



INTERNATIONAL ATOMIC ENERGY AGENCY

INTERNATIONAL CENTRE FOR THEORETICAL  
PHYSICS

P AND C PROPERTIES  
OF THE  $\tilde{U}(12)$  MULTIPLETS

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## P AND C PROPERTIES OF THE $\tilde{U}(12)$ MULTIPLETS

A dynamical  $U(12)$  symmetry scheme has been proposed for strong interactions recently<sup>1</sup> (Salam, Delbourgo, and Strathdee 1965). This scheme has definite implications regarding parity and charge-conjugation properties of strong-interaction physics and some of these are listed below.

(1) A  $\tilde{U}(12)$  invariant Lagrangian or S-matrix element must conserve parity. To prove this, remark that all  $\tilde{U}(12)$  multiplets are composed from basic quarks  $\psi_{\alpha p}$  which transform as

$$\delta \psi_{\alpha p} = i \left( \epsilon^j + \epsilon^j_5 \gamma_5 + \epsilon^j_\mu \gamma_\mu + \epsilon^j_{\mu 5} i \gamma_\mu \gamma_5 + \frac{1}{2} \epsilon^j_{\mu\nu} \sigma_{\mu\nu} \right)_\alpha^{\beta} (T^j)_p^q \psi_{\beta q}$$

The parity transformation (P),  $\psi \rightarrow \gamma_0 \psi$  is part of the general  $\tilde{U}(12)$  transformation. Even after the Bargmann-Wigner equations<sup>2</sup> have been imposed, parity is conserved, since the operator  $(\not{x} - m)$  is P-invariant.

(2) The theory preserves charge-conjugation (C) invariance. To see this, note that in this theory the charge-conjugating anti-symmetrical matrix  $C_{\alpha\beta}$  (defined by  $(C^{-1})^{\alpha\beta} (\gamma_\mu)_\beta^\gamma C_{\gamma\delta} = -(\gamma_\mu)_\delta^\alpha$ ,  $(C^{-1})^{\alpha\beta} C_{\beta\gamma} = \delta_\gamma^\alpha$ ) acts as a lowering operator, and  $(C^{-1})^{\alpha\beta}$  as a raising operator, for example

$$\psi_{\alpha p} \rightarrow C_{\alpha\alpha'} \bar{\psi}^{\alpha' p}, \quad (1)$$

$$\phi_{\alpha p}^{\beta q} \rightarrow \pm C_{\alpha\alpha'} (C^{-1})^{\beta\beta'} \phi_{\beta' q}^{\alpha' p} \quad (2)$$

under charge conjugation. A  $\tilde{U}(12)$  invariant expression such as  $\phi_A^B(1) \phi_B^C(2) \phi_C^A(3)$  transforms into  $\mp \phi_A^C(3) \phi_C^B(2) \phi_B^A(1)$  so that  $\pm \text{Tr}(\phi(1)[\phi(2), \phi(3)])$  is an invariant. This particular interaction was considered in a previous paper (1965 b), where the mesons were further assumed to be composites of basic quarks transforming according to (1). This assumption allows only for the + sign in (2), and this is the so-called case of "normal" C-parity ( $C \pi^0 C^{-1} = +\pi^0$ ,  $C \rho^0 C^{-1} = -\rho^0$ ).

(3) A consistent assignment of parities for the  $\tilde{U}(12)$  multiplets is obtained if the irreducible multispinor  $\Psi_{AB\dots}^{A'B'\dots}$  transforms under P according to the rule,

$$\Psi_{AB\dots}^{A'B'\dots}(\underline{p}) \rightarrow (-1)^{\bar{n}} (\gamma_0 \dots \gamma_0 \Psi(-\underline{p}))_{AB\dots}^{A'B'\dots} \quad (3)$$

where  $\bar{n}$  denotes the number of upper indices (A', B', .....). This gives for the basic multiplets of type (3) the assignments:

$$\underline{143}^-, \quad \underline{4212}^+, \quad \underline{5940}^+, \dots \text{ for mesons,}$$

and

$$\underline{220}^+, \quad \underline{364}^+, \quad \underline{572}^+, \quad \underline{5720}^-, \dots \text{ for baryons.}$$

(These basic multiplets correspond<sup>3</sup> respectively to 36, 189, 405 for mesons and 20, 56, 70 and again 70 for baryons in the SU(6) language.) The  $\underline{572}^+$  with multispinor form  $\Psi_{[A9]C}$  (symmetry type [2, 1]) has been considered by Salam, Delbourgo, Rashid and Strathdee<sup>4</sup>(1965). The  $\underline{5720}^-$  is a 4-quark, one anti-quark composite with the multispinor form  $\Psi_{[BCDE]A}^A$ . It is a very much more complicated  $\tilde{U}(12)$  entity than the  $\underline{572}^+$  though the SU(6) structure is identical. We shall consider the decomposition of this new basic multiplet in a separate paper. This is the multiplet which seems to be appearing experimentally.

Experimentally also there seems some evidence for  $O^+$  particles. These could belong to a  $\tilde{U}(12)$ -violating P-wave quark - anti-quark system in which case they form parts of an incomplete 4212 or 5940 multiplet occurring together with  $2^+$  particles. Alternatively there is the possibility that there exists a  $\tilde{U}(12)$  multiplet which is not a simple quark - anti-quark ( $q\bar{q}$ ) composite but is a basic multiplet arising as a  $qq\bar{q}\bar{q}$  system (with zero relative momenta as for multiplets in (3)).

## REFERENCES

1. DELBOURGO R., SALAM A., and STRATHDEE J. 1965a  
ICTP 64/11 (to be published); 1965b Proc. Roy. Soc. A (1965)
2. The Bargmann-Wigner equations are discussed together with other aspects of the multispinor formalism in its application to  $\tilde{U}(12)$  calculations in a paper by Salam, Delbourgo, and Strathdee (1965b).
3. The correspondence between  $\tilde{U}(12)$  multiplets and  $SU(6)$  multiplets is established through the Bargmann-Wigner equations. As it happens, the  $143^-$  collapses to  $35^- \oplus 1^-$  under this process.
4. DELBOURGO R., SALAM A., RASHID M., and STRATHDEE J. 1965  
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