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INTERNATIONAL CENTRE FOR THEORETICAL PHYSICS

$\tilde{U}(12)$ AND THE BARYON FORM FACTOR*

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In place of Eq. (3.6) read:

$$\Psi_{\alpha p\,\beta q\,\gamma r} = \frac{\sqrt{3}}{2\sqrt{2}}\, D_{\alpha\beta\gamma,\,pqr} + \cdots$$

In place of Eq. (3.8) read:

$$J^{Ri} = \frac{3}{5}\,\bar{D}^{\alpha\beta\gamma,\,pqr}\,(\gamma^R)_\alpha^{\alpha'}\,(T^i)_p^{p'}\,D_{\alpha'\beta\gamma,\,p'qr}$$
$$+ \frac{1}{4}\left[\bar{D}^{\alpha\beta\gamma}(\gamma^R)_\alpha^{\alpha'}(T^i)_p^{p'}\,\epsilon_{p'qs}\,N_{[\alpha'\beta]\gamma,\,r}^{\;s} + \cdots\right] + \cdots$$

In place of Eqs. (5.10 – 5.16) read:

$$J^i = \frac{P^2}{4m^2}(\bar{N}N)_F + \frac{3P^2}{4m^2}\bar{D}_\lambda D_\lambda + \frac{3}{2m^2}q_\lambda\bar{D}_\lambda q_\kappa D_\kappa$$

$$J^i_\mu = \frac{P_\mu}{2m}(\bar{N}N)_F + \left(\bar{N}\frac{\gamma_\mu}{4m^2}N\right)_{D+\frac{2}{3}F}$$
$$+ \frac{1}{m^2}\left(\epsilon_{\mu\nu\kappa\lambda}P_\kappa q_\lambda \bar{D}_\nu N + h.c\right)$$
$$+ \frac{3P^2}{4m^2}\bar{D}_\lambda\gamma_\mu D_\lambda + \frac{3}{2m^2}q_\lambda\bar{D}_\lambda\gamma_\mu q_\kappa D_\kappa$$

$$J^i_{\mu\nu} = \frac{P^2}{4m^2}\left[(\bar{N}\sigma_{\mu\nu}N)_{D+\frac{2}{3}F} + 3\bar{D}_\lambda\sigma_{\mu\nu}D_\lambda\right] + \frac{3}{2m^2}q_\lambda\bar{D}_\lambda\sigma_{\mu\nu}q_\kappa D_\kappa$$
$$+ \frac{i}{4m^2}(P_\mu q_\nu - P_\nu q_\mu)(\bar{N}N)_{\frac{1}{3}F-D} + \frac{i}{m}\epsilon_{\mu\nu\kappa\lambda}(P_\lambda\bar{D}_\kappa N + h.c)$$

$$J^i_{\mu 5} = \frac{P^2}{4m^2}(i\bar{N}\gamma_\mu\gamma_5 N)_{D+\frac{2}{3}F} + \frac{3i}{4m^2}\bar{D}_\lambda\gamma_\mu\gamma_5 D_\lambda$$
$$+ \frac{3i}{2m^2}q_\lambda\bar{D}_\lambda\gamma_\mu\gamma_5 q_\kappa D_\kappa - \frac{iP^2}{2m^2}(\bar{D}_\mu N + h.c.)$$
$$+ \frac{i}{m^2}(p'_\mu q_\lambda\bar{D}_\lambda N + h.c.)$$

$$J^i_5 = \frac{P^2}{4m^2}(\bar{N}\gamma_5 N)_{D+\frac{2}{3}F} + \frac{1}{m}(q_\lambda\bar{D}_\lambda N + h.c)$$
$$+ \frac{3P^2}{4m^2}\bar{D}_\lambda\gamma_5 D_\lambda + \frac{3}{2m^2}q_\lambda\bar{D}_\lambda\gamma_5 q_\kappa D_\kappa$$

$$J_5 = \left(1+\frac{2m}{\mu}\right)\frac{P^2}{4m^2}\left[\left(\bar{N}\gamma_5 N\right)_{D+\frac{2}{3}F} + 3\,\bar{D}_\lambda \gamma_5 D_\lambda\right]$$
$$+ \frac{3}{2m^2}\left(1+\frac{2m}{\mu}\right) q_\lambda \bar{D}_\lambda \gamma_5 q_\kappa D_\kappa + \frac{1}{m}\left(1+\frac{2m}{\mu}\right)\left[q_\lambda \bar{D}_\lambda N + h.c\right]$$

$$J_\mu = \frac{P_\mu}{2m}\left(1+\frac{q^2}{2\mu m}\right)\left(\bar{N}N\right)_F + \left(1+\frac{2m}{\mu}\right)\left(\bar{N}\frac{\gamma_\mu}{4m^2}N\right)_{D+\frac{2}{3}F}$$
$$+ \frac{3P^2}{4m^2}\bar{D}_\lambda\left[\left(1+\frac{2m}{\mu}\right)\gamma_\mu - \frac{P_\mu}{\mu}\right]D_\lambda + \frac{3}{2m^2} q_\lambda \bar{D}_\lambda\left[\left(1+\frac{2m}{\mu}\right)\gamma_\mu - \frac{P_\mu}{\mu}\right] q_\kappa D_\kappa$$
$$- \frac{1}{m^2}\left(1+\frac{2m}{\mu}\right)\left[\epsilon_{\mu\nu\kappa\lambda} P_\nu q_\kappa \bar{D}_\lambda N + h.c\right]$$

For Eq. (6.6) read:

$$J_\mu^i(p,p') = f^{ijk}\left[\;\ldots\;\right]$$
$$+ \frac{i}{\mu} d^{ijk}\left(\ldots\ldots\right)$$

ABSTRACT

A classification of particles is suggested based on a $\tilde{U}(12)$ symmetry scheme. This is a relativistic generalization of the U(6) symmetry. The spin $\frac{1}{2}$ and $^3/2$ baryons are each described by 20-component spinors which satisfy Bargmann-Wigner equations and belong to the $\underset{\sim}{364}$ representation of the $\tilde{U}(12)$ group while the vector and p.s. mesons belong to the representation $\underset{\sim}{143}$. The procedure for writing fully relativistic form factors is worked out in detail for Baryon-Meson and Meson-Meson cases.

The new results are the following:

(1) $\dfrac{F^c(q^2)}{F^M(q^2)} \propto \left(1 + \dfrac{q^2}{2\langle\mu\rangle m}\right)$ where F^c and F^M are (Sachs) electromagnetic form factors.

(2) $\mu_p = 1 + \dfrac{2m}{\langle\mu\rangle}$, where $\langle\mu\rangle$ is the mean mass of the 1^- multiplet.

(3) $\mu_{\rho, K^*} = 3$.

The conventional results can be recovered by projecting to the positive energy sub-space in the rest system for each particle.

1. INTRODUCTION

The problem of finding a relativistic generalization of the U(6) group structure has engaged considerable attention recently [1]. In an earlier paper [1] (I) it was suggested that one way to write relativistic S-matrix elements is to embed U(6) in a $\tilde{U}(12)$ group-structure. The present paper gives the detailed formalism for writing relativistic S-matrix elements in this theory. In particular we compute the two basic baryon-meson and meson-meson form factors. The generalization of the symmetry from U(6) to $\tilde{U}(12)$ gives the following new results:

1. There is essentially just one <u>relativistic</u> form factor in strong interaction physics of the octet baryons and the (1^-) and (0^-) mesons. The relevance of this result to the conventional nucleon electric and magnetic form factors is discussed in Section 6.

2. In Bohr magnetons the magnetic moments of the proton and the neutron are:

$$\mu_p = 1 + 2x$$
$$\mu_n = -\frac{2}{3}(1 + 2x)$$

where $x = \frac{m_N}{\langle\mu\rangle}$ and $\langle\mu\rangle$ is the mean-mass of the (1^-) multiplet. The experimental magnetic moment values are well reproduced if $\langle\mu\rangle \approx 1000$ MeV. This mass-value is not far from the mean of m_ρ, m_ω, m_ϕ, etc.

The formalism describes both the spin $\frac{1}{2}$ and 3/2 baryons as 20-component [2] composite entities made from the basic 4-component (Dirac) quark [3]. In Section 2 we describe the $\tilde{U}(12)$ Algebra; in Section 3 are computed the form factors incorporating full $\tilde{U}(12)$ symmetry. In Section 4 we specialize to the reduced symmetry group U(3) x $\mathcal{L}_+$

(where $\mathcal{L}_4$ refers to the homogeneous Lorentz group), the inhomogeneous Lorentz Group ($I\mathcal{L}_4$)being considered in Section 5. The reduction of the $\widetilde{U}(12)$ symmetry to $U(3) \times (I\mathcal{L}_4)$ in Sections 4 and 5 correctly reproduces the final physical symmetry situation, and gives the relativistic expressions for the baryon form factors in a fundamentally broken U(6) symmetry scheme. The exact U(6) symmetry can be recovered from our expressions in the limit of zero momenta.

2. $\widetilde{U}(12)$ AND ITS SUBGROUPS

We assume that the fundamental entity for strong interactions is a 12-component (Dirac) quark. The group structure $\widetilde{U}(12)$ is defined by the algebra of the 144 matrices $F^{Ri} = \gamma^R T^i$, $R = 1, \ldots, 16;\ i = 0, \ldots. 8$. Here,

$$\gamma^R = 1,\ \gamma_\mu,\ \sigma_{\mu\nu} = \frac{i}{2}[\gamma_\mu, \gamma_\nu],\ i\gamma_\mu\gamma_5,\ \gamma_5$$

with γ_0 hermitian and γ antihermitian and the metric (1, -1, -1, -1). The general $\widetilde{U}(12)$ transformation on the quark field $\psi_A = \psi_{p\alpha}$, ($p = 1,2,3$; $\alpha = 1, 2, 3, 4$) will be assumed to be

$$\delta\psi_{p\alpha} = i\left(\epsilon^j + \epsilon^j_5\gamma_5 + \epsilon^j_\mu\gamma_\mu + i\epsilon^j_{\mu5}\gamma_\mu\gamma_5 + \frac{1}{2}\epsilon^j_{\mu\nu}\sigma_{\mu\nu}\right)^\beta_\alpha \times \times (T^j)^q_p\,\psi_{q\beta} \tag{2.1}$$

where all 144 ϵ's are real and this property leaves $\bar\psi\psi = \psi^\dagger\gamma_0\psi$ invariant.

For higher representations of $\widetilde{U}(12)$ (made up compositively from quarks) the transformation (2.1) will take the form

$$\delta\Psi = i\left(\epsilon^j F^j + \epsilon^j_5 F^j_5 + \epsilon^j_\mu F^j_\mu + \epsilon^j_{\mu5} F^j_{\mu5} + \frac{1}{2}\epsilon^j_{\mu\nu} F^j_{\mu\nu}\right)\Psi \tag{2.2}$$

The general commutation rules of the generators F are listed in the appendix. The quadratic Casimir operator is

$$F^j F^j - F_5^j F_5^j + \frac{1}{2} F_{\mu\nu}^j F_{\mu\nu}^j + F_\mu^j F_\mu^j - F_{\mu 5}^j F_{\mu 5}^j$$

Inspection of the commutators for $\tilde{U}(12)$ reveals that a 72-component subalgebra is generated by the operators F^j, F_5^j, $F_{\mu\nu}^j$. This is the subgroup [1] W(6) and in the fundamental representation has the generators T^j, $\gamma_5 T^j$, and $\sigma_{\mu\nu} T^j$. The expressions

$$F_\mu^j F_\mu^j - F_{\mu 5}^j F_{\mu 5}^j \qquad \text{and}$$

$$F^j F^j - F_5^j F_5^j + \frac{1}{2} F_{\mu\nu}^j F_{\mu\nu}^j$$

are now separately invariant under W(6). Note that W(6) possesses the important 36-parameter sub-group U(6) $(T^i \sigma_{ab}\, ; T^i\ ;\ a, b = 1,2,3)$. This will later be identified with the U(6) of Gürsey, Radicati and Sakita.

3. SOME REPRESENTATIONS OF $\tilde{U}(12)$ AND THEIR DECOMPOSITION

A. The fundamental representation of $\tilde{U}(12)$ is the 12-component quark discussed above. Following the usual procedure we assign to this quark the baryon number B = 1/3. The baryons are then to be constructed from three quark states and the mesons from quark-antiquark states.

These states decompose under $\tilde{U}(12)$ in the following way

$$\begin{aligned} \underline{12} \otimes \underline{12}^* &= \underline{1} + \underline{143} \\ \underline{12} \otimes \underline{12} \otimes \underline{12} &= \underline{220} + \underline{364} + \underline{572} + \underline{572} \end{aligned} \qquad (3.1)$$

The 220 is completely antisymmetrical, the 364 completely symmetrical and the 572 is of the mixed symmetry type [2,1]

Under the subgroup W(6) these states reduce according to

$$\begin{aligned}
\underline{143} &= (35,1) + (6,6^*) + (6^*,6) + (1,35) + (1.1) \\
\underline{220} &= (20,1) + (15,6) + (6,15) + (1,20) \\
\underline{364} &= (56,1) + (21,6) + (6,21) + (1,56) \\
\underline{572} &= (70,1) + (21,6) + (6,21) + (1,70)
\end{aligned} \tag{3.2}$$

Under the subgroup SU(3)$\otimes$ SU(4), where SU(3) refers to the space of unitary spin matrices T^j, and SU(4) to that of the Dirac matrices γ^R, the contents are given by

$$\begin{aligned}
\underline{143} &= (8,15) + (1,15) + (8,1) \\
\underline{220} &= (8,20) + (10,4) + (1,20) \\
\underline{364} &= (10,20') + (8,20'') + (1,4) \\
\underline{572} &= (10,20'') + (8,20) + (8,20') + (1,20'') + (8,4)
\end{aligned} \tag{3.3}$$

where in each bracket the first number denotes the SU(3) representation and the second the SU(4) representation. The three SU(4) representations denoted here by $\underline{20}$, $\underline{20}'$ and $\underline{20}''$ are of symmetry types $[1^3]$, $[3]$ and $[2,1]$ respectively.

We note also the reduction of the products

$$\begin{aligned}
\underline{143} \otimes \underline{143} &= \underline{1} + \underline{143}_F + \underline{143}_D + \underline{4212} + \underline{5005} + \underline{5005}^* + \underline{5940} \\
\underline{364} \otimes \underline{364}^* &= \underline{1} + \underline{143} + \underline{5940} + \underline{126412} \\
\underline{143} \otimes \underline{364} &= \underline{364} + \underline{572} + \underline{16016} + \underline{35100}
\end{aligned} \tag{3.4}$$

B. Since it is our intention to assign the baryon to the $\underline{364}$, we compute the expectation values of the 144-vector $(\gamma^R T^j)_A^B$ between 364 states, namely

$$J^{Ri} = \bar{\Psi}^{ABC} (\gamma^R T^i)_A^{A'} \Psi_{A'BC} \tag{3.5}$$

where Ψ_{ABC} is fully symmetric and has the SU(3) x SU(4) decomposition

$$\Psi_{\alpha p \beta q \gamma r} = D_{\alpha\beta\gamma, pqr} + \epsilon_{pqr} V_{[\alpha\beta\gamma]} + \frac{1}{2\sqrt{6}} \left(\epsilon_{pqs} N_{[\alpha\beta]\gamma,r}^{\;\;s} + \epsilon_{qrs} N_{[\beta\gamma]\alpha,p}^{\;\;s} + \epsilon_{rps} N_{[\gamma\alpha]\beta,q}^{\;\;s} \right) \tag{3.6}$$

Here (α, β, γ) take the values 1, 2, 3, 4 and (p, q, r) the values 1, 2, 3. $V_{[\alpha\beta\gamma]}$ is completely antisymmetric, $D_{\alpha\beta\gamma, pqr}$ is completely symmetric in both $\alpha\beta\gamma$ and pqr and $N_{[\alpha\beta]\gamma, r}^{\;\;s}$ is of the symmetry type $[2,1]$, i.e.

$$\begin{aligned} N_{[\alpha\beta]\gamma} + N_{[\beta\alpha]\gamma} &= 0 \;, \\ N_{[\alpha\beta]\gamma} + N_{[\beta\gamma]\alpha} + N_{[\gamma\alpha]\beta} &= 0 \end{aligned} \tag{3.7}$$

After some algebra we find (specializing to $i \neq 0$ so that $(T^i)_p^p = 0$),

$$\begin{aligned} J^{Ri} = {} & \bar{D}^{\alpha\beta\gamma, pqr} (\gamma^R)_\alpha^{\alpha'} (T^i)_p^{p'} D_{\alpha'\beta\gamma, p'qr} \\ & + \frac{1}{\sqrt{6}} \Big[\bar{D}^{\alpha\beta\gamma} (\gamma^R)_\alpha^{\alpha'} (T^i)_p^{p'} \epsilon_{p'qs} N_{[\alpha'\beta]\gamma, r}^{\;\;s} + \\ & \qquad + \bar{N}^{[\alpha\beta]\gamma, r}_{\;\;s} \epsilon^{pqs} (\gamma^R)_\alpha^{\alpha'} (T^i)_p^{p'} D_{\alpha'\beta\gamma, p'qr} \Big] \\ & + \Big[\bar{V}^{[\alpha\beta\gamma]} (\gamma^R)_\alpha^{\alpha'} (T^i)_p^{p'} N_{[\beta\gamma]\alpha', p'}^{\;\;p} + \bar{N}^{[\beta\gamma]\alpha, p}_{\;\;p'} (T^i)_p^{p'} (\gamma^R)_\alpha^{\alpha'} V_{[\alpha'\beta\gamma]} \Big] \\ & - \frac{1}{24} \left(\bar{N}^{[\beta\alpha]\gamma} (\gamma^R)_\alpha^{\alpha'} N_{[\alpha'\beta]\gamma} \right)^i_{3D+5F} \\ & + \frac{1}{12} \left(\bar{N}^{[\beta\alpha]\gamma} (\gamma^R)_\alpha^{\alpha'} N_{[\alpha'\gamma]\beta} \right)^i_{3D+2F} \end{aligned} \tag{3.8}$$

where

$$(\bar{N}N)^i_F = \bar{N}^p_r (T^i)^q_p N^r_q - \bar{N}^p_r N^q_p (T^i)^r_q$$
$$(\bar{N}N)^i_D = \bar{N}^p_r (T^i)^q_p N^r_q + \bar{N}^p_r N^q_p (T^i)^r_q \qquad (3.9)$$

Notice the appearance at this stage of the characteristic combinations 3D + 2F and 3D + 5F for the form factors involving the 8-fold baryons.

4 . REDUCTION OF $\tilde{U}(4)$ TO THE HOMOGENEOUS LORENTZ GROUP $\mathcal{L}_4$.

A . We now specifically consider the space-time symmetries. So far it has been assumed that in its space-time behaviour, the fundamental 4-component $\tilde{U}(4)$ entity ψ_α transforms as

$$\psi_\alpha \rightarrow S^\beta_\alpha \psi_\beta$$

where

$$S^\beta_\alpha = 1 + i\,(\epsilon_5\gamma_5 + \epsilon_\mu\gamma_\mu + i\epsilon_{\mu 5}\gamma_\mu\gamma_5 + \tfrac{1}{2}\epsilon_{\mu\nu}\sigma_{\mu\nu})^\beta_\alpha\,\psi_\beta \qquad (4.1)$$

with $\bar{\psi}$ transforming as $\bar{\psi}^\alpha \rightarrow \bar{\psi}^\beta [S^{-1}]^\alpha_\beta$

With ϵ 's real, these transformations preserve the invariance of $\bar{\psi}^\alpha \psi_\alpha$. The higher representations $\Psi^{\gamma\delta\cdots}_{\alpha\beta\cdots}$ of $\tilde{U}(4)$ transform as

$$\Psi^{\gamma\delta\cdots}_{\alpha\beta\cdots} \rightarrow S^{\alpha'}_\alpha S^{\beta'}_\beta \cdots (S^{-1})^\gamma_{\gamma'} (S^{-1})^\delta_{\delta'} \cdots \qquad (4.2)$$

The symmetry represented by (4.1), however, is too general.

In space-time terms it corresponds, as is well known, to the full conformal group symmetry C_4. To make contact, however, with physical space-time symmetries of (at this stage) the homogeneous Lorentz group we must descend from $\widetilde{U}(4)$ to $\mathcal{L}_4$. ere are a number of ways of doing this which are not all necessarily equivalent so far as the underlying physics is concerned as will be discussed in Section 5.

Disregarding the problems connected with unitary spin, clearly the most direct symmetry reduction is achieved of course by taking

$$\epsilon_5 = \epsilon_\mu = \epsilon_{\mu 5} = 0$$

Now as is well-known for this case (though not for $\widetilde{U}(4)$) one can define an anti-symmetric matrix $(C^{-1})^{\alpha\beta}$ (within the Dirac Algebra) with the defining property that $C^{-1}\psi^T$ transforms similarly to $\bar{\psi}$. (Here ψ^T is the transpose of ψ.) In particular, $C^{-1}\psi^T\psi$ just like $\bar{\psi}\psi$ is an invariant. Clearly from the definition above, the anti-symmetric matrix $C^{-1} = (C^{-1})^{\alpha\beta} = -(C^{-1})^{\beta\alpha}$ plays the role for $\mathcal{L}_4$ of the metric tensor. We may regard $(C^{-1})^{\alpha\beta}$ as a contra-variant quantity (with two upper indices) and its inverse $C_{\alpha\beta}$ as the corresponding co-variant ($C_{\alpha\beta}(C^{-1})^{\beta\gamma} = \delta_\alpha^\gamma$).

It is easy to show [4] that the matrix C with the defining property above [5] can be realized by finding a matrix satisfying

$$(\gamma_\mu C)_{\alpha\beta} = (\gamma_\mu C)_{\beta\alpha}$$

where $$(\gamma_\mu C)_{\alpha\beta} = (\gamma_\mu)_\alpha^\gamma C_{\gamma\beta}$$

It is also easy to show that the 16 Dirac matrices $(\gamma^R C)_{\alpha\beta}$ fall into two distinct classes; the matrices $(\gamma_\mu C)_{\alpha\beta}$ and $(\sigma_{\mu\nu} C)_{\alpha\beta}$ are symmetric, and $C_{\alpha\beta}$, $(\gamma_5 C)_{\alpha\beta}$, $(i\gamma_\mu\gamma_5 C)_{\alpha\beta}$ are anti-symmetric. For writing symmetric and anti-symmetric higher rank "spinors" in $\mathcal{L}_4$ these are the primary quantities one needs.

To illustrate consider two examples:

(1) Multi-Spinor of Rank 2

A second-rank symmetric spinor must have the form:

$$\Phi_{\{\alpha\beta\}} = \left[(\gamma_\mu C)\phi_\mu + \tfrac{1}{2}(\sigma_{\mu\nu} C)\phi_{\mu\nu}\right]_{\alpha\beta} \tag{4.3}$$

Likewise the general anti-symmetric spinor has the form:

$$\Phi_{[\alpha\beta]} = \left[C\phi + (\gamma_5 C)\phi_5 + i(\gamma_\mu\gamma_5 C)\phi_{\mu 5}\right]_{\alpha\beta} \tag{4.4}$$

(2) Fully Symmetric Spinor of Rank 3

Consider $\Psi_{\alpha\beta\gamma}$ with full symmetry in α, β, γ. From symmetry in γ and β, one may write Ψ in the form

$$\Psi_{\alpha\beta\gamma} = \psi_{\alpha\mu}(\gamma_\mu C)_{\beta\gamma} + \tfrac{1}{2}\psi_{\alpha\mu\nu}(\sigma_{\mu\nu} C)_{\beta\gamma} \tag{4.5}$$

We now show that full symmetry in α, β, γ is realized provided

$$\gamma_\mu \psi_\mu = 0 \tag{4.6}$$

$$\gamma_\mu \psi_{\mu\nu} + i\psi_\nu = 0 \tag{4.7}$$

For, the three anti-symmetric tensors $(C^{-1})^{\gamma\alpha}$, $(C^{-1}\gamma_5)^{\gamma\alpha}$ and $(iC^{-1}\gamma_\mu\gamma_5)^{\gamma\alpha}$ must annihilate $\Psi_{\alpha\beta\gamma}$.

This gives:

$$\psi_{\alpha\mu}(\gamma_\mu)^\alpha_\beta + \tfrac{1}{2}\psi_{\alpha\mu\nu}(\sigma_{\mu\nu})^\alpha_\beta = 0$$

$$\psi_{\alpha\mu}(\gamma_\mu\gamma_5)^\alpha_\beta + \tfrac{1}{2}\psi_{\alpha\mu\nu}(\sigma_{\mu\nu}\gamma_5)^\alpha_\beta = 0$$

$$\psi_{\alpha\mu}(\gamma_\mu\gamma_\lambda\gamma_5)^\alpha_\beta + \tfrac{1}{2}\psi_{\alpha\mu\nu}(\sigma_{\mu\nu}\gamma_\lambda\gamma_5)^\alpha_\beta = 0$$

Suppressing Dirac indices, the first two equations give (4.6) and $\sigma_{\mu\nu}\psi_{\mu\nu} = 0$; and the last is equivalent to (4.7). Note that as a result of (4.6) and (4.7) the 40-component entity on the right side of (4.5) contains only 20 independent components.

(3) Mixed Spinor of Rank 3

The 20-component tensor $\Psi_{[\alpha\beta]\gamma}$ which satisfies the "trace" condition [6]

$$\Psi_{[\alpha\beta]\gamma} + \Psi_{[\gamma\alpha]\beta} + \Psi_{[\beta\gamma]\alpha} = 0 \qquad (4.8)$$

can necessarily be written in the form

$$\Psi_{[\alpha\beta]\gamma} = (\gamma_5 C)_{\alpha\beta}\psi_\gamma + i(\gamma_\mu\gamma_5 C)_{\alpha\beta}\psi_\mu + C_{\alpha\beta}K_\gamma \qquad (4.9)$$

The mixed symmetry character of Ψ yields the constraint

$$\gamma_5\psi + i\gamma_\mu\gamma_5\psi_\mu - K = 0 \qquad (4.10)$$

This follows on multiplying (4.8) by $(C^{-1})^{\alpha\beta}$.

B. To return to $\widetilde{U}(12)$, we now decompose all irreducible $\widetilde{U}(12)$ higher representations relative to the representations of $U(3)\otimes\mathcal{L}_4$, maintaining the over-all symmetry. Thus for the (8 x 20") part of the 364 representation, we write

$$N_{[\alpha\beta]\gamma,p}^{\;q} = C_{\alpha\beta}K_{\gamma.p}^{\;q} + (\gamma_5 C)_{\alpha\beta}N_{\gamma,p}^{\;q} + (i\gamma_\mu\gamma_5 C)_{\alpha\beta}N_{\gamma\mu,p}^{\;q}$$

The contribution of $N_{[\alpha\beta]\gamma,p}^{\;q}$ to the currents (3.8), namely

$$J^{Ri}(N) = -\frac{1}{24}\left(\overline{N}^{[\beta\alpha]\gamma}(\gamma^R)_\alpha^{\alpha'}N_{[\alpha'\beta]\gamma}\right)_{3D+5F} + \frac{1}{12}\left(\overline{N}^{[\beta\alpha]\gamma}(\gamma^R)_\alpha^{\alpha'}N_{[\alpha'\gamma]\beta}\right)_{3D+2F}$$

may be now written out in terms of N and N_μ. Thus

$$J^i(N) = \frac{1}{2}\left[2\bar{N}N + i\left(\bar{N}_\mu\gamma_\mu N - \bar{N}\gamma_\mu N_\mu\right) + \bar{N}_\lambda\gamma_\lambda\gamma_\mu N_\mu + \bar{N}_\mu N_\mu\right]_F$$

$$J^i_\mu(N) = \frac{i}{6}\left(\bar{N}_\mu N - \bar{N}N_\mu\right)^i_{F-3D} - \frac{1}{6}\left[i\left(\bar{N}_\lambda\gamma_\lambda\gamma_\mu N - \bar{N}\gamma_\mu\gamma_\lambda N_\lambda\right) - \bar{N}_\lambda\gamma_\mu N_\lambda + \bar{N}_\lambda\gamma_\lambda\gamma_\mu\gamma_\nu N_\nu\right]^i_{3D+2F}$$

$$J^i_{\mu\nu}(N) = \frac{i}{6}\left(\bar{N}_\mu N_\nu - \bar{N}_\nu N_\mu\right)^i_{F-3D} + \frac{1}{6}\left[2\bar{N}\sigma_{\mu\nu}N + i\left(\bar{N}_\lambda\gamma_\lambda\sigma_{\mu\nu}N - \bar{N}\sigma_{\mu\nu}\gamma_\lambda N_\lambda\right) + \bar{N}_\lambda\sigma_{\mu\nu}N_\lambda + \bar{N}_\lambda\gamma_\lambda\sigma_{\mu\nu}\gamma_\kappa N_\kappa\right]^i_{3D+2F}$$

$$J^i_{\mu 5}(N) = -\frac{1}{6}\left(\bar{N}\gamma_5 N_\mu + \bar{N}_\mu\gamma_5 N + i\bar{N}_\lambda\gamma_\lambda\gamma_5 N_\mu - i\bar{N}_\mu\gamma_5\gamma_\lambda N_\lambda\right)^i_{3D+5F}$$
$$-\frac{1}{6}\left[\begin{array}{l} 2\left(\bar{N}_\mu\gamma_5 N + \bar{N}\gamma_5 N_\mu\right) + \left(\bar{N}_\lambda\gamma_\lambda\gamma_5\gamma_\mu N + \bar{N}\gamma_\mu\gamma_5\gamma_\lambda N_\lambda\right) - \\ - 2i\left(\bar{N}_\mu\gamma_5\gamma_\lambda N_\lambda - \bar{N}_\lambda\gamma_\lambda\gamma_5 N_\mu\right) - i\bar{N}_\lambda\gamma_\mu\gamma_5 N_\lambda + \\ + i\bar{N}_\lambda\gamma_\lambda\gamma_\mu\gamma_5\gamma_\nu N_\nu \end{array}\right]^i_{3D+2F}$$

$$J_5^i(N) = -\frac{1}{6}\left(2\bar{N}\gamma_5 N + i\bar{N}_\mu\gamma_\mu\gamma_5 N - i\bar{N}\gamma_5\gamma_\mu N_\mu\right)^i_{6D+7F} + \frac{1}{6}\left(\bar{N}_\lambda\gamma_5 N_\lambda + \bar{N}_\lambda\gamma_\lambda\gamma_5\gamma_\mu N_\mu\right)^i_{3D+2F}$$

Note now that already the sacrosanct combinations 3D + 2F and 3D + 5F of U(12) have disappeared, being replaced by their various linear combinations.

5 THE INHOMOGENEOUS LORENTZ GROUP AND THE FINAL EXPRESSIONS FOR THE FORM FACTORS

A. The work so far has been concerned with purely static considerations. We have computed the expectation values of matrices γ^R between (the homogeneous Lorentz group) multi-spinors of various symmetry properties. The formalism can have no physical content till these spinors are made to represent physical particles, i.e., till the formalism assures that they correspond to the representations of the inhomogeneous Poincaré group. One needs therefore, at this stage, some equations of motion which the spinors $\Psi_{\alpha\beta\gamma\ldots}$ must satisfy. This essentially is the point of departure of our work in comparison with other approaches to the problem and was stressed strongly [illegible]

Among the variety of higher spin equations available, we shall choose the simplest, and the least restrictive (though in many ways the most profound) set of equations: We generalize the Bargmann-Wigner[7] approach to the representations of the inhomogeneous Lorentz group. The approach works with the equations

$$(\not{p})_{\alpha}^{\alpha'} \Psi_{\alpha'\beta\gamma\ldots}(p) = m \Psi_{\alpha\beta\gamma\ldots}(p)$$

$$(\not{p})_{\beta}^{\beta'} \Psi_{\alpha\beta'\gamma\ldots}(p) = m \Psi_{\alpha\beta\gamma\ldots}(p) \quad . \qquad (5.1)$$

These describe particles of (a) one definite mass m, (b) one definite spin (provided $\Psi_{\alpha\beta\gamma\ldots}$ is a spinor of a definite symmetry type), (c) the solutions of the equations pose no problems of negative energies or indefinite metrics and (d) the higher spinors transform "visibly" as direct products of the fundamental quark which satisfies Dirac's equation $\not p \psi = \kappa \psi$.

We now examine the implications of applying these equations to the spinors of rank 2 and 3 considered previously:

(1) <u>Spinor of Rank 2</u>

Write

$$\Phi_\alpha^\beta = \left[\phi + \gamma_5 \phi_5 + i\gamma_\mu \gamma_5 \phi_{\mu 5} + \gamma_\mu \phi_\mu + \tfrac{1}{2} \sigma_{\mu\nu} \phi_{\mu\nu} \right]_\alpha^\beta \qquad (5.2)$$

From

$$(\not p)_\alpha^{\alpha'} \Phi_{\alpha'}^\beta = m \Phi_\alpha^\beta \ , \quad (\not p)_{\beta'}^\beta \Phi_\alpha^{\beta'} = m \Phi_\alpha^\beta$$

we deduce

$$\begin{aligned} &\phi = 0 \\ &p_\mu \phi_5 = i m \phi_{\mu 5} \ , \quad p_\mu \phi_{\mu 5} = -i m \phi_5 \\ &p_\mu \phi_\nu - p_\nu \phi_\mu = i m \phi_{\mu\nu} \ , \quad p_\nu \phi_{\nu\mu} = - i m \phi_\mu \end{aligned} \qquad (5.3)$$

Thus (ϕ_5, $\phi_{\mu 5}$) together describe [8] a 0^- particle [9] and (ϕ_μ, $\phi_{\mu\nu}$) describe a 1^- particle. The relation of the second-rank spinor with what is essentially the Kemmer theory of spin zero and spin one particles was first pointed out by Belinfante.[10] Note that the assumption that the tensor $\Phi_\alpha^\beta(p)$ transforms as a quark-antiquark composite $\psi_\alpha(p)\,\bar\psi^\beta(p)$ (with no relative momentum) fixes the parities of the mesons unambiguously [9].

(2) Fully Symmetric Spinor of Rank 3

On account of full symmetry the three equations can be collapsed into a single equation

$$(\not{p})_{\alpha}^{\alpha'} \Psi_{\alpha'\beta\gamma} = m \Psi_{\alpha\beta\gamma}$$

Substituting the expression (4.5) into this equation and contracting it with $(C^{-1}\gamma_{\mu})^{\beta\gamma}$, $(C^{-1}\gamma_{\mu})^{\alpha\beta}$ and $(C^{-1}\sigma_{\mu\nu})^{\alpha\beta}$ we find

$$(\not{p} - m)\psi_{\mu} = 0 \tag{5.4}$$

$$p_{\nu}\psi_{\nu\mu} = -im\psi_{\mu}, \quad p_{\mu}\psi_{\nu} - p_{\nu}\psi_{\mu} = im\psi_{\mu\nu} \tag{5.5}$$

It is simple to show, moreover, that ψ_{μ}, $\psi_{\mu\nu}$ satisfying (4.6) and (4.7) give a fully symmetrical $\Psi_{\alpha\beta\gamma}$. Thus the system is entirely equivalent to the Rarita-Schwinger formalism [11] for a particle of spin $\frac{3}{2}^{+}$

(3) Mixed Spinor of Rank 3

Applied to Eq. (4.9), the first equation of motion

$$(\not{p})_{\gamma}^{\gamma'} \Psi_{\alpha\beta\gamma'} = m \Psi_{\alpha\beta\gamma}$$

gives simply

$$(\not{p} - m)\psi_{\mu} = (\not{p} - m)\psi = (\not{p} - m)K = 0 \tag{5.6}$$

while the other pair of equations $(\not{p})_{\alpha}^{\alpha'} \Psi_{\alpha'\beta\gamma} = m\Psi_{\alpha\beta\gamma}$ give the relations

$$\begin{aligned} K &= 0, \quad p_{\mu}\psi_{\nu} - p_{\nu}\psi_{\mu} = 0 \\ p_{\mu}\psi &= im\psi_{\mu}, \quad p_{\mu}\psi_{\mu} = -im\psi. \end{aligned} \tag{5.7}$$

Taken together the system clearly describes a particle of spin $\frac{1}{2}^+$.

3. With the Bargmann-Wigner equations our identification with physical particles of the $\tilde{U}(12)$ quantities Φ^A_B and Ψ_{ABC} is complete.

We here summarize the results:

(1) The regular representation 12 x 12* decomposes as:

$$\Phi^A_B = \left(\dot{\phi}^j + \gamma_5 \dot{\phi}^j_5 + i\gamma_\mu\gamma_5 \dot{\phi}^j_{\mu 5} + \gamma_\mu \dot{\phi}^j_\mu + \frac{1}{2}\sigma_{\mu\nu}\dot{\phi}^j_{\mu\nu}\right)^\alpha_\beta (T^i)^p_q$$

If the mesons are free with mass m, the implication of the equations of motion are:

$$p_\mu \dot{\phi}^j_5 = im\,\dot{\phi}^j_{\mu 5} \quad , \quad p_\mu \dot{\phi}^j_{\mu 5} = -im\,\dot{\phi}^j_5 \qquad \text{for } (0^-) \text{ particles}$$

$$p_\mu \dot{\phi}^j_\nu - p_\nu \dot{\phi}^j_\mu = im\,\dot{\phi}^j_{\mu\nu} \quad , \quad p_\nu \dot{\phi}^j_{\nu\mu} = -im\,\dot{\phi}^j_\mu \qquad \text{for } (1^-) \text{ particles}$$

$$\dot{\phi}^j \equiv 0 \qquad \text{for } (0^+)$$

Thus spin zero particles are represented by 5-component entities; spin one by ten components and

144 = 143 + 1 = 9 x 10 + 9 x 5 + 9

The last nine are the so-called trivial components.

(2) <u>Rank 3</u>

The 364 components of the fully symmetric $\tilde{U}(12)$ tensor Ψ_{ABC} decompose as

$\underline{364}$ = (10 , 20) + (8 , 20) + (1 , 4)

In detail:

$$\Psi_{\alpha p \beta q \gamma r} = D_{\alpha\beta\gamma, pqr} + \epsilon_{pqr} V_{[\alpha\beta\gamma]} + $$
$$+ \frac{1}{2\sqrt{6}} \left(\epsilon_{pqs} N^{s}_{[\alpha\beta]\gamma, r} + \epsilon_{qrs} N^{s}_{[\beta\gamma]\alpha, p} + \epsilon_{rps} N^{s}_{[\gamma\alpha]\beta, q} \right)$$

where D is completely symmetric both in its spinor and unitary spin indices, $V_{[\alpha\beta\gamma]}$ is completely anti-symmetric and $N^{s}_{[\alpha\beta]\gamma, r}$ has mixed symmetry in spinor indices and is traceless in unitary spin indices.

The equations of motion ensure that D (with its 20 components) describes a particle of spin 3/2, N (with its 20 components) a particle of spin $\frac{1}{2}$ and V vanishes identically because of complete anti-symmetry. The relative parities of the decimet and the octet are the same (and for the quark ψ transforming as $P\psi P^{-1} = \gamma_0 \psi$ the same as the quark). The detailed consequences of the equations are to allow us to write D and N in the forms:

$$D_{\alpha\beta\gamma, pqr}(p) = (\gamma_\mu C)_{\alpha\beta} D_{\gamma\mu, pqr}(p) +$$
$$+ \frac{i}{2m} (\sigma_{\mu\nu} C)_{\alpha\beta} \left[p_\mu D_{\gamma\nu, pqr}(p) - p_\nu D_{\gamma\mu, pqr}(p) \right]$$

with

$$(\not p - m) D_{\mu, pqr}(p) = 0 , \quad \gamma_\mu D_{\mu, pqr}(p) = 0 \qquad (5.8)$$

and for the eight-fold baryon N:

$$m N^{s}_{[\alpha\beta]\gamma, r}(p) = [(\not p + m)\gamma_5 C]_{\alpha\beta} N^{s}_{\gamma, r}(p)$$

$$(\not p - m) N^{s}_{r}(p) = 0 \qquad (5.9)$$

It is important to remember that the $\tilde{U}(12)$ multiplets (which were decomposed relative to U(3) x $\mathcal{L}_4$ before the Bargmann-Wigner equations were applied) now no longer possess the full symmetry.[12] The exact U(6) limit, however, can still be recovered, by going for every particle to its rest frame ($\underset{\sim}{p} = 0$) and projecting out to the positive energy subspace.[13] The demands of "relativistic completion" are incompatible with exact symmetry at any (but zero) momenta.

One point is worth emphasizing again at this stage. With the Bargmann-Wigner equations the 364 multiplet of $\tilde{U}(12)$ has exactly the content of the 56 of U(6); likewise for the 143 of $\tilde{U}(12)$ which corresponds with the 35 of U(6); no more and no less. This will happen for all multiplets. In a future paper we shall treat the 572 multiplet and show how its Algebra provides the relativistic completion of the 70 of U(6) in a one-one manner.

C. With (5.8) and (5.9) inserted into (3.8) and p and p' denoting the incoming and outgoing baryon momenta, we can now give the final expressions for the form factors. These are

$$J^i = \tfrac{1}{4} m^{-2} P^2 \left[(\bar{N} N)_F + \bar{D}_\lambda D_\lambda \right] + \tfrac{1}{2} m^{-2} q_\lambda \bar{D}_\lambda q_\kappa D_\kappa \tag{5.10}$$

$$\begin{aligned} J^i_\mu = {} & \tfrac{1}{2} m^{-1} P_\mu (\bar{N} N)_F + \tfrac{1}{4} m^{-2} (\bar{N} \gamma_\mu N)_{D+\frac{2}{3}F} + \\ & + (3)^{-\frac{1}{2}} m^{-1} (q_\lambda \bar{D}_\lambda N + h.c) + \\ & + \tfrac{1}{4} m^{-2} P^2 \bar{D}_\lambda \gamma_\mu D_\lambda + \tfrac{1}{2} m^{-2} q_\lambda \bar{D}_\lambda \gamma_\mu q_\kappa D_\kappa \end{aligned} \tag{5.11}$$

$$\begin{aligned} J^i_{\mu\nu} = {} & \tfrac{1}{4} m^{-2} P^2 \left[(\bar{N} \sigma_{\mu\nu} N)_{D+\frac{2}{3}F} + \bar{D}_\lambda \sigma_{\mu\nu} D_\lambda \right] + \\ & + \tfrac{i}{2} m^{-2} q_\lambda \bar{D}_\lambda \gamma_\mu q_\kappa D_\kappa + \tfrac{i}{4} m^{-2} (P_\mu q_\nu - P_\nu q_\mu)(\bar{N} N)_{\frac{1}{3}F-D} \\ & + i (3)^{-\frac{1}{2}} m^{-1} (\epsilon_{\mu\nu\kappa\lambda} q_\lambda \bar{D}_\kappa N + h.c.) \end{aligned} \tag{5.12}$$

$$J^{i}_{\mu 5} = \tfrac{1}{4} m^{-2} P^2 \left(i \bar{N} \gamma_\mu \gamma_5 N \right)_{D+\frac{2}{3}F} + $$
$$+ \tfrac{i}{4} m^{-2} P^2 \bar{D}_\lambda \gamma_\mu \gamma_5 D_\lambda + \tfrac{i}{2} m^{-2} q_\lambda \bar{D}_\lambda \gamma_\mu \gamma_5 q_\nu D_\nu -$$
$$- \tfrac{1}{2} (3)^{-\frac{1}{2}} m^{-2} q^2 \left(\bar{D}_\mu N + h.c \right) - 2i (3)^{-\frac{1}{2}} m^{-2} \left(p'_\mu p_\lambda \bar{D}_\lambda N + h.c \right) \quad (5.13)$$

$$J^{i}_{5} = \tfrac{1}{4} m^{-2} P^2 \left(\bar{N} \gamma_5 N \right)_{D+\frac{2}{3}F} + (3)^{-\frac{1}{2}} m^{-1} \left(q_\lambda \bar{D}_\lambda N + h.c \right) +$$
$$+ \tfrac{1}{4} m^{-2} P^2 \bar{D}_\lambda \gamma_5 D_\lambda + \tfrac{1}{2} m^{-2} q_\lambda \bar{D}_\lambda \gamma_5 q_\kappa D_\kappa \quad (5.14)$$

where $P = p + p'$, $q = p - p'$; $r_\mu \equiv \epsilon_{\mu\nu\kappa\lambda} P_\nu q_\kappa \gamma_\lambda \gamma_5$

Note that the coefficient multiplying $P_\mu / 2m$ and $r_\mu / 4m^2$ are the Sachs form factors[14] F_C and F_M respectively.

To recover the conventional results for the form factors[15] take $\not{p} = \not{p}' = 0$.

If we further make the identifications

$$i\mu \phi^{i}_{\kappa\lambda} = p_\kappa \phi^{i}_\lambda - p_\lambda \phi^{i}_\kappa$$
$$i\mu \phi^{i}_{\lambda 5} = p_\lambda \phi^{i}_5 , \quad \mu = \text{meson mass},$$

for the meson field interaction $\Phi_A^B J_B^A$, we arrive at the following predictions for the pseudoscalar and vector currents.

$$J_5 = \left(1 + \frac{2m}{\mu} \right) \frac{P^2}{4m^2} \left[\left(\bar{N} \gamma_5 N \right)_{D+\frac{2}{3}F} + \bar{D}_\lambda \gamma_5 D_\lambda \right] +$$
$$+ \frac{1}{2m^2} \left(1 + \frac{2m}{\mu} \right) q_\lambda \bar{D}_\lambda \gamma_5 q_\kappa D_\kappa + \frac{1}{m (3)^{\frac{1}{2}}} \left(1 + \frac{2m}{\mu} \right) \left[q_\lambda \bar{D}_\lambda N + h.c \right] \quad (5.15)$$

$$J_\mu = \frac{P_\mu}{2m}\left(1+\frac{q^2}{2\mu m}\right)(\bar{N}N)_F + \left(1+\frac{2m}{\mu}\right)\left(\bar{N}\frac{\gamma_\mu}{4m^2}N\right)_{D+\frac{2}{3}F} +$$
$$+ \frac{P^2}{4m^2}\bar{D}_\lambda\left[\left(1+\frac{2m}{\mu}\right)\gamma_\mu - \frac{P_\mu}{\mu}\right]D_\lambda +$$
$$+ \frac{1}{2m^2} q_\lambda \bar{D}_\lambda\left[\left(1+\frac{2m}{\mu}\right)\gamma_\mu - \frac{P_\mu}{\mu}\right] q_\kappa D_\kappa -$$
$$- \frac{1}{m^2(3)^{\frac{1}{2}}}\left(1+\frac{2m}{\mu}\right)\left[\epsilon_{\mu\nu\kappa\lambda} P_\nu q_\kappa \bar{D}_\lambda N + h.c\right] \qquad (5.16)$$

6. RESULTS AND CONCLUSIONS

As shown in Section 5 C. the baryon-meson (N^+NM) vertex contains just one form factor. Its relation with the electric and the magnetic form factors introduced by Sachs and Wali is obvious. Assuming that the photon ($F_{\mu\nu}$) couples with the composite structure of the baryon[16] through an effective gauge-invariant "interaction" $f(q^2)F_{\mu\nu}M^{u}_{\mu\nu}$ where $M^{u}_{\mu\nu}$ is the U-scalar meson combination ($M^{u} = M^3 + (3)^{-\frac{1}{2}}M^8$), one would get for the electromagnetic form factors the expressions

$$J_\mu^{e} = \frac{P_\mu}{2m}\left(1+\frac{q^2}{2\mu m}\right)(\bar{N}N)_F \; F(q^2) \qquad (6.1)$$

$$J_\mu^{M} = \left(1+\frac{2m}{\mu}\right)\left(\bar{N}\frac{\gamma_\mu}{4m^2}N\right)_{D+\frac{2}{3}F} \qquad (6.2)$$

where $F(q^2) \propto f(q^2)/(q^2-\mu^2)$

As stated in the introduction we note that

(i) $\mu_p = \left(1+\frac{2m}{\mu}\right)$ (Bohr magnetons)

$$\mu_n = -\frac{2}{3}\left(1+\frac{2m}{\mu}\right) \qquad (6.3)$$

(2) $$\frac{J^C}{J^M} = \frac{1 + q^2/2\mu m}{1 + 2m/\mu} \qquad (6.4)$$

For small q^2 essentially there is therefore just one electromagnetic (Sachs) form factor.

(3) The U_6 limit of $\tilde{U}(12)$ taken at the stage of equations (5.10) - (5.14) results in losing the "anomalous" magnetic moment of the proton. Stated otherwise, the extension of the (U(6)) 35-fold of mesons to a ($\tilde{U}(12)$) 143-fold is essential to obtain the results (6.3).

(4) For weak interactions (5.11) and (5.13) give the vector and axial-vector form factors. These reproduce the well-known U(6) result $g_A/g_V = -5/3$ at zero momentum transfer. $\tilde{U}(12)$ does not seem to improve on it except to indicate that the problem lies more at the level of the effective weak interaction vertex $(f_1(q^2)\, W_\mu M_\mu + f_2(q^2) W_{\mu\nu} M_{\mu\nu})$ and with the precise forms of $f_1(q^2)$ and $f_2(q^2)$.

(5) <u>Meson-Meson Vertex</u>

In general <u>143</u> x <u>143</u> contains <u>143</u> twice but for the special case of identical <u>143</u>'s, there is just one coupling of the type

$$\mathcal{L} = \Phi_A^B \bar{\Phi}_B^C \bar{\Phi}_C^A$$

Written out in terms of ϕ_5^i, ϕ_μ^i, etc., introduced in Eq. (5.2), $\mathcal{L}$ equals

$$f^{ijk}\left[3\phi_\mu^i \phi_\nu^j \phi_{\mu\nu}^k + 3\phi_{\mu 5}^i \phi_{\mu\nu}^j \phi_{\nu 5}^k + 6\phi_5^i \phi_\mu^j \phi_{\mu 5}^k + \phi_{\mu\nu}^i \phi_{\nu\lambda}^j \phi_{\lambda\mu}^k\right]$$

$$+ \frac{3}{2} d^{ijk} \epsilon_{\kappa\lambda\mu\nu}\left[\phi_\kappa^i \phi_{\lambda\mu}^j \phi_{\nu 5}^k + \frac{1}{2}\phi_{\kappa\lambda}^i \phi_{\mu\nu}^j \phi_5^k + (\text{terms in } \phi^i)\right]$$

(6.5)

Introducing Eqs. (5.3), we get for the effective vector current

$$
\begin{aligned}
J_\mu^i(p,p') = f^{ijk} \Big[& -P_\mu \Big\{ \Big(1+\frac{q^2}{2\mu^2}\Big)\phi_\nu^j(p')\phi_\nu^k(-p) - \Big(1-\frac{q^2}{\mu^2}\Big)\phi_5^j(p')\phi_5^k(-p) \Big\} \\
& + 3\Big\{ q_\nu \phi_\nu^j(p')\phi_\mu^k(-p) + q_\nu \phi_\nu^j(-p)\phi_\mu^k(p') \Big\} \\
& - P_\mu q_\nu \phi_\nu^j(p') q_\lambda \phi_\lambda^k(-p) / \mu^2 \Big] + \\
& - \frac{1}{\mu} d^{ijk} \epsilon_{\kappa\lambda\mu\nu} P_\lambda q_\nu \Big\{ \phi_\kappa^j(p')\phi_5^k(-p) + \phi_5^j(p')\phi_\kappa^k(-p) \Big\}
\end{aligned}
\tag{6.6}
$$

Introducing the gauge-invariant electromagnetic coupling $f(q^2)\, F_{\mu\nu}\phi_{\mu\nu}$, we obtain a total magnetic moment of 3 Bohr magnetons and a quadrupole moment of -4 (in units of e/μ^2), for positively charged $1-$ particles.

(6) The Outlook

With the effective baryon-meson and meson-meson vertices available it is a trivial step to write pole approximations for the strong interaction four-particle process. With this approximation as the starting point all S-matrix techniques (like Mandelstam representation, Reggeisation, analytic continuation now both in angular momentum and unitary spin) are available for determining the complete ($\tilde{U}(12)$) relativistic S-matrix theory. This is so because as a rule all that the S-matrix theory requires are "Born approximations" as the "input". The higher "fundamental" 4-particle vertices involving 4-particles (like $\bar{N}\ N\ M\ M$) which are analogous to 4-field Lagrangians (and knowledge of which is necessary for peripheral interactions) are no harder to write down, using the methods and techniques of this paper. Every one of these "fundamental" interactions will now be momentum dependent - this dependence coming in a determinate manner from $\tilde{U}(12)$ symmetry. The conventional field theory Lagrangians are but local approximations of these "fundamental" interactions.

It would seem that the $\widetilde{U}(12)$ ideas fulfil completely the dream of the U(6) theorist, i.e., to write S-matrix elements whose momentum dependence is dictated by the internal symmetry group of SU_3.

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2. Of all the approaches to the relativistic U(6) theory listed in Ref. 1) the formulation nearest in spirit to our own is that of P. Roman and J.J. Agasshi. The mesons are described similarly in both approaches but the baryon states (and their underlying symmetries) are different. Thus the well-known 8-fold of baryons appears in the Agasshi-Roman theory with four components contrasted with the 20 components for our case. These essential differences come about because we use Bargmann-Wigner equations of Section 5 to make sure we are dealing with particles having a definite mass as well as definite spin.

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5. One could have made the transition $\tilde{U}(4) \rightarrow \mathcal{L}_4$ via the intermediate symmetry stage $\widetilde{Sp}(4)$ $(\epsilon_5 = \epsilon_{\mu 5} = 0)$. The $\widetilde{Sp}(4)$ transformation with 10 parameters ($\epsilon_\mu, \epsilon_{\mu\nu}$) also admits of the existence of this matrix C. All statements of Section 4 apply equally to theories with $\widetilde{Sp}(4)$ symmetry.

6. $\Psi_{[\alpha\beta]\gamma}$ has 24 components; Eq. (4.8) states that the fully anti-symmetric part of $\Psi_{[\alpha\beta]\gamma}$ vanishes. We are thus left with 20 independent components if Eq. (4.8) is satisfied.

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12. The residual symmetry will always depend on the type of equation applied to the spinor $\Psi_{\alpha\beta\gamma\ldots}$. For example if in place of equations (5.1) , one had used Eq. (8) of paper I $(\{\not{p} + i\gamma^5 \not{W}\}\psi = M\psi)$ the W(6) symmetry would have formally survived.

13. We shall here prove that the positive energy projection of all irreducible representations $\Psi_{\alpha\beta\gamma\ldots}(p)$ of $\tilde{U}(12)$ remain irreducible representations of U(6) even after Bargmann-Wigner equations are applied provided $\underline{p} = 0$. Here by U(6) we mean the group generated

by the matrices $\tau^j, \tau^j \underset{\sim}{\sigma}$ ($\underset{\sim}{\sigma}$ are 2 x 2 Pauli matrices). The proof is elementary. For $\underset{\sim}{p} = 0$ the positive energy projection of $\not{p}$ equals $m\underset{\sim}{1}$, and α, β, γ ..... run over 1, 2 rather than 1, ..., 4. Thus the symmetry character of any Ψ is unaltered by the application of the set of equations and carries itself from $\widetilde{U}(12)$ to U(6).

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APPENDIX

Our convention for the unitary spin matrices is the following: $T^i = \frac{1}{2}\lambda^i$, with the λ^i defined by M. Gell-Mann, Phys. Rev., 125, 1067 (1962). Thus $\mathrm{Tr}\,(T^i T^j) = \frac{1}{2}\delta^{ij}$, $f^{0jk} = 0$, and $d^{0jk} = \delta^{jk}(2/3)^{\frac{1}{2}}$.

$$[T^i, T^j] = i f^{ijk} T^k \quad , \quad \{T^i, T^j\} = d^{ijk} T^k$$

From the fundamental representation, $F^{Ri} \equiv \gamma^R T^i$ we deduce the following commutators ,

$$[F^i, F^j] = i f^{ijk} F^k$$
$$[F^i, F^j_5] = i f^{ijk} F^k_5$$
$$[F^i_5, F^j_5] = -i f^{ijk} F^k$$
$$[F^i, F^j_{\mu\nu}] = i f^{ijk} F^k_{\mu\nu}$$
$$[F^i_5, F^j_{\mu\nu}] = \frac{i}{2} f^{ijk} \epsilon_{\mu\nu\kappa\lambda} F^k_{\kappa\lambda}$$
$$[F^i_{\kappa\lambda}, F^j_{\mu\nu}] = i d^{ijk} (g_{\kappa\nu} F^k_{\lambda\mu} + g_{\lambda\mu} F^k_{\kappa\nu} - g_{\kappa\mu} F^k_{\lambda\nu} - g_{\lambda\nu} F^k_{\kappa\nu}) + i f^{ijk} \{(g_{\kappa\mu} g_{\lambda\nu} - g_{\lambda\mu} g_{\kappa\nu}) F^k - \epsilon_{\kappa\lambda\mu\nu} F^k_5)$$
$$[F^i_\mu, F^j_\nu] = i f^{ijk} g_{\mu\nu} F^k - i d^{ijk} F^k_{\mu\nu}$$
$$[F^i_\mu, F^j_{\nu 5}] = i d^{ijk} g_{\mu\nu} F^k_5 + \frac{i}{2} f^{ijk} \epsilon_{\mu\nu\kappa\lambda} F^k_{\kappa\lambda}$$
$$[F^i_{\mu 5}, F^j_{\nu 5}] = -i f^{ijk} g_{\mu\nu} F^k - i d^{ijk} F^k_{\mu\nu}$$

and

$$[F^i, F^j_\mu] = i f^{ijk} F^k_\mu$$
$$[F^i, F^j_{\mu 5}] = i f^{ijk} F^k_{\mu 5}$$
$$[F^i_5, F^j_\mu] = i d^{ijk} F^k_{\mu 5}$$
$$[F^i_5, F^j_{\mu 5}] = i d^{ijk} F^k_\mu$$
$$[F^i_\lambda, F^j_{\mu\nu}] = i d^{ijk} (g_{\lambda\mu} F^k_\nu - g_{\lambda\nu} F^k_\mu) - i f^{ijk} \epsilon_{\lambda\mu\nu\kappa} F^k_{\kappa 5}$$
$$[F^i_{\lambda 5}, F^j_{\mu\nu}] = i d^{ijk} (g_{\lambda\mu} F^k_{\nu 5} - g_{\lambda\nu} F^k_{\mu 5}) + i f^{ijk} \epsilon_{\lambda\mu\nu\kappa} F^k_\kappa \; .$$