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THE SCALAR BEHAVIOR OF VECTOR ELECTRODYNAMICS AT HIGH ENERGY

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ABSTRACT

Using Salam's gauge approximation technique to the electrodynamics of vector mesons, it is shown using two-particle unitarity, that in its high energy aspects, the theory is analogous to that for scalar mesons only for particular values of the constants of the theory.

I. INTRODUCTION

The renormalizable theory of vector mesons has for some time attracted the attention of various authors[1] who have shown that the use of conventional Feynman rules leads to an unrenormalizable theory. In an attempt to obtain a (covariant) renormalizable theory, Lee and Yang[1] introduced the ξ-limiting formalism and obtained a renormalized theory of vector mesons with arbitrary anomalous magnetic moment on the assumption that after summing all contributions to the Feynman diagram, the limit $\xi \to 0$ exists and can therefore be taken safely.

Using a completely different approach from the usual procedure, Salam and Delbourgo[2] have recently proposed a theory (of the electrodynamics of vector mesons with arbitrary magnetic moment), which is based rather on Dyson's form of the equations for the renormalized propagator and the vertex function. In this theory, the inhomogeneous term Γ^A [3] of the full vertex function Γ is considered as the first approximation to Γ, the process termed the gauge approximation. It is then shown that with conventional subtractions, vector electrodynamics is completely renormalizable.

In their treatment of this theory, SD conjectured that the equations of vector electrodynamics give rise to two distinct solutions depending on the boundary conditions imposed on the propagators. These two distinct solutions are called the "vector" and "scalar" alternatives by the authors. A solution was found for the "vector" alternative, valid for the arbitrary gauge, which is non-perturbative in methodology but still possesses the same limits as the free propagator. However the authors did not consider the scalar alternative i.e. the case when the vector propagators and vertex function behave as they do for scalar electrodynamics. A problem therefore arises : Does this other alternative exist physically? to wit, are there any special values of the constants of the theory for which a different solution could be obtained? In particular Lee and Yang[1] have suggested that the meson propagator $\Delta_{\mu\nu}$ behaves not like the free propagator $\Delta_0^{\mu\nu}$ but like the scalar propagator Δ. It is the purpose of this paper to show, using two-particle unitarity and the gauge approximation, that the scalar alternative might hold for special values of the constants of the theory.

Section (2) deals briefly with these two distinct solutions and the boundary conditions to be imposed. In Section (3) we consider the meson self-energy equations and obtain the conditions necessary for this other behavior to hold. Section (4) deals at some length with the photon and the vertex function equations, obtaining the conditions to be satisfied by the form-factor equations in the gauge approximation.

II. THE STABILITY CRITERION

Defining the meson propagator as[4]

$$\Delta^{-1}_{\mu\nu}(p) = (p^2-m^2)Z_1(p^2)d_{\mu\nu} + m^2 Z_2(p^2)e_{\mu\nu}(p) \tag{2.1}$$

where

$$Z_1(p^2) = 1 - (p^2-m^2)\int \frac{G_1(x)}{p^2-x+i\epsilon}dx\,, \tag{2.2}$$

$$Z_2(p^2) = 1 - m^2\int\frac{G_1(x)}{x}dx - p^2\int\frac{G_2(x)\,dx}{p^2-x+i\epsilon}\,, \tag{2.3}$$

the Dyson equations for Eq. (2.1) are

$$Z_1(p^2) = Z\left(\frac{p^2-m_0^2}{p^2-m^2}\right) + \frac{1}{3}\mathrm{Tr}\left(\frac{\underline{d}\cdot t_a K_a}{p^2-m^2}\right) \tag{2.4}$$

$$Z_2(p^2) = Z m_0^2 + \mathrm{Tr}(\underline{e}\cdot t_a K_a) \tag{2.5}$$

where the kernel K, is defined as

$$K[\Gamma, \Delta] = e^2 \int \Gamma \Delta \Gamma \Delta \Gamma D + e^4 \cdots \qquad (2.6)$$

and the wave function renormalization constant Z and the bare mass constant m_0^2 are given by the relations

$$Z = \lim_{p^2 \to \infty} Z_1(p^2) = 1 - \int G_1(x)\, dx \qquad (2.7)$$

$$\frac{m_0^2 Z}{m^2} = \lim_{p^2 \to \infty} Z_2(p^2)$$
$$= 1 - m^2 \int \frac{G_1(x)}{x}\, dx - \int G_2(x)\, dx \qquad (2.8)$$

It follows from the definitions (2.7) and (2.8) that the second terms on the right-hand side of Equations (2.4) and (2.5) must approach zero in this limit.

Now a sufficient condition for

$$\Gamma^A \Delta \approx \frac{1}{p} \,, \quad D \approx \frac{1}{K^2} \,, \qquad (2.9)$$

to hold is given by

$$\lim_{p^2 \to \infty} p^2 Z_1(p^2) \approx Z_2(p^2) \qquad (2.10)$$

Eq. (2.9) is the stability criterion of any approximation procedure for computing any Green's function based on Dyson formalism and from it there arise two possible behaviors of the propagator and the vertex function, Δ and Γ^A respectively :

(A) Either $Z m_0^2$ is finite (apart from logarithmic factors) so that we have the boundary condition

$$Z_1(p^2) \approx \frac{1}{p^2} \qquad \text{(i.e. } Z = 0 \text{ automatically)}$$

or

(B) $Z \neq 0$ and $Z_2(p^2) \approx p^2$ In this case m_0^2 must be intrinsically quadratically infinite.

For case (A) we have the corresponding propagators behaving as

$$\Delta \approx 1 , \quad \Gamma^A \approx \frac{1}{p} \tag{2.11a}$$

and for case (B)

$$\Delta \approx \frac{1}{p^2} , \quad \Gamma^A \approx p \tag{2.11b}$$

Cases (A) and (B) are respectively the "vector" and "scalar" alternative solutions for vector electrodynamics as conjectured by SD. The former is investigated in detail by the authors and we now consider the latter in the following sections.

III. THE MESON SELF-ENERGY EQUATIONS

The meson self-energy equations obtained by SD may be reduced to the form

$$\begin{aligned}\frac{1}{\pi} \operatorname{Im} Z_1(p^2) = -\frac{\alpha\, Q(p^2-m^2)}{24 m^2 p^4}\Big[& \{3a(p^2+m^2)^3 - 5(p^2-m^2)^2 - 36 m^2 p^2\} |Z_1(p^2)|^2 \\ & + 2(p^2-m^2)^2\{(p^2-m^2)^2 + 12 m^2 p^2\} \operatorname{Re} Z_1^*(p^2) Z_1'(p^2) \\ & + 2(p^2-m^2)^2(p^2+m^2) \operatorname{Re} Z_1^*(p^2) M(p^2) \\ & + 2(p^2-m^2)^2(p^2+m^2) |M(p^2)|^2 \Big]\end{aligned} \tag{3.1}$$

$$\frac{1}{\pi}\,\mathrm{Im}\,Z_2(p^2) = \frac{\alpha(p^2-m^2)\,\theta(p^2-m^2)}{8m^2p^4}$$
$$\times\Big[3a\,m^4(p^2+m^2)\,|Z_2(p^2)|^2 - 3(p^2-3m^2)|F_1(p^2)|^2$$
$$- 2(p^2-m^2)(2p^2+m^2)\,\mathrm{Re}\,F_1^*(p^2)\,F_2(p^2)$$
$$- 5/4\,(p^2-m^2)^2(p^2+m^2)\,|F_2(p^2)|^2\Big] \qquad (3.2)$$

where $F_1(p^2)$, $F_2(p^2)$ are defined as

$$F_1(p^2) = -(p^2-m^2)\left[Z_1(p^2) - \tfrac{1}{2}M(p^2)\right] - m^2 Z_2(p^2) \qquad (3.2a)$$

$$F_2(p^2) = 2Z_1(p^2) - M(p^2) \qquad (3.2b)$$

and the prime denotes differentiation with respect to p^2

For large p^2, Equations (3.1) and (3.2) reduce to the forms

$$\mathrm{Im}\,Z_1(p^2)\left[1 + \alpha p^2\{\mathrm{Im}\,Z_1'(p^2) + \mathrm{Im}\,M(p^2)\}\right]$$
$$\approx -\alpha p^2\Big[(3a-5)|Z_1(p^2)|^2 + \mathrm{Re}\,Z_1^*(p^2)M(p^2) \qquad (3.3)$$
$$+ 2p^2\,\mathrm{Re}\,Z_1^*(p^2)\,Z_1'(p^2) + 2|M(p^2)|^2\Big]$$

i.e. $$\mathrm{Im}\,Z_1^{-1\prime}(p^2) = U(p^2) + \tan\theta(p^2)\,\mathrm{Re}\,Z_1^{-1\prime}(p^2) \qquad (3.4)$$.

where

$$U(p^2) = \frac{\alpha p^2\left[(3a-5) + 2|Z_1^{-1}(p^2)M(p^2)|^2\right]}{1 + \alpha p^2\left[\mathrm{Im}\,Z_1'(p^2) + \mathrm{Im}\,M(p^2)\right]} \qquad (3.5)$$

and

$$\tan \theta(p^2) = \frac{\alpha p^2 [\mathrm{Re}\, M(p^2) + p^2 \mathrm{Re}\, Z_1'(p^2)]}{1 + \alpha p^2 [\mathrm{Im}\, Z_1'(p^2) + \mathrm{Im}\, M(p^2)]} \qquad (3.6)$$

Assuming for simplicity that $Z_1'(p^2)$ is small compared to $M(p^2)$ and that $p^2 \mathrm{Re}\, Z_1'(p^2) \sim \mathrm{Re}\, M(p^2)$, equations (3.5) and (3.6) can be written as

$$U(p^2) \approx \frac{\alpha p^2 [(3a-5) + 2|Z_1^{-1}(p^2) M(p^2)|^2]}{1 + \alpha p^2 \mathrm{Im}\, M(p^2)} \qquad (3.7)$$

$$\tan \theta(p^2) \approx \frac{\alpha p^2 \mathrm{Re}\, M(p^2)}{1 + \alpha p^2 \mathrm{Im}\, M(p^2)} \qquad (3.8)$$

Similarly

$$\mathrm{Im}\, Z_2^{-1}(p^2) = V(p^2) + \tan\delta(p^2)\, \mathrm{Re}\, Z_2^{-1}(p^2), \qquad (3.9)$$

where

$$V(p^2) = \frac{3\alpha(a-1)}{1 + \alpha p^2 \{2 \mathrm{Im}\, Z_1(p^2) - \mathrm{Im}\, M(p^2)\}} \qquad (3.10)$$

$$\tan\delta(p^2) = \frac{\alpha p^2 \{2 \mathrm{Re}\, Z_1(p^2) - \mathrm{Re}\, M(p^2)\}}{1 + \alpha p^2 \{2 \mathrm{Im}\, Z_1(p^2) - \mathrm{Im}\, M(p^2)\}} \qquad (3.11)$$

Consider now Equation (3.4). This is the inhomogeneous Riemann-Hilbert equation which has been fully investigated by Muskhelishvili[5]. Applying Muskhelishvili's results, Equation (3.4) has the solution, with the subtraction $Z_1^{-1}(m^2) = 1$, [6]

$$Z_1^{-1}(p^2) = \left[C + \frac{p^2 - m^2}{\pi} \int \frac{U(x)\, dx}{X(x)(x-p^2)(x-m^2)} \right] X(p^2), \qquad (3.12)$$

where

$$X(p^2) = \exp\left\{ \frac{(p^2 - m^2)}{\pi} \int_{m^2}^{\infty} \frac{\theta(x)\, dx}{(x-m^2)(x-p^2)} \right\}. \qquad (3.13)$$

and C is a constant.

Now for Γ to behave as $\Gamma \approx p$, $M(p^2)$ must behave as $M(p^2) \approx 1$ so that the term $\alpha p^2 \mathrm{Im} M(p^2)$ dominates over 1 in the denominators of Equations (3.7) and (3.8). Hence equation (3.4) would admit of solutions behaving like $Z_1(p^2) \approx 1$ if and only if the phase change

$$\left[\theta(x)\right]_{m^2}^{\infty} = 0 \qquad (3.14)$$

Similarly Equation (3.9) would admit of solutions behaving at infinity as $Z_2(p^2) \approx p^2$ if and only if[7]

$$\left[\delta(x)\right]_{m^2}^{\infty} = -\pi \qquad (3.15)$$

In this case in order to satisfy all the conditions of the Riemann-Hilbert equation we should introduce a subtraction constant $Z_2(0) = Z(0)$, the computation of which is given by SD. Also it is to be noted that equation (3.9) is homogeneous for the Fermi-Stuckelberg gauge and then $Z_2^{-1}(p^2)$ vanishes identically.

Equations (3.14) and (3.15) are the necessary and sufficient conditions to be satisfied by the constants of the theory (such as α and the observed magnetic moment κ). These conditions are clearly gauge-independent.

IV. THE PHOTON SELF-ENERGY AND FORM-FACTOR EQUATIONS

The photon propagator $D_{\mu\nu}$ may be expressed as[8]

$$D_{\mu\nu}^{-1}(t) = (t^2-\mu^2) Z_3(t^2) d_{\mu\nu}(t) - Z_3 \frac{\mu_0^2}{\lambda^2}(t^2-\lambda^2) e_{\mu\nu}(t) \qquad (4.1)$$

where

$$Z_3(t^2) = 1 - \int \frac{(t^2-\mu^2)\, G_3(x)\, dx}{t^2 - x + i\epsilon}, \qquad (4.2)$$

$$\frac{\mu_0^2}{\mu^2} Z_3 = 1 - \int \frac{G_3(x)}{x}\, dx . \qquad (4.3)$$

The photon self-energy equation in the two-particle unitarity approximation then becomes

$$t^4 G_3(t^2) = \frac{-e^2}{3(2\pi)^2}\, d_{ab}(t) \int d^4p\, \Gamma_{a\mu\rho}(p,p')\, d_{\mu\nu}(p)\, d_{\rho\sigma}(p') \times \Gamma^{*}_{b\nu\sigma}(p,p')\, \delta_+(p^2-m^2)\, \delta_+(p'^2-m^2), \qquad (4.4)$$

where

$$\Gamma_{a\mu\rho}(p,p') = (p+p')_a\left(-g_{\mu\rho}\mathcal{E}(t^2) + t_\mu t_\rho \mathcal{Q}(t^2)\right) + (g_{a\mu}t_\rho - g_{a\rho}t_\mu)\,\mathcal{M}(t^2), \qquad (4.5)$$

$t = p - p'$, $\delta_\pm(p) = \theta(\pm p_0)\,\delta(p^2-m^2)$ and $\mathcal{E}(t^2)$, $\mathcal{Q}(t^2)$ and $\mathcal{M}(t^2)$ are as defined in SD.

From Equation (4.4) the photon self-energy equation reduces to the form[9]

$$G_3(t^2) = \frac{\alpha}{24m^2t^2}\left(1 - \frac{4m^2}{t^2}\right)^{3/2} \theta(t^2-4m^2) \times \Big[(t^4 - 4m^2t^2 + 12m^4)|\mathcal{E}(t^2)|^2 - 2t^2(t^2-2m^2)\,\mathrm{Re}\,\mathcal{E}^*(t^2)\mathcal{M}(t^2) - t^2(t^2-2m^2)(t^2-4m^2)\,\mathrm{Re}\,\mathcal{E}^*(t^2)\mathcal{Q}(t^2) + t^2(t^2+4m^2)|\mathcal{M}(t^2)|^2 + t^4(t^2-4m^2)\,\mathrm{Re}\,\mathcal{M}^*(t^2)\mathcal{Q}(t^2) + \frac{t^4}{4}(t^2-4m^2)^2|\mathcal{Q}(t^2)|^2\Big] \qquad (4.6)$$

That equation (4.6) is positive definite is clearly exhibited if we define a function $\chi(t^2)$ as

$$\chi(t^2) = -\left(1 - \frac{t^2}{2m^2}\right)\mathcal{E}(t^2) - \frac{t^2}{2m^2}\mathcal{M}(t^2) + t^2\left(1 - \frac{t^2}{4m^2}\right)\mathcal{Q}(t^2) \qquad (4.7)$$

Then eliminating $Q(t^2)$, equation (4.6) becomes simply

$$G_3(t^2) = \frac{\alpha}{3t^2}\left(1-\frac{4m^2}{t^2}\right)^{3/2}\left\{|\mathcal{E}(t^2)|^2+\frac{t^2}{2m^2}|\mathcal{M}(t^2)|^2+\frac{1}{2}|\chi(t^2)|^2\right\} \tag{4.8}$$

The physical significance of $\chi(t^2)$ is easily shown if we define a function $\psi(t^2)$ such that

$$\psi(t^2) = \mathcal{E}'(t^2) + \chi'(t^2), \tag{4.9}$$

where the prime denotes differentiation with respect to t^2 Then in the static limit, we have the interesting **relation**[9].

$$\psi(0) = \mathcal{E}'(0) + \chi'(0) \tag{4.10}$$

$$= \frac{q}{2m^2} + \text{Constant}$$

where q is the quadrupole moment in units of e/m^2

Now the stability criterion requires that $Z_3 \sim 1$ (or alternatively $G_3(t^2)$ must be at least as convergent as $1/t^2$ for large t^2) if the photon propagator is to behave like $D \sim 1/t^2$. Hence from Equation (4.8) $\mathcal{E}(t^2) \sim 1$, $\mathcal{M}(t^2) \sim \frac{1}{|t|}$, $\chi(t^2) \sim 1$ (apart from logarithmic factors), which is consistent with the fact that for $\Gamma^A \sim p$, the form-factors must behave like $\mathcal{E}(t^2) \sim a$, $\mathcal{M}(t^2) \sim b$, $Q(t^2) \sim \frac{2c}{t^2}$ say, where a, b, and c are constants (since for large t^2, $\mathcal{E}$ and χ will dominate in Equation (4.8)). An examination of Equation (4.7) then reveals that $\chi(t^2) \sim 1$ only if a = b + c.

Consider now the exact two-particle unitarity equation for the vertex function. For simplicity we shall deal with the photon unphysical since the meson unphysical is more complicated, involving a large number of form-factors, and requiring a proper treatment of the Compton parts in order to preserve gauge invariance.

We are concerned with $\Gamma_{a\mu\nu}(p,p')$ such that the mesons are physical. Hence we deal with the gauge invariant quantity

$$F_{a\mu\nu}(p,p') = d_{\mu\rho}(p)\,\Gamma_{a\rho\sigma}(p,p')\,d_{\sigma\nu}(p') , \tag{4.11}$$

where $\Gamma_{a\rho\sigma}$ is as defined in Equation (4.5).
The three form-factors $\mathcal{E}(t^2)$, $\mathcal{M}(t^2)$ and $\mathcal{Q}(t^2)$ are linearly related to the three possible contractions

$$F_1(t^2) = (p+p')_a F_{a\mu\mu} \tag{4.12a}$$

$$F_2(t^2) = (p+p')_a F_{a\mu\nu} t_\mu t_\nu \tag{4.12b}$$

$$F_3(t^2) = F_{\mu\mu\nu} t_\nu \tag{4.12c}$$

and from these we obtain $\mathcal{E}(t^2)$, $\mathcal{M}(t^2)$ and $\mathcal{Q}(t^2)$ in terms of $F_1(t^2)$, $F_2(t^2)$ and $F_3(t^2)$.

Now one of the contributions to the vertex function is the one-photon-exchange diagram. This, in the two-particle unitarity approximation, gives

$$\frac{1}{\pi}\,\mathrm{Im}\,F_{a\mu\nu}(p,p') = \frac{e^2}{(2\pi)^3}\, d_{\mu\rho}(p)\int d^4k\, \delta_-\{(p-k)^2-m^2\}\,\delta_+\{(p'-k)^2-m^2\}$$
$$\times\, d_{\sigma\nu}(p')\,\Gamma_{a\alpha\delta}(p-k,p'-k)\,d_{\alpha\beta}(p-k) \tag{4.13}$$
$$\times\, d_{\delta\gamma}(p'-k)\,\Gamma^*_{b\rho\beta}(p,p-k)\,D_{bc}(k)\,\Gamma^c_{\gamma\sigma}(p'-k,p') .$$

From Equation (4.13), the form factor equations are obtained as

$$\frac{1}{\pi}\,\mathrm{Im}\,\mathcal{E}(t^2)(-1)\,8m^2t^4\left(1-\frac{t^2}{4m^2}\right)^3 = \frac{1}{\pi}\,\mathrm{Im}\left\{I_2^2 F_1(t^2) + I_9 F_2(t^2)\right\} \tag{4.14a}$$

$$\frac{1}{\pi}\,\mathrm{Im}\,\mathcal{M}(t^2)\,8m^2t^2\left(1-\frac{t^2}{4m^2}\right)^2 = \frac{1}{\pi}\,\mathrm{Im}\left\{F_2(t^2) + (4m^2-t^2)F_3(t^2)\right\} \tag{4.14b}$$

$$\frac{1}{\pi}\,\mathrm{Im}\left\{\mathcal{E}(t^2)+\mathcal{X}(t^2)\right\}8m^2t^2\left(1-\frac{t^2}{4m^2}\right)^2$$
$$= \frac{1}{\pi}\,\mathrm{Im}\left\{I_2 F_1(t^2) + \left(3-\frac{t^2}{2m^2}\right)F_2(t^2)\right\}. \tag{4.14c}$$

where $\mathrm{Im}\, F_i(t^2)\ (i=1,2,3)$ are given in the Appendix and I_2, I_q are to be found in SD.

Eqs. (4.14) together with Eq. (4.8) give us four equations involving four unknowns $\mathcal{E}(t^2)$, $\mathcal{M}(t^2)$, $\chi(t^2)$ and $Z_3^{-1}(t^2)$. They are, however, very complicated ones, solutions of which cannot easily be obtained. We therefore make use of the approximation $\Gamma = \Gamma^A$ (i.e. $\mathcal{E}(t^2) = \mathcal{E}^A(t^2)$) and a first approximation of $\mathcal{M}^A(t^2) = K\mathcal{E}^A(t^2)$, $Q(t^2) = Q^A(t^2) = \frac{1}{2m^2t^2}(K-1)\mathcal{E}^A(t^2)G(t^2)$ where we set $G(0) = 0$, $G'(0) = 1$ and $G(\infty) = \rho$, a constant proportional to the fine structure constant $2\pi\alpha = e^2/4\pi$. Thus for large t^2, $Q^A(t^2) = \frac{1}{2m^2t^2}(K-1)\rho = \frac{\beta}{t^2}(K-1)$.

In this approcimation, Eq. (4.14a) reduces to the homogeneous Riemann-Hilbert form, viz.,

$$\mathrm{Im}\,\mathcal{E}(t^2) \sim \tan\gamma(t^2)\, \mathrm{Re}\,\mathcal{E}(t^2), \tag{4.15}$$

where

$$\tan\gamma(t^2) \sim \frac{\mathrm{Im}}{\pi t'^c}\left[I_2^2 F_1(t^2) + I_q F_2(t^2)\right]. \tag{4.15a}$$

Similar equations are obtained for $\mathrm{Im}\,\mathcal{M}(t^2)$ and $\mathrm{Im}\left[\mathcal{E}(t^2) + \chi(t^2)\right]$

Thus $\mathcal{E}(t^2)$ behaves for large t^2 as $\mathcal{E}(t^2) \sim 1$ if

$$\left[\gamma(x)\right]_{-t^2}^{0} = 0 . \tag{4.16}$$

It follows from Eq. (4.15) that for large t^2, $\tan\gamma(t^2)$ must behave like unity. A tedious but straightforward calculation, using the I's in SD and in the Appendix, gives two cubic equations in K and β. A oonsistency requirement enables us to impose a constraint on β, which is, however, too complex an equation (of the 12th order!) to be solved. Taking a special value of $K = 1$ (which simplifies the calculations considerably) leads to an inconsistency, implying that the Bernstein and Lee suggestion[10)] that the vector mesons have no anomalous magnetic moment is not reproduced at high energy in this approximation.

CONCLUSION

From the stability criterion, which serves as the boundary condition to be satisfied by the three Green's functions Δ, Γ and $\mathcal{D}$, it has been conjectured that two alternative solutions (one of which has already been discussed by $S\mathcal{D}$) to vector electrodynamics, exist which exhibit distinct behaviours at high energy. The power of this criterion in specifying acceptable high energy behaviour of the propagators enables us to obtain conditions for the various spectral functions. Exploiting these, we have been able to show, using two-particle unitarity and the gauge approximation technique, that for some particular values of the constants of the theory, which owing to the complexity of the equations concerned we were unable to obtain, solution to vector electrodynamics might exist which behaves at high energy as in scalar electrodynamics.

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APPENDIX

Using the phase space integrals listed in SD, Equations (4.12) reduce to the form

$$\frac{1}{\pi}\,\mathrm{Im}\,F_1(t^2) = \frac{\alpha\,\mathcal{Q}(t^2-4m^2)}{2(t^4-4m^2t^2)^{1/2}}\int_{4m^2-t^2}^{0}\frac{d\kappa^2}{\kappa^2}\,Z_3^{-1}(\kappa^2)\,(t^2+2\kappa^2-4m^2)$$

$$\times\Big\{\mathrm{Re}\,\mathcal{E}(t^2)\Big[(2t^2+\kappa^2-4m^2)\big(I_{18}\mathcal{E}^2(\kappa^2)-2I_{21}\mathcal{E}(\kappa^2)\mathcal{Q}(\kappa^2)+I_6^2\mathcal{Q}^2(\kappa^2)\big)$$

$$+4\mathcal{M}(\kappa^2)\mathcal{E}(\kappa^2)(I_{23}-I_{22})+4\mathcal{M}(\kappa^2)\mathcal{Q}(\kappa^2)I_6(I_8-I_7)$$

$$+2\mathcal{M}^2(\kappa^2)(I_{24}-I_5I_6)\Big]$$

$$-\mathrm{Re}\,\mathcal{Q}(t^2)\Big[(2t^2+\kappa^2-4m^2)\big(I_{20}\mathcal{E}^2(\kappa^2)-2I_3I_{12}\mathcal{E}(\kappa^2)\mathcal{Q}(\kappa^2)$$

$$+I_3^2I_6\,\mathcal{Q}^2(\kappa^2)\big)$$

$$+4\mathcal{M}(\kappa^2)\mathcal{E}(\kappa^2)(I_3I_{13}-I_4I_{12})$$

$$+4\mathcal{M}(\kappa^2)\mathcal{Q}(\kappa^2)I_3(I_4I_6-I_3I_7)$$

$$+\mathcal{M}^2(\kappa^2)(2I_3I_{14}-I_6^2-I_3^2I_5)\Big]\Big\} \qquad (A.1)$$

$$\frac{1}{\pi}\,\mathrm{Im}\,F_2(t^2) = -\frac{\alpha\,\mathcal{Q}(t^2-4m^2)}{2(t^4-4m^2t^2)^{1/2}}\int_{4m^2-t^2}^{0}\frac{d\kappa^2}{\kappa^2}\,Z_3^{-1}(\kappa^2)\,(t^2+2\kappa^2-4m^2)$$

$$\times\Big\{\mathrm{Re}\,\mathcal{E}(t^2)\Big[(2t^2+\kappa^2-4m^2)\big(I_{19}\mathcal{E}^2(\kappa^2)-2I_1I_{15}\mathcal{E}(\kappa^2)\mathcal{Q}(\kappa^2)+I_1^2I_6\mathcal{Q}^2(\kappa^2)\big)$$

$$+4\mathcal{M}(\kappa^2)\mathcal{E}(\kappa^2)(I_2I_{15}-I_1I_{16})+4\mathcal{M}(\kappa^2)\mathcal{Q}(\kappa^2)I_1(I_1I_5-I_2I_6)$$

$$+\mathcal{M}^2(\kappa^2)(2I_1I_{17}-I_6I_9-I_1^2I_5)\Big]$$

$$-\mathrm{Re}\,\mathcal{Q}(t^2)\Big[(2t^2+\kappa^2-4m^2)\big(I_{10}^2\mathcal{E}^2(\kappa^2)-2I_1I_3I_{10}\mathcal{E}(\kappa^2)\mathcal{Q}(\kappa^2)$$

$$+I_1^2I_3^2\mathcal{Q}^2(\kappa^2)\big)$$

$$+4\mathcal{M}(\kappa^2)\mathcal{E}(\kappa^2)I_{10}(I_2I_3-I_1I_4)$$

$$+4\mathcal{M}(\kappa^2)\mathcal{Q}(\kappa^2)I_1I_3(I_1I_4-I_2I_3)$$

$$+\mathcal{M}^2(\kappa^2)(2I_1I_3I_{11}-I_1^2I_6-I_3^2I_9)\Big]\Big\}, \qquad (A.2)$$

$$
= \mathrm{Im}\, F_3(t^2) = \frac{\alpha Q(t^2-4m^2)}{2(t^4-4m^2t^2)^{1/2}} \int_{4m^2-t^2}^{0} \frac{dK^2}{K^2}\, Z_3^{-1}(K^2)
$$

$$
\begin{aligned}
\times \Big\{ -\mathrm{Re}\,\xi(t^2) \Big[& (2t^2+K^2-4m^2) \big\{ (I_{19}-2I_{47})\xi^2(K^2) \\
& \quad - (I_1 I_{15} - I_1 I_{39} + 2I_3 I_{39})\xi(K^2)Q(K^2) \\
& \quad + I_3 I_6 (I_1 - 2I_3) Q^2(K^2) \big\} \\
& + 2\mathfrak{M}(K^2)\xi(K^2) \big\{ I_{15}(I_2 - 2I_1) - (I_1 - 2I_3) I_{16} \\
& \qquad - I_1 (I_{16} - 2I_{40}) + I_2 (I_{15} - 2I_{39}) \big\} \\
& + 2\mathfrak{M}(K^2) Q(K^2) \big\{ (I_1 - 2I_3)(2I_1 I_8 - I_2 I_6) \\
& \qquad - I_1 I_6 (I_2 - 2I_1) \big\} \\
& + \mathfrak{M}^2(K^2) \big\{ I_1 (I_{17} - 2I_{45}) + (I_1 - 2I_3)(I_{17} - I_1 I_5) \\
& \qquad - I_8 (I_9 - 2I_7) \big\} \Big] \\
+ \mathrm{Re}\, Q(t^2) \Big[& (2t^2+K^2-4m^2) \big\{ (I_{10} - 2I_{28})(I_{10}\xi^2(K^2) - I_1 I_3 \xi(K^2) Q(K^2)) \\
& \qquad - (I_1 - 2I_3)(I_3 I_{10} \xi(K^2) Q(K^2) - I_1 I_3^2 Q^2(K^2)) \big\} \\
& + 2\mathfrak{M}(K^2)\xi(K^2) \big\{ I_3 I_{10} (I_2 - 2I_1) - I_4 I_{10} (I_1 - 2I_3) \\
& \qquad - (I_{10} - 2I_{28})(I_1 I_4 - I_2 I_3) \big\} \\
& + 2\mathfrak{M}(K^2) Q(K^2) \big\{ (I_1 - 2I_3) I_3 (2I_4 I_4 - I_3 I_2) \\
& \qquad - I_3^2 I_1 (I_2 - 2I_1) \big\} \\
& + \mathfrak{M}^2(K^2) \big\{ I_1 I_3 (I_{11} - I_{31}) - I_3^2 (I_9 - 2I_7) \\
& \qquad + (I_1 - 2I_3)(I_3 I_{11} - I_1 I_6) \big\} \Big]
\end{aligned}
$$

$$
\begin{aligned}
&+\operatorname{Re} M(t^2)\Big[(2t^2+k^2-4m^2)\Big[\{I_{26}(I_{30}-I_{10})-(I_{49}-I_{48})\}\xi^2(k^2)\\
&\qquad -\{(I_4-I_3)(I_1 I_{26}-I_{35})-I_1(I_{43}-I_{41})\\
&\qquad\qquad +I_{28}(I_{30}-I_{10})\}\xi(k^2)Q(k^2)\\
&\qquad +I_1(I_4-I_3)(I_{28}-I_{31})Q(k^2)\Big]\\
&\quad +2M(k^2)\xi(k^2)\{(I_4-I_3)(I_2 I_{26}-I_{36})\\
&\qquad +(I_{25}-I_4)(I_{35}-I_{26}I_1)\\
&\qquad +(I_{30}-I_{10})(I_{10}-I_{29})\\
&\qquad +I_1(I_{44}-I_{42})-I_3(I_{43}-I_{41})\}\\
&\quad +2M(k^2)Q(k^2)\{(I_4-I_3)\big[I_1(I_{30}-I_{10}-I_{11}-I_{32})\\
&\qquad\qquad +I_2(I_{31}-I_{28})\big]\\
&\qquad +(I_{25}-I_4)I_1(I_{31}+I_{28})\}\\
&\quad +M^2(k^2)\{I_1\big[(I_{42}-I_{41})-I_{27}(I_4-I_3)\\
&\qquad\qquad -(I_{40}-I_{39})+(I_{46}-I_{45})\big]\\
&\qquad +(I_4-I_3)(I_{38}+I_{34}-I_{37})\\
&\qquad -I_{31}(I_{33}-I_{11})\}\Big]\Big\} \qquad (A\cdot 3)
\end{aligned}
$$

where $I_1 - I_{24}$ can be found in SD except that I_{24} should read

$$I_{24} = k\, d(p')\, d(p)\, d(p'-k)\, d(p-k)\, k$$

and $I_{25} - I_{49}$ are as follows, with $I_i(p, p', k) = I_i(p', p, k)$

$$I_{25} = p'\, d(p-k)\, p'$$

$$I_{26} = \operatorname{Tr} d(p)\, d(p-k)$$

$$I_{27} = \operatorname{Tr} d(p)\, d(p'-k)$$

$$I_{28} = k\, d(p)\, d(p-k)\, k$$

$$I_{29} = k\, d(p)\, d(p-k)\, p'$$

$$I_{30} = p'\, d(p)\, d(p-k)\, p'$$

$$I_{31} = k\, d(p)\, d(p'-k)\, k$$

$$I_{32} = k\, d(p)\, d(p'-k)\, p$$

$$I_{33} = p'\, d(p)\, d(p'-k)\, p$$

$$I_{34} = k\, d(p)\, d(p-k)\, d(p')\, p$$

$$I_{35} = p'\, d(p)\, d(p-k)\, d(p')\, k$$

$$I_{36} = p'\, d(p)\, d(p-k)\, d(p')\, p$$

$$I_{37} = k\, d(p-k)\, d(p)\, d(p')\, p$$

$$I_{38} = k\, d(p'-k)\, d(p)\, d(p')\, p$$

$$I_{39} = k\, d(p-k)\, d(p'-k)\, d(p')\, k$$

$$I_{40} = p'\, d(p-k)\, d(p'-k)\, d(p')\, k$$

$$I_{41} = k\, d(p-k)\, d(p)\, d(p'-k)\, k$$

$$I_{42} = k\, d(p-k)\, d(p)\, d(p'-k)\, p$$

$$I_{43} = p'\, d(p-k)\, d(p)\, d(p'-k)\, k$$

$$I_{44} = p'\, d(p-k)\, d(p)\, d(p'-k)\, p$$

$$I_{45} = k\, d(p-k)\, d(p'-k)\, d(p)\, k$$

$$I_{46} = p'\, d(p-k)\, d(p'-k)\, d(p)\, k$$

$$I_{47} = k\,d(p)\,d(p-k)\,d(p'-k)\,d(p')\,p$$

$$I_{48} = k\,d(p-k)\,d(p)\,d(p'-k)\,d(p')\,p$$

$$I_{49} = p'\,d(p-k)\,d(p)\,d(p'-k)\,d(p')\,p$$

REFERENCES:

1) T. D. Lee and C. N. Yang : Phys. Rev. 128, 885 (1962)
References to earlier work are also cited.

2) A. Salam : Phys. Rev. 130, 1287 (1963)
A. Salam and R. Delbourgo : Phys. Rev. 135, B1398 (1964)
(hereafter referred to as SD).

3) We are using the same notation as SD.

4) The transverse and longitudinal projection operators are defined respectively, as

$$d_{\mu\nu}(p) = -g_{\mu\nu} + \frac{p_\mu p_\nu}{p^2} \quad , \quad e_{\mu\nu} = p_\mu p_\nu / p^2$$

5) N. Muskhelishvili, Singular Integral Equations
(P. Noordhoff Ltd., Groningen Holland, 1953) p.111

6) Equations (3.12) and (3.13) contain the usual infra-red divergence which we shall ignore since it does not affect the high-energy limits of $Z_i^{-1}(p^2)$

7) The condition (3.15) implies the existence of possible bound states and CDD poles. For a discussion of this see J. M. Charap, to be published, and R. Omnes, Nuovo Cimento, 8, 316 (1958)

8) G. Feldman and P. T. Matthews, Phys. Rev. 130, 1633, (1963).
Here λ^2 is the mass of a "time-like" photon.
The constant $a = \frac{\lambda^2}{\mu^2}$ specifies a particular covariant gauge.

9) Note here that a misprint in SD has been corrected. Also Equation 11.55 of SD should read : In the static limit, we obtain

$$Q(0) = \frac{1}{2m^2}(q + \kappa - 1) \qquad (\ q \text{ is the quadrupole moment in units of } e/m^2)$$

10) J. Bernstein and T. D. Lee : Phys. Rev. Letters 11, 512 (1963)