} (0) \approx K^{(\nu)}_{\rho NN} (-m^2_{\rho}) = 1$ and $q_{\rho NN} = \frac{1}{2} q_{\rho NN} \pi^0 / 4\pi \approx 2$ (the "universality" value for $q_{\rho NN} \pi^0$).

Now, consider the decays $\gamma \rightarrow \pi^0 + \gamma$, where $\gamma = \rho^0$, $\omega$, $\phi$. Denoting by a and b one of the vector particles $\gamma$, $\rho^0$, $\omega$, $\phi$, we can write

$$\langle \pi^0 | \langle \nu | e \mu \rangle (x) \rangle$$

$$\langle \pi^0 | \langle \nu | e \mu \rangle (x) \rangle$$

$$\langle \pi^0 | \langle \nu (0) | e \mu \rangle (x) \rangle = \frac{1}{(2\pi)^3} \frac{1}{\sqrt{4 m^2_{\pi}}} \frac{f_{\pi ba} K_{\pi ba} (0)}{m^2_{\pi}} e^{\nu (0) e a} e^{\nu (0) e a}$$

(23)

where $e_a$ is a polarization of a and $k = p_{\pi^0} - p_{\omega}$. Here $K_{\pi ba} (-m^2_{b}) = 1$.

Then from (7) we obtain

$$f_{\pi \gamma} = e D (\omega) \left[ \frac{C_\omega f_{\pi \omega} K_{\pi \omega} (0)}{m^2_{\omega}} + \frac{C_\phi f_{\pi \phi} K_{\pi \phi} (0)}{m^2_{\phi}} \right]$$

and

$$f_{\pi \gamma} = e D (\omega) \frac{f_{\pi \omega} K_{\pi \omega} (0)}{m^2_{\omega}} \right] + \frac{f_{\pi \phi} K_{\pi \phi} (0)}{m^2_{\phi}}.$$

(25)
The decay rate for $\gamma \to \pi^0 + \gamma$ is given by the formula

$$\Gamma (\gamma \to \pi^0 \gamma) = \frac{1}{3} \frac{\bar{f}_{\pi\gamma\gamma}}{4\pi} m_\pi \left( \frac{m_\pi^2 - m_{\pi^0}^2}{2m_\pi m_{\pi^0}} \right)^3. \tag{26}$$

Thus, from the first equation (25) we get

$$\Gamma (\omega \to \pi^0 \gamma) = \frac{1}{3} \times \alpha \left( \frac{3}{2} g_{\gamma NN} K_{\gamma NN}^{(0)} \right)^{-1} \bar{f}_{\pi\rho\omega} K_{\pi\omega}^{(0)} m_\pi \left( \frac{m_\pi^2 - m_{\pi^0}^2}{2m_\pi m_{\pi^0}} \right)^3. \tag{27}$$

This formula with $K$'s replaced by $1$ was derived in ref. \cite{8} under the assumption that the decay $\omega \to \pi^0 + \gamma$ goes via the virtual process $\omega \to \pi^0 + \rho^0 \to \pi^0 + \gamma$.

Finally, consider the decay $\pi^0 \to \gamma + \gamma$. Then from (7), making use of (23), we obtain

$$f_{\pi\gamma\gamma} = e_\omega^3 D(0) f_{\pi\rho\omega} K_{\pi\rho\omega}^{(0)} m_\omega^2 + e_\omega^3 D(0) \left[ \frac{C_{\omega} f_{\pi\omega\gamma} K_{\pi\omega}^{(0)}}{m_\omega^2} + \frac{C_{\phi} f_{\pi\phi\gamma} K_{\pi\phi}^{(0)}}{m_\phi^2} \right]. \tag{28}$$

Using the relations $f_{\pi\alpha\beta} = f_{\pi\beta\alpha}$ we can insert (24) and (25) into (28). Then we get

$$f_{\pi\gamma\gamma} = e_\omega^3 D(0) D(0) \left\{ \frac{C_{\omega} f_{\pi\omega\gamma} K_{\pi\omega}^{(0)}}{m_\omega^2} \frac{K_{\omega}(0)K_{\omega}(0) + K_{\phi}(0)K_{\phi}(0)}{m_{\omega}^2 m_{\phi}^2} \right\} +$$

$$+ \frac{C_{\phi} f_{\pi\phi\gamma} K_{\pi\phi}^{(0)}}{m_\phi^2} \left[ K_{\omega}(0)K_{\phi}(0) + K_{\phi}(0)K_{\omega}(0) \right]. \tag{29}$$

The decay rate for $\pi^0 \to \gamma + \gamma$ is given by

$$\Gamma (\pi^0 \to 2 \gamma) = \frac{1}{m_\pi} \frac{\bar{f}_{\pi\gamma\gamma}}{4\pi} m_\pi. \tag{30}$$
Thus the equation (29) leads to the formula

\[ (\pi^0 \rightarrow 2\gamma) = \frac{1}{6\pi} \left( \frac{\alpha^2}{\pi} \right) \left( \frac{\sin^2 \frac{\theta}{2}}{\sin \frac{\theta}{2}} \right) \int_{4\pi}^{2\pi} C_\omega \frac{m}{\pi} \frac{\lambda^2}{\mu^2}, \]

where

\[ \lambda = \frac{K_{\pi\omega\gamma}(0) K_{\pi\rho\gamma}(0) + K_{\pi\rho\omega}(0) K_{\pi\omega\gamma}(0)}{K_{\pi\rho\gamma}(0) K_{\pi\omega\gamma}(0)} \]

and

\[ \mu = \frac{m^2}{m^2} + \frac{m^2}{m^2} \frac{C_\omega \int_{\pi} \rho \omega}{C_\omega \int_{\pi} \rho \omega} \frac{K_{\pi\rho\omega}(0) K_{\pi\rho\gamma}(0) + K_{\pi\rho\gamma}(0) K_{\pi\omega\gamma}(0)}{K_{\pi\omega\gamma}(0) K_{\pi\omega\gamma}(0)} \]

The formula (31) would be identical with that found on the base of a pole approximation by DASHEN and SHARP [9], if \( \lambda \) were equal to 1 and \( \mu \) equal to 1. Taking from ref. [9] an estimate of the coefficient in (31), we have

\[ \Gamma(\pi^0 \rightarrow 2\gamma) = (24.8 \text{ eV}) \times \lambda^2 \mu^2, \]

whereas the experimental figure for \( \Gamma(\pi^0 \rightarrow 2\gamma) \) is near 6\text{ eV} [12]. Thus the factor \( \lambda^2 \mu^2 \) should seriously diminish the number 24.8 eV. Unfortunately the form factors K's involved in \( \lambda \) and \( \mu \) are unknown. In the case of an extremely "gentle" behaviour of K's the factor \( \lambda^2 \) would be 4 (notice that there is no counterpart of this factor in ref. [9]). On the contrary, the factor \( \mu^2 \) is presumably much less than 1 (\( C_\omega / C_\phi = -1/\sqrt{2} < 0 \)).

From the formulae (27) and (31) we have the following ratio:

\[ \frac{\Gamma(\pi^0 \rightarrow 2\gamma)}{\Gamma(\omega \rightarrow \pi^0\gamma)} = (1.7 \times 10^{-5}) \lambda^2 \mu^2 \]
The estimate of the coefficient in (35) is taken from ref. [9]. The ratio (35) would be equal to that in ref. [9], if the product $\lambda \mu \nu$ were equal to $1 + \frac{C_\rho \pi \pi}{C_\omega \pi \omega}$. Concluding, we can say that PCTC leads essentially to the same results for meson electromagnetic decays as the polology based on dispersion relations and the Sakurai hypothesis. A difference is introduced by form factors. One gets from PCTC exactly a pole approximation if one puts form factors (taken at zero momentum transfer) equal to 1. This situation turns out to be closely analogous to the well-known connection between "the partial conservation of the axial current" (PCAC) [13] and the polology operating on dispersion relations. So, in both cases one can wonder whether there is something more in the partial conservation approach than in the polology. The recent successful application of PCAC to intermediate states in commutation relations leading to the Adler-Weisberger formula seems a point in favour of this possibility. Besides, one should stress the fact that both PCTC and PCAC introduce some universalities to interactions of vector and pseudoscalar mesons respectively.

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