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The proposed relativistic higher symmetry theories for elementary particles[1] based on the SU(6) scheme have the following two general characteristics:

1) They give remarkable agreement with experiment for certain quantities.

2) They seem to have in them various difficulties in principle[2]. In addition there seem to be certain cases where predictions of the theory are in disagreement with experiment[3].

We should like to point out the existence of a soluble model containing many of the essential features of these higher symmetry schemes. The results of the model are extremely instructive. They indicate that certain predictions of the symmetry are indeed correct, such as the structure of certain multiplets and their properties. However, other predictions are not correct, and it is quite clear why the symmetry breaks down in this domain. The S-matrix is unitary in this model, and there are no difficulties in principle, other than the standard ones associated with conventional field theory.

The essential features of the relativistic higher symmetry schemes containing SU(6) are the following. Invariance is assumed under a group which is the direct product of two non-compact groups $G_i \times G_{st}$, where G_i acts only on the internal degrees of freedom of particles, including the spinor components, and G_{st} acts only on space time[4]. Both these groups contain groups isomorphic to the homogeneous Lorentz group (which is non-compact) as subgroups. Although the interactions are invariant under this group, the free field equations are not, and must break the symmetry.

For our soluble model, we choose the minimum group having the above properties, namely the direct product SL(2C) x SL(2C) of two groups isomorphic to the homogeneous Lorentz group. We choose as fundamental entities two four-component Dirac spinor fields of finite mass coupled to a massless vector boson field.

The interaction Lagrangian is invariant under SL(2C) x SL(2C); however, the free Lagrangian for the Diract fields is not. In the same way as in the case of elementary particles, the coupling $\gamma_\mu p^\mu$ breaks the separation of internal degrees of freedom and space time.

It turns out that solutions are already available in the literature[5] for certain versions of this model with specific values of the masses and coupling constants of the Dirac fields. These indicate that there exists a considerable number of quantum states which are well described by the assumption of complete symmetry, i. e., by the classification of states into multiplets using the representations of the group SL(2C) x SL(2C). However, there are also a wide range of phenomena in which the symmetry breaks down[6]. Although difficulties of principle would arise if the symmetry described by this non-compact group were rigorously valid[6], these do not arise in the complete theory.

The investigators of this model were not aware of the relation of their work to the field of subnuclear particles, and did not use the invariance of their interaction under the group SL(2C) x SL(2C). Thus the literature may be difficult reading for those presently working in the field of particle symmetries. For this reason the following glossary of terms is presented, in which the modern equivalents are given of terms used to treat this model.

Modern Terms	Terms used in the Model
Elementary particle	Atom
Quarks (Dirac fields)	Electrons and Nuclei
Invariance under the group SL(2C)xSL(2C)	Russell-Saunders or LS-coupling
Gell-Mann-Okubo mass formula	Landé interval rule
Symmetry-breaking	Spin-orbit interaction
Mass splittings	Fine structure; hyperfine structure

The electrons and nuclei are taken to have charge -e and +Ze and mass m and AM respectively. The interaction $\bar{\psi}\gamma^{\mu}\psi A_{\mu}$ is clearly invariant under SL(2C) x SL(2C), since it is Lorentz invariant and has no explicit space-time dependence (contains no derivitives[7]).

By a more detailed study of this model, one can hope to obtain some insight into the behaviour of a relativistic higher symmetry. In particular one can hope to gain an understanding of where the predicitions from the symmetry can be expected to be valid, and where they should break down. The following general results are immediately evident.

1. The symmetry holds to good approximation in describing the bound states of particles moving non-relativistically. This means for the particle case that the size of a meson or baryon should be large in comparison with the Compton Wave Length of the constituent quarks.

2. The exact symmetry holds for scattering processes in the Born approximation. An example of particular relevance is the scattering of relativistic electrons by a Coulomb potential, where there is no polarization in Born approximation, and the symmetry also predicts zero polarization[3]. The physically observed polarizations are obtained in calculations which go beyond the Born approximation and break the symmetry with the introduction of the electron propagator in the intermediate state.

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FOOTNOTES AND REFERENCES

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3. R. BLANKENBECLER, M. L. GOLDBERGER, K. JOHNSON and S. B. TREIMAN, Phys. Rev. Letters 14, 518 (1965).

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5. See for example E.U. CONDON and G.H. SHORTLEY, Theory of Atomic Spectra, (Cambridge University Press, 1957).

6. Note that the probability density for a Dirac particle is not invariant under a Lorentz transformation. This is normally not a source of difficulty, since the density should change under a transformation which changes the velocity of the particle. However, the group SL(2C) x SL(2C) includes simultaneous independent transformations of the spinor components and of space time. These change the probability density and the velocity independently of one another, and can clearly lead to non-conservation of probability.

7. This symmetry is not related to the peculiar symmetry of the spinless treatment of the non-relativistic hydrogen atom, which results in a degeneracy of states having different orbital angular momentum. The symmetry discussed here is just the decoupling of the spins from the orbital angular momenta in some atomic spectra, to an exceedingly good approximation.