



INTERNATIONAL ATOMIC ENERGY AGENCY

INTERNATIONAL CENTRE FOR THEORETICAL
PHYSICS

P AND C PROPERTIES
OF THE $\tilde{U}(12)$ MULTIPLETS

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A dynamical $U(12)$ symmetry scheme has been proposed for strong interactions recently¹ (Salam, Delbourgo, and Strathdee 1965). This scheme has definite implications regarding parity and charge-conjugation properties of strong-interaction physics and some of these are listed below.

(1) A $\tilde{U}(12)$ invariant Lagrangian or S-matrix element must conserve parity. To prove this, remark that all $\tilde{U}(12)$ multiplets are composed from basic quarks $\psi_{\alpha p}$ which transform as

$$\delta \psi_{\alpha p} = i \left(\epsilon^j + \epsilon^j_5 \gamma_5 + \epsilon^j_\mu \gamma_\mu + \epsilon^j_{\mu 5} i \gamma_\mu \gamma_5 + \frac{1}{2} \epsilon^j_{\mu\nu} \sigma_{\mu\nu} \right)_\alpha^{\beta} (T^i)_p^q \psi_{\beta q}$$

The parity transformation (P), $\psi \rightarrow \gamma_0 \psi$ is part of the general $\tilde{U}(12)$ transformation. Even after the Bargmann-Wigner equations² have been imposed, parity is conserved, since the operator $(\not{x} - m)$ is P-invariant.

(2) The theory preserves charge-conjugation (C) invariance. To see this, note that in this theory the charge-conjugating anti-symmetrical matrix $C_{\alpha\beta}$ (defined by $(C^{-1})^{\alpha\beta} (\gamma_\mu)_\beta^\gamma C_{\gamma\delta} = -(\gamma_\mu)_\delta^\alpha$, $(C^{-1})^{\alpha\beta} C_{\beta\gamma} = \delta_\gamma^\alpha$) acts as a lowering operator, and $(C^{-1})^{\alpha\beta}$ as a raising operator, for example

$$\psi_{\alpha p} \rightarrow C_{\alpha\alpha'} \bar{\psi}^{\alpha' p}, \quad (1)$$

$$\phi_{\alpha p}^{\beta q} \rightarrow \pm C_{\alpha\alpha'} (C^{-1})^{\beta\beta'} \phi_{\beta' q}^{\alpha' p} \quad (2)$$

under charge conjugation. A $\tilde{U}(12)$ invariant expression such as $\phi_A^B(1) \phi_B^C(2) \phi_C^A(3)$ transforms into $\mp \phi_A^C(3) \phi_C^B(2) \phi_B^A(1)$ so that $\pm \text{Tr}(\phi(1)[\phi(2), \phi(3)])$ is an invariant. This particular interaction was considered in a previous paper (1965 b), where the mesons were further assumed to be composites of basic quarks transforming according to (1). This assumption allows only for the + sign in (2), and this is the so-called case of "normal" C-parity ($C \pi^0 C^{-1} = +\pi^0$, $C \rho^0 C^{-1} = -\rho^0$).

(3) A consistent assignment of parities for the $\tilde{U}(12)$ multiplets is obtained if the irreducible multispinor $\Psi_{AB\dots}^{A'B'\dots}$ transforms under P according to the rule,

$$\Psi_{AB\dots}^{A'B'\dots}(\underline{p}) \rightarrow (-1)^{\bar{n}} (\gamma_0 \dots \gamma_0 \Psi(-\underline{p}))_{AB\dots}^{A'B'\dots} \quad (3)$$

where \bar{n} denotes the number of upper indices (A', B',). This gives for the basic multiplets of type (3) the assignments:

$$\underline{143}^-, \quad \underline{4212}^+, \quad \underline{5940}^+, \dots \text{ for mesons,}$$

and

$$\underline{220}^+, \quad \underline{364}^+, \quad \underline{572}^+, \quad \underline{5720}^-, \dots \text{ for baryons.}$$

(These basic multiplets correspond³ respectively to 36, 189, 405 for mesons and 20, 56, 70 and again 70 for baryons in the SU(6) language.) The $\underline{572}^+$ with multispinor form $\Psi_{[A9]C}$ (symmetry type [2, 1]) has been considered by Salam, Delbourgo, Rashid and Strathdee⁴(1965). The $\underline{5720}^-$ is a 4-quark, one anti-quark composite with the multispinor form $\Psi_{[BCDE]A}^A$. It is a very much more complicated $\tilde{U}(12)$ entity than the $\underline{572}^+$ though the SU(6) structure is identical. We shall consider the decomposition of this new basic multiplet in a separate paper. This is the multiplet which seems to be appearing experimentally.

Experimentally also there seems some evidence for O^+ particles. These could belong to a $\tilde{U}(12)$ -violating P-wave quark - anti-quark system in which case they form parts of an incomplete 4212 or 5940 multiplet occurring together with 2^+ particles. Alternatively there is the possibility that there exists a $\tilde{U}(12)$ multiplet which is not a simple quark - anti-quark ($q\bar{q}$) composite but is a basic multiplet arising as a $qq\bar{q}\bar{q}$ system (with zero relative momenta as for multiplets in (3)).

REFERENCES

1. DELBOURGO R., SALAM A., and STRATHDEE J. 1965a
ICTP 64/11 (to be published); 1965b Proc. Roy. Soc. A (1965)
2. The Bargmann-Wigner equations are discussed together with other aspects of the multispinor formalism in its application to $\tilde{U}(12)$ calculations in a paper by Salam, Delbourgo, and Strathee (1965b).
3. The correspondence between $\tilde{U}(12)$ multiplets and $SU(6)$ multiplets is established through the Bargmann-Wigner equations. As it happens, the 143^- collapses to $35^- \oplus 1^-$ under this process.
4. DELBOURGO R., SALAM A., RASHID M., and STRATHDEE J. 1965
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